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ON THE GEOMETRY OF PSEUDO-SLANT SUBMANIFOLDS OF A KENMOTSU MANIFOLD

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ABSTRACT. In this paper, the geometry of pseudo-slant submanifold of a Kenmotsu manifold are studied. We research integrability conditions for the distributions which are involved in the definition of a pseudo-slant submanifold. The necessary and sufficient conditions are given for a pseudo-slant submanifold to be pseudo-slant product.

1. Introduction and preliminaries

The differential geometry of slant submanifolds has shown an increasing development since B-Y. Chen defined slant submanifolds in complex manifolds as a natural generallization of both holomorphic and totally real submanifolds [8]. Since then many research articles have been appeared on the existence of these submanifolds in different knows spaces. The slant submanifolds of an almost contact metric manifolds were defined and studied by A. Lotta [5]. After, these submanifolds were studied by J.L Cabrerizo et. al in the setting of Sasakian manifolds [2].

The notion of semi-slant submanifolds of an almost Hermitian manifold was introduced by N. Papagiuc [10]. Hemi-slant submanifolds first were indroduced by A.Carrizo [6] and he called them pseudo-slant submanifolds. Recently, M. Atceken and S. K. Hui studied pseudo-submanifolds in $(LCS)_n$ manifolds[1] and warped product pseudo-slant submanifolds have been studied in [13].

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In this paper, we study pseudo-slant submanifolds of a Kenmotsu manifold. In section 2, we review basic formulas and definitions for a Kenmotsu manifold and their submanifolds, which will be used later. In section 3, we recall the definition and some basic results of a pseudo-slant submanifold of almost contact metric manifold. We deal with the integrability of the distributions on the pseudo-slant submanifolds of Kenmotsu manifold and then we obtain some results for these submanifolds in the setting of Kenmotsu manifolds.The necessary and sufficient

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conditions are given for a pseudo-slant submanifold to be pseudo-slant product.

Let $\widetilde{M}$ be a $(2n+1)$-dimensional almost contact metric manifold together with a metric tensor g, a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η on $\widetilde{M}$ which satisfy

$$\phi^2 X = -X + \eta(X)\xi, \tag{1.1}$$

$$\phi\xi = 0,\ \eta(\phi X) = 0,\ \eta(\xi) = 1,\ \eta(X) = g(X,\xi) \tag{1.2}$$

and

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(\phi X, Y) + g(X, \phi Y) = 0 \tag{1.3}$$

for any vector fields X, Y on $\widetilde{M}$.

If in addition to above relations

$$(\widetilde{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{1.4}$$

then $\widetilde{M}$ is called a Kenmotsu manifold, where $\widetilde{\nabla}$ is the Levi-Civita connections of g. We have also on Kenmotsu manifold $\widetilde{M}$

$$\widetilde{\nabla}_X \xi = X - \eta(X)\xi, \tag{1.5}$$

for any $X, Y \in \Gamma(T\widetilde{M})$.

Now, let M be a submanifold of a contact metric manifold $\widetilde{M}$ with the induced metric g and ξ be tangent to M. Also, let ∇ and $\nabla^{\perp}$ be the induced connections on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M, respectively. Then the Gauss and Weingarteen formulas are, respectively, given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{1.6}$$

and

$$\widetilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{1.7}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, where h and A_V are, respectively, the second fundamental form and the shape operator (corresponding to the normal vector field V) for the immersion of M into $\widetilde{M}$. The second fundamental form h and shape operator A_V are related by

$$g(A_V X, Y) = g(h(X,Y), V), \tag{1.8}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

In (1.6), $Y = \xi$ is written, we obtain

$$\widetilde{\nabla}_X \xi = \nabla_X \xi + h(X,\xi).$$

Using (1.5), the tangential and normal parts of the last equation give, respectively, us

$$\nabla_X \xi = X - \eta(X)\xi \tag{1.9}$$

and

$$h(X,\xi) = 0. \tag{1.10}$$

The mean curvature vector H of M is given by

$$H = \frac{1}{n} trace(h) = \frac{1}{n} \sum_{i=1}^{n} h(e_i, e_i), \tag{1.11}$$

where n is the dimension of M and $\{e_1, e_2, ..., e_n\}$ is a local orthonormal frame of M.

A submanifold M of an contact metric manifold $\widetilde{M}$ is said to be totally umbilical if

$$h(X, Y) = g(X, Y)H, \tag{1.12}$$

where H is the mean curvature vector. A submanifold M is said to be totally geodesic if $h(X, Y) = 0$, for each $X, Y \in \Gamma(TM)$ and M is said to be minimal if $H = 0$.

Let M be a submanifold of an almost contact metric manifold $\widetilde{M}$. Then for any $X \in \Gamma(TM)$, we can write

$$\phi X = TX + NX, \tag{1.13}$$

where TX is the tangential component and NX is the normal component of ϕX.

Similarly, for $V \in \Gamma(T^{\perp}M)$, we can write

$$\phi V = tV + nV, \tag{1.14}$$

where tV is the tangential component and nV is the normal component of ϕV.

Thus by using (1.1), (1.13) and (1.14), we obtain

$$T^2 = -I + \eta \otimes \xi - tN, \quad NT + nN = 0 \tag{1.15}$$

and

$$Tt + tn = 0, \quad Nt + n^2 = -I, \tag{1.16}$$

where the covariant derivatives of the tensor field T, N, t and n are, respectively, defined by

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{1.17}$$

$$(\nabla_X N)Y = \nabla^{\perp}_X NY - N\nabla_X Y, \tag{1.18}$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla^{\perp}_X V \tag{1.19}$$

and

$$(\nabla_X n)V = \nabla^{\perp}_X nV - n\nabla^{\perp}_X V, \tag{1.20}$$

for any $X, Y \in \Gamma(TM)$.

By direct calculations, we obtain the following equalities

$$(\nabla_X T)Y = A_{NY}X + th(X, Y) + g(TX, Y)\xi - \eta(Y)TX \tag{1.21}$$

and

$$(\nabla_X N)Y = nh(X, Y) - h(X, TY) - \eta(Y)NX. \tag{1.22}$$

Similarly, for any $V \in \Gamma(T^{\perp}M)$, we obtain

$$(\nabla_X t)V = A_{nV}X - TA_V X + g(NX, V)\xi \tag{1.23}$$

and

$$(\nabla_X n)V = -h(tV, X) - NA_V X. \tag{1.24}$$

Putting $Y = \xi$ in (1.4) and using (1.2), we have

$$(\widetilde{\nabla}_X \phi)\xi = g(\phi X, \xi)\xi - \eta(\xi)\phi X$$

$$(\widetilde{\nabla}_X \phi)\xi = -\phi X, \tag{1.25}$$

that is,

$$-\phi\widetilde{\nabla}_X \xi = -TX - NX$$

and

$$T\nabla_X \xi + N\nabla_X \xi + \phi h(X, \xi) = TX + NX. \tag{1.26}$$

Using (1.10) and (1.26), the tangential and normal parts of the equation (1.26) give, respectively, us

$$T\nabla_X \xi = TX \tag{1.27}$$

and

$$N\nabla_X \xi = NX. \tag{1.28}$$

Definition 1.1. Let M be a submanifold of a Kenmotsu manifold $\widetilde{M}$. For each non-zero vector X tangent to M at x, the angle $\theta(x)$, $\theta(x) \in \left[0, \frac{\pi}{2}\right]$, between ϕX and $T_x M$ is called the slant angle or the Wirtinger angle of M. If the slant angle is constant, for vector field $X \in \Gamma(TM)$ and for each point $x \in M$, then the submanifold is also called the slant submanifold. If $\theta = 0$ the submanifold is *invariant submanifold*. If $\theta = \frac{\pi}{2}$ then it is called anti-invariant submanifold. If $\theta(x) \in (0, \frac{\pi}{2})$, then it is called proper-slant submanifold[1].

If M is a slant submanifold of an almost contact metric manifold, then the tangent bundle TM of M can be decomposed as

$$TM = D_\theta \oplus \langle \xi \rangle, \tag{1.29}$$

where $\langle \xi \rangle$ denotes the distribution spanned by the structure vector field ξ and D_θ is the complementary of distribution of $\langle \xi \rangle$ in TM, known as the slant distribution on M. For a proper-slant submanifold M of an almost contact manifold $\widetilde{M}$ with a slant angle θ, in [5], Lotta proved that

$$T^2 X = -\cos^2\theta(X - \eta(X)\xi), \tag{1.30}$$

for any $X \in \Gamma(TM)$.

Recently, Cabrerizo et al. extended the above result in to a characterization for a slant submanifold in a contact metric manifold[9]. In fact, they given the following crucial theorem.

Theorem 1.2. *Let M be a slant submanifold of an almost contact metric manifold $\widetilde{M}$ such that $\xi \in \Gamma(TM)$. Then M is slant submanifold if and only if there exists a constant $\lambda \in [0,1]$ such that*

$$T^2 = -\lambda(I - \eta \otimes \xi), \tag{1.31}$$

Furthermore, if θ is the slant angle of M, then $\lambda = \cos^2\theta$[9].

Corollary 1.3. *Let M be a slant submanifold of an almost contact metric manifold $\widetilde{M}$ with slant angle θ. Then for any $X, Y \in \Gamma(TM)$, we have*

$$g(TX, TY) = \cos^2\theta \{g(X,Y) - \eta(X)\eta(Y)\} \tag{1.32}$$

and

$$g(NX, NY) = \sin^2\theta \{g(X,Y) - \eta(X)\eta(Y)\}. \tag{1.33}$$

[9]

2. Pseudo-slant submanifolds of a Kenmotsu manifold

In this section we will obtain the integrability conditions of the distributions of pseudo-slant submanifold a Kenmotsu manifold. Also, the necessary and sufficient conditions are given for a pseudo-slant submanifold to be pseudo-slant product.

Definition 2.1. We say that M is a pseudo-slant submanifold of an almost contact metric manifold $\widetilde{M}$ if there exist two orthogonal distributions D_θ and $D^\perp$ on M such that

1) TM admits the orthogonal direct decomposition $TM = D^\perp \oplus D_\theta$, $\xi \in \Gamma(D_\theta)$,
2) The distribution $D^\perp$ is anti-invariant(totally-real) i.e., $\phi D^\perp \subset (T^\perp M)$,
3) The distribution D_θ is a slant with slant angle $\theta \neq \frac{\pi}{2}$, that is, θ the angle between D_θ and $\phi(D_\theta)$ is a constant[1, 4].

From the definition, it is clear that if $\theta = 0$, then the pseudo-slant submanifold is a semi-invariant submanifold, for $\theta = \frac{\pi}{2}$, the submanifold becomes an anti-invariant.

In the rest of this paper, we suppose that M is a pseudo-slant submanifold of an almost contact metric manifold $\widetilde{M}$.

On the other hand, if we denote the dimensions of distributions $D^\perp$ and D_θ by d_1 and d_2, respectively, then we have the following cases:
1) If $d_2 = 0$, then M is an anti-invariant submanifold,
2) If $d_1 = 0$ and $\theta = 0$, then M is an invariant submanifold,
3) If $d_1 = 0$ and $\theta \neq 0$, then M is a proper slant submanifold with slant angle θ,
4) If $d_1.d_2 \neq 0$ and $\theta \in (0, \frac{\pi}{2})$ then M is a proper pseudo -slant submanifold.

If we denote the projections on $D^\perp$ and D_θ by P_1 and P_2, respectively, then for any vector field , $X \in \Gamma(TM)$, we can write.

$$X = P_1X + P_2X + \eta(X)\xi. \tag{2.1}$$

Now applying ϕ on both sides of equation (2.1), we have

$$\phi X = \phi P_1 X + \phi P_2 X,$$

or

$$TX + NX = NP_1X + TP_2X + NP_2X. \tag{2.2}$$

We can easily see

$$TX = TP_2X, \quad NX = NP_1X + NP_2X \tag{2.3}$$

and

$$\phi P_1 X = NP_1X, \ \ TP_1X = 0, \qquad \phi P_2X = TP_2X + NP_2X \tag{2.4}$$
$$TP_2X \in \Gamma(D_\theta).$$

If we denote the orthogonal complementary of ϕTM in $T^\perp M$ by μ, then the normal bundle $T^\perp M$ can be decomposed as follows

$$T^\perp M = N(D^\perp) \oplus N(D_\theta) \oplus \mu, \tag{2.5}$$

where μ is an invariant subbundle of $T^\perp M$ as $N(D^\perp)$ and $N(D_\theta)$ are orthogonal distribution on $\widetilde{M}$. $g(Z, X) = 0$ for each $Z \in \Gamma(D^\perp)$ and $X \in \Gamma(D_\theta)$. Thus, by equation (1.3) and (1.13), we write

$$g(NZ, NX) = g(\phi Z, \phi X) = g(Z, X) = 0, \tag{2.6}$$

that is, the distributions $N(D^\perp)$ and $N(D_\theta)$ are mutually perpendicular. In fact, the decomposition (2.5) is an orthogonal direct decomposition.

Lemma 2.2. *Let M be a submanifold of an almost contact metric manifold $\widetilde{M}$. Then D_θ is slant distribution on M if only and if there is a constant $\lambda \in [0, 1]$ such that*

$$(TP_2)^2 X = -\lambda X, \tag{2.7}$$

for all $X \in \Gamma(D_\theta)$, in such case, if θ is the slant angle of M, then $\lambda = \cos^2\theta$[9].

Theorem 2.3. *Let M be a pseudo-slant of a Kenmotsu manifold $\widetilde{M}$. Then the anti-invariant distribution $D^\perp$ is integrable if and only if*

$$A_{NX}Y = A_{NY}X \tag{2.8}$$

for any $X, Y \in \Gamma\ (D^\perp)$.

Proof. By using (1.4), (1.6), (1.7), (1.12) and (1.13), we have

$$(\widetilde{\nabla}_X \phi)Y = \widetilde{\nabla}_X \phi Y - \phi \widetilde{\nabla}_X Y,$$

that is,

$$\begin{aligned} g(\phi X, Y)\xi - \eta(Y)\phi X &= \widetilde{\nabla}_X NY - \phi(\nabla_X Y + h(X, Y)) \\ 0 &= -A_{NY}X + \nabla^\perp_X NY - T\nabla_X Y - N\nabla_X Y \\ &- th(X, Y) - nh(X, Y), \end{aligned}$$

for any $X, Y \in \Gamma\ (D^\perp)$. From the tangent components of this last equation, we obtain

$$A_{NY}X + T\nabla_X Y + th(X, Y) = 0. \tag{2.9}$$

By interchange roles of X and Y in (2.9), we have

$$A_{NX}Y + T\nabla_Y X + th(X,Y) = 0 \tag{2.10}$$

which is equivalent to

$$T[X,Y] = A_{NY}X - A_{NX}Y.$$

This proves our assertion. □

Lemma 2.4. *Let M be a pseudo-slant of a Kenmotsu manifold $\widetilde{M}$. Then the anti-invariant distribution $D^{\perp}$ is integrable if and only if*

$$(\nabla_Z T)W = (\nabla_W T)Z, \tag{2.11}$$

for any $Z, W \in \Gamma\ (D^{\perp})$.

Proof. Since the ambient manifold $\widetilde{M}$ is a Kenmotsu, for any $Z, W \in \Gamma\ (D^{\perp})$, we have

$$(\widetilde{\nabla}_Z \phi)W = g(\phi Z, Y)\xi - \eta(Y)\phi Z$$

which is equivalent to

$$\widetilde{\nabla}_Z \phi W - \phi \widetilde{\nabla}_Z W = g(NZ, Y)\xi - \eta(Y)NZ,$$

or

$$\widetilde{\nabla}_Z NW - \phi(\nabla_Z W + h(W,Z)) = 0.$$

So we have

$$-A_{NW}Z + \nabla^{\perp}_Z NW - T\nabla_Z W - N\nabla_Z W - th(W,Z) - nh(X,Y) = 0.$$

From the tangent components of the last equation, we obtain

$$A_{NW}Z + T\nabla_Z W + th(W,Z) = 0.$$

This yields to

$$T[W,Z] = A_{NW}Z + T\nabla_W Z + th(W,Z).$$

For $[Z,W] \in \Gamma(D^{\perp})$, $\phi[Z,W] = N[Z,W]$ because of the tangent component of $\phi[Z,W]$ is zero. So we have

$$A_{NW}Z + T\nabla_W Z + th(W,Z) = 0. \tag{2.12}$$

Similarly, we obtain

$$A_{NZ}W + T\nabla_Z W + th(W,Z) = 0. \tag{2.13}$$

Here, by using Theorem 2.3, (2.12) and (2.13), we obtain

$$(\nabla_Z T)W = (\nabla_W T)Z.$$

Thus the anti-invariant distribution $D^{\perp}$ is integrability if and only if (2.11) is satisfied. □

Lemma 2.5. *Let M be pseudo slant submanifold of Kenmotsu manifold $\widetilde{M}$, then*

$$\eta([X,Y]) = 0,$$

for any $X, Y \in \Gamma(D^{\perp} \oplus D_\theta)$.

Proof. Since the ambient space is a Kenmotsu manifold, for any $X, Y \in \Gamma(D^{\perp} \oplus D_{\theta})$, we have

$$g([X,Y],\xi) = g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, \xi).$$

By using (1.9), (1.6) and (1.7), we obtain

$$\begin{aligned} g([X,Y],\xi) &= g(\nabla_X Y - \nabla_Y X, \xi) \\ &= g(\nabla_X Y, \xi) - g(\nabla_Y X, \xi) \\ &= -g(\nabla_X \xi, Y) + g(\nabla_Y \xi, X) \\ &= -g(-X + \eta(X)\xi, Y) + g(-Y + \eta(Y)\xi, X) \\ &= g(X,Y) - \eta(X)\eta(Y) - g(Y,X) + \eta(X)\eta(Y), \end{aligned}$$

that is,

$$g([X,Y],\xi) = \eta([X,Y]) = 0.$$

Thus the proof is complete. □

Theorem 2.6. *Let M be a pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the slant distribution D_{θ} is integrable if and only if*

$$TA_{NU}X + A_{NU}TX = 0,$$

for any $U \in \Gamma(D^{\perp})$ and $X \in \Gamma(D_{\theta})$.

Proof. For any $U \in \Gamma\ (D^{\perp})$ and $X, Y \in \Gamma(D_{\theta})$, by direct calculation, we have

$$\begin{aligned} g([X,Y],U) &= g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, U) = g(\widetilde{\nabla}_Y U, X) - g(\widetilde{\nabla}_X U, Y) \\ &= g(\phi\widetilde{\nabla}_Y U, \phi X) - g(\phi\widetilde{\nabla}_X U, \phi Y) \\ &= -g((\widetilde{\nabla}_Y \phi)U, \phi X) + g(\widetilde{\nabla}_Y \phi U, \phi X) + g((\widetilde{\nabla}_X \phi)U, \phi Y) \\ &- g(\widetilde{\nabla}_X \phi U, \phi Y) \\ &= -g(g(\phi Y, U)\xi - \eta(U)\phi Y, \phi X) + g(\widetilde{\nabla}_Y NU, \phi X) \\ &+ g(g(\phi X, U)\xi - \eta(U)\phi X, \phi Y) - g(\widetilde{\nabla}_X NU, \phi Y) \\ &= g(\widetilde{\nabla}_Y NU, \phi X) - g(\widetilde{\nabla}_X NU, \phi Y) \\ &= g(\widetilde{\nabla}_Y NU, TX) + g(\widetilde{\nabla}_Y NU, NX) \\ &- g(\widetilde{\nabla}_X NU, TY) - g(\widetilde{\nabla}_X NU, NY). \end{aligned}$$

On the other hand, from (1.4), (1.6) and (1.7), we reach

$$\begin{aligned} (\widetilde{\nabla}_X \phi)U &= \widetilde{\nabla}_X \phi U - \phi\widetilde{\nabla}_X U \\ g(\phi X, U)\xi - \eta(U)\phi X &= \widetilde{\nabla}_X NU - T\nabla_X U - N\nabla_X U - th(X,U) - nh(X,U) \\ -A_{NU}X + \nabla^{\perp}_X NU &= T\nabla_X U + N\nabla_X U + th(X,U) + nh(X,U). \end{aligned}$$

From the tangential and normal components of this equality, we obtain

$$-A_{NU}X = T\nabla_X U + th(X,U) \tag{2.14}$$

and

$$(\nabla_X N)U = nh(X,U). \tag{2.15}$$

Also, by using (1.18) and (2.15) we reach

$$\begin{aligned} g([X,Y],U) &= g(A_{NU}X,TY) - g(A_{NU}Y,TX) + g(\nabla^{\perp}_{Y}NU,NX) \\ &- g(\nabla^{\perp}_{X}NU,NY) \\ &= -g(TA_{NU}X,Y) - g(A_{NU}TX,Y) + g((\nabla_{Y}N)U + N\nabla_{Y}U,NX) \\ &- g((\nabla_{X}N)U + N\nabla_{X}U,NY) \\ &= -g(TA_{NU}X,Y) - g(A_{NU}TX,Y) + g(nh(Y,U),NX) \\ &+ g(N\nabla_{Y}U,NX) - g(nh(X,U),NY) + g(N\nabla_{X}U,NY) \\ &= -g(TA_{NU}X,Y) - g(A_{NU}TX,Y) + g(N\nabla_{Y}U,NX) \\ &- g(N\nabla_{X}U,NY) \\ &= -g(TA_{NU}X + A_{NU}TX,Y) + \sin^2\theta\,\{g(\nabla_{Y}U,X) - g(\nabla_{X}U,Y)\} \\ &= -g(TA_{NU}X + A_{NU}TX,Y) + \sin^2\theta\,\{g(\nabla_{X}Y,U) - g(\nabla_{Y}X,U)\} \\ &= -g(TA_{NU}X + A_{NU}TX,Y) + \sin^2\theta\,\{[X,Y],U\}. \end{aligned}$$

So we conclude

$$\cos^2\theta\,\{[X,Y],U\} = -g(TA_{NU}X + A_{NU}TX,Y),$$

which verifies our assertion. ☐

Theorem 2.7. *Let M be a pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the slant distribution D_θ is integrable if and only if*

$$\nabla^{\perp}_{Z}NW - \nabla^{\perp}_{W}NZ + h(Z,TW) - h(W,TZ) \in \mu \oplus N(D_\theta)$$

for any $Z, W \in \Gamma(D_\theta)$.

Proof. For any $Z, W \in \Gamma(D_\theta)$ and $X \in \Gamma(D^{\perp})$, we have

$$\begin{aligned} g([Z,W],X) &= g(\widetilde{\nabla}_{Z}W,X) - g(\widetilde{\nabla}_{W}Z,X) \\ &= g(\phi\widetilde{\nabla}_{Z}W,\phi X) + \eta(\widetilde{\nabla}_{Z}W)\eta(X) \\ &- g(\phi\widetilde{\nabla}_{W}Z,\phi X) - \eta(\widetilde{\nabla}_{W}Z)\eta(X). \end{aligned}$$

We have $\eta(X) = 0$ for $X \in \Gamma(D^{\perp})$. Then we obtain

$$g([Z,W],X) = g(\widetilde{\nabla}_{Z}\phi W,\phi X) - g((\widetilde{\nabla}_{Z}\phi)W,\phi X) - g(\widetilde{\nabla}_{W}\phi Z,\phi X) + (\widetilde{\nabla}_{W}\phi)Z,\phi X).$$

Making use of (1.13), we have

$$\begin{aligned} g([Z,W],X) &= g(\widetilde{\nabla}_{Z}TW + \widetilde{\nabla}_{Z}NW,\phi X) - g(\widetilde{\nabla}_{W}TZ + \widetilde{\nabla}_{W}NZ,\phi X) \\ &- g(g(\phi Z,W)\xi - \eta(W)\phi Z,\phi X) + g(g(\phi W,Z)\xi - \eta(Z)\phi W,\phi X). \end{aligned}$$

From the Gauss and Weingarten formulas, the above equation takes the form

$$\begin{aligned} g([Z,W],X) &= g(h(Z,TW),\phi X) + g(\nabla^{\perp}_{Z}NW,\phi X) - g(h(W,TZ),\phi X) \\ &- g(\nabla^{\perp}_{W}NZ,\phi X) + \eta(W)g(Z,X) \\ &- \eta(W)\eta(Z)\eta(X) - \eta(Z)g(W,X) + \eta(Z)\eta(W)\eta(X), \end{aligned}$$

that is,

$$\begin{aligned} g([Z,W],X) &= g(h(Z,TW),\phi X) + g(\nabla^{\perp}_{Z} NW,\phi X) - g(h(W,TZ),\phi X) \\ &- g(\nabla^{\perp}_{W} NZ,\phi X). \end{aligned}$$

Since $\phi X \in \phi(D^{\perp}) \subseteq T^{\perp}M$, $N(D_\theta)$ and $N(D^{\perp})$ are orthogonal subbundle in $T^{\perp}M$, we conclude $[Z,W] \in D_\theta$ if and only if

$$\nabla^{\perp}_{Z} NW - \nabla^{\perp}_{W} NZ + h(Z,TW) - h(W,TZ) \in \mu \oplus N(D_\theta).$$

□

Theorem 2.8. *Let M be a pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the slant distribution D_θ is integrable if and only if*

$$\eta(Y)P_1TX = P_1\{T\nabla_Y X - \nabla_X TY + A_{NY}X + th(X,Y)\} \tag{2.16}$$

for any $X, Y \in \Gamma(D_\theta)$.

Proof. By using (1.21) and considering the tangential component, we obtain

$$\begin{aligned} T[X,Y] &= \nabla_X TY - T\nabla_Y X - A_{NY}X - th(X,Y) - g(TX,Y)\xi \\ &+ \eta(Y)TX, \end{aligned} \tag{2.17}$$

for any $X, Y \in \Gamma(D_\theta)$. Applying P_1 to (2.17), we get (2.16) □

Theorem 2.9. *Let M be a pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then the distribution $D^{\perp} \oplus \langle\xi\rangle$ is always integrable.*

Proof. By a direct calculation, we have

$$\begin{aligned} g([X,\xi],TZ) &= g(\nabla_X \xi - \nabla_\xi X, TZ) \\ &= g(\widetilde{\nabla}_X \xi - \widetilde{\nabla}_\xi X, TZ) \\ &= g(\widetilde{\nabla}_X \xi, TZ) - g(\widetilde{\nabla}_\xi X, TZ), \end{aligned} \tag{2.18}$$

for all $X \in \Gamma(D^{\perp})$ and $Z \in \Gamma(D_\theta)$, where $D^{\perp}$ and D_θ are two orthogonal distributions and $D^{\perp}$ is an anti-invariant, in view of (1.9) and (2.18), we obtain

$$g([X,\xi],TZ) = g(\nabla_X \xi, TZ) - g(\widetilde{\nabla}_\xi X, TZ).$$

By using (1.10), we have

$$\begin{aligned} g([X,\xi],TZ) &= g(X - \eta(X)\xi, TZ) - g(\widetilde{\nabla}_\xi X, TZ) \\ &= g(\widetilde{\nabla}_\xi TZ, X) = g(\nabla_\xi TZ, X) \\ &= g((\nabla_\xi T)Z + T\nabla_\xi Z, X) = g(T\nabla_\xi Z, X) = 0. \end{aligned}$$

Hence $[X,\xi] \in \Gamma(D^{\perp})$ for $X \in \Gamma(D^{\perp})$. Therefore, the distribution $D^{\perp} \oplus \langle\xi\rangle$ is always integrable. □

Theorem 2.10. *Let M be a pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then $\nabla T^2 = 0$ if and only if M is anti- invariant submanifold.*

Proof. Since M is a slant submanifold, we can write

$$T^2Y = -\cos^2\theta(Y - \eta(Y))\xi, \tag{2.19}$$

for any $Y \in \Gamma(TM)$, where θ denotes the slant angle of M. Replacing Y by $\nabla_X Y$ in (2.19), we get

$$T^2\nabla_X Y = -\cos^2\theta(\nabla_X Y) + \cos^2\theta\eta(\nabla_X Y)\xi. \tag{2.20}$$

Taking the covariant derivative of (2.19) with respect to $X \in (D_\theta)$, we obtain

$$\nabla_X T^2 Y = \cos^2\theta\{-(\nabla_X Y) + \eta(\nabla_X Y)\xi + g(\nabla_X\xi, Y)\xi + \eta(Y)\nabla_X\xi\} \tag{2.21}$$

for $X\eta(Y) = Xg(Y,\xi) = g(\nabla_X Y, \xi) + g(\nabla_X\xi, Y)$. Since M is a submanifold of a Kenmotsu manifold $\widetilde{M}$ and ξ is tangent to M, we have $\nabla_X\xi = X - \eta(X)\xi$ for any $X \in \Gamma(TM)$. Putting the value of $\nabla_X\xi$ in (2.21), we conclude

$$\begin{aligned} \nabla_X T^2 Y &= \cos^2\theta\{-\nabla_X Y + \eta(\nabla_X Y)\xi + g(X,Y)\xi \\ &- 2\eta(X)\eta(Y)\xi + \eta(Y)X\}. \end{aligned} \tag{2.22}$$

Combining (2.20) and (2.22), we find

$$(\nabla_X T^2)Y = \cos^2\theta(g(X,Y)\xi - 2\eta(X)\eta(Y)\xi + \eta(Y)X), \tag{2.23}$$

for any $X, Y \in \Gamma(D_\theta) \oplus \langle\xi\rangle$. Here, we note that

$$g(X,Y)\xi - 2\eta(X)\eta(Y)\xi + \eta(Y)X \neq 0.$$

Otherwise, it implies that $g(\phi X, \phi Y) = 0$. This is impossible. Hence, from (2.23), we reach $\nabla T^2 = 0$ if and only if $\theta = \frac{\pi}{2}$ holds in $\Gamma(D_\theta) \oplus \langle\xi\rangle$. □

For a pseudo-slant submanifold M of $\widetilde{M}$, the invariant and anti- invariant distributions are totally geodesic in M, then M is called pseudo-slant product. The following theorem characterizes the pseudo-slant products.

Theorem 2.11. *Let M be a pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. Then M is a proper pseudo-slant product if and only if the second fundamental form h satisfies*

$$th(X,Z) = 0, \tag{2.24}$$

for all $X \in \Gamma(D_\theta)$ and $Z \in \Gamma(TM)$.

Proof. For all $X, Y \in \Gamma(D_\theta)$ and $U, V \in \Gamma(D^\perp)$, we have

$$\begin{aligned} g(\nabla_X Y, U) &= -g(\nabla_X U, Y) = -g(\widetilde{\nabla}_X U, Y) \\ &= -g(\phi\widetilde{\nabla}_X U, \phi Y) + \eta(\widetilde{\nabla}_X U)\eta(Y) \\ &= g((\widetilde{\nabla}_X\phi)U - \widetilde{\nabla}_X\phi U, \phi Y) + g(\nabla_X U, \xi)\eta(Y) \\ &= g(g(\phi X, U)\xi - \eta(U)\phi X, \phi Y) - g(\widetilde{\nabla}_X\phi U, \phi Y) + g(\nabla_X U, \xi)\eta(Y) \\ &= -g(\widetilde{\nabla}_X\phi U, \phi Y) - g(\nabla_X\xi, U)\eta(Y) \\ &= -g(\widetilde{\nabla}_X\phi U, TY) - g(\widetilde{\nabla}_X\phi U, NY) - g(\nabla_X\xi, U)\eta(Y). \end{aligned}$$

Here, $\phi U = NU$ and using (1.9), we obtain

$$\begin{aligned} g(\nabla_X Y, U) &= -g(\widetilde{\nabla}_X NU, TY) - g(\widetilde{\nabla}_X NU, NY) + g(X - \eta(X)\xi, U)\eta(Y) \\ &= -g(\widetilde{\nabla}_X NU, TY) - g(\widetilde{\nabla}_X NU, NY) \\ &+ g(X, U)\eta(Y) - \eta(X)\eta(U)\eta(Y) \\ &= -g(\widetilde{\nabla}_X NU, TY) - g(\widetilde{\nabla}_X NU, NY). \end{aligned}$$

By using (1.6) and (1.7), we have

$$\begin{aligned} g(\nabla_X Y, U) &= g(A_{NU}X, TY) - g(\nabla^{\perp}_X NU, NY) \\ &= g(A_{NU}X, TY) - g((\nabla_X N)U, NY) - g(N\nabla_X U, NY) \\ &= g(A_{NU}X, TY) - g(N\nabla_X U, NY) \\ &- g(nh(X, U) - \eta(U)NX, NY). \end{aligned}$$

Making use of (1.33), we reach

$$\begin{aligned} g(\nabla_X Y, U) &= g(A_{NU}X, TY) - g(N\nabla_X U, NY) \\ &= g(A_{NU}X, TY) - \sin^2\theta\, \{g(\nabla_X U, Y) - \eta(\nabla_X U)\eta(Y)\} \\ &= g(h(X, TY), NU) - \sin^2\theta g(\nabla_X U, Y) + \sin^2\theta g(\nabla_X U, \xi)\eta(Y) \\ &= g(h(X, TY), NU) + \sin^2\theta g(\nabla_X Y, U) - \sin^2\theta g(\nabla_X \xi, U)\eta(Y) \end{aligned}$$

and

$$\begin{aligned} \cos^2\theta g(\nabla_X Y, U) &= g(h(X, TY), NU) - \sin^2\theta g(X - \eta(X)\xi, U)\eta(Y) \\ &= g(h(X, TY), NU) - \sin^2\theta g(X, U)\eta(Y) \\ &+ \sin^2\theta \eta(X)\eta(U)\eta(Y), \end{aligned}$$

that is,

$$\cos^2\theta g(\nabla_X Y, U) = g(h(X, TY), NU) = -g(th(X, TY), U). \tag{2.25}$$

In the same way, we obtain

$$\begin{aligned} g(\nabla_V U, X) &= g(\widetilde{\nabla}_V U, X) = -g(\widetilde{\nabla}_V X, U) \\ &= -g(\phi\widetilde{\nabla}_V X, \phi U) - \eta(\widetilde{\nabla}_V X)\eta(U) \\ &= g((\widetilde{\nabla}_V \phi)X, \phi U) - g(\widetilde{\nabla}_V \phi X, \phi U). \end{aligned}$$

Since the tangent component of ϕU and ϕU are zero, for any $U, V \in \Gamma(D^{\perp})$, we have

$$\begin{aligned} g(\nabla_V U, X) &= -g(\widetilde{\nabla}_V \phi X, NU) + g((\widetilde{\nabla}_V \phi)X, NU) \\ &= -g(\widetilde{\nabla}_V \phi X, NU) + g(g(\phi V, X)\xi - \eta(X)\phi V, NU) \\ &= -g(\widetilde{\nabla}_V \phi X, NU) - \eta(X)g(NV, NU) \\ &= -g(\widetilde{\nabla}_V \phi X, NU) - \eta(X)\sin^2\theta g(V, U) \\ &= -g(\widetilde{\nabla}_V TX, NU) - g(\widetilde{\nabla}_V NX, NU) - \eta(X)\sin^2\theta g(V, U) \\ &= -g(h(TX, V), NU) - g(\nabla^{\perp}_V NX, NU) \\ &- \eta(X)\sin^2\theta g(V, U). \end{aligned}$$

By using (1.18) and (1.22), we have

$$\begin{aligned} g(\nabla_V U, X) &= -g(h(TX,V),NU) - g(\nabla^{\perp}_V NX, NU) - \eta(X)\sin^2\theta g(V,U) \\ &= -g(h(TX,V),NU) - g((\nabla_V N)X + N\nabla_V X, NU) \\ &- \eta(X)\sin^2\theta g(V,U) \\ &= -g(h(TX,V),NU) - g(N\nabla_V X, NU) - \eta(X)\sin^2\theta g(V,U) \\ &- g(-h(V,TX) + nh(V,X) - \eta(X)NV, NU) \\ &= -g(h(V,TX),NU) - g(N\nabla_V X, NU) - \eta(X)\sin^2\theta g(V,U) \\ &+ g(h(V,TX),NU) + \eta(X)g(NV,NU) \\ &= -g(N\nabla_V X, NU) - \eta(X)\sin^2\theta g(V,U) \\ &- g(h(V,X),NU) + \eta(X)\sin^2\theta g(V,U) \\ &- \sin^2\theta\eta(X)\eta(V)\eta(U) \\ &= -g(h(V,X),NU) - g(N\nabla_V X, NU) \\ &= -g(h(V,X),NU)) + sin^2\theta g(\nabla_V U, X), \end{aligned}$$

from which

$$\cos^2\theta g(\nabla_V U, X) = -g(h(V,X),NU) = g(th(X,V),U). \tag{2.26}$$

Thus, from (2.25) and (2.26), we can infer M is a pseudo-slant product if and only if (2.24) is satisfied. □

Theorem 2.12. *Let M be a proper pseudo-slant submanifold of a Kenmotsu manifold $\widetilde{M}$. If N is parallel on D_θ, then either M is a $D_\theta-$ geodesic submanifold or $h(X,Y)$ is an eigenvector of n^2 with eigenvalue $-\cos^2\theta$.*

Proof. Since $(\nabla_X N)Y = 0$ for any $X, Y \in \Gamma(D_\theta)$, from (1.22) we have

$$nh(X,Y) - h(X,TY) - \eta(Y)NX = 0. \tag{2.27}$$

Taking into account of D_θ being slant distribution, $T\xi = 0$ and $h(X,\xi) = 0$ we obtain

$$nh(X, Y - \eta(Y)\xi) - h(X, T(Y - \eta(Y)\xi)) - \eta(Y - \eta(Y)\xi)NX = 0,$$

that is,

$$nh(X, Y - \eta(Y)\xi) - h(X,TY) - \eta(Y)NX + \eta(Y)NX = 0,$$

or

$$nh(X, Y - \eta(Y)\xi) = h(X,TY). \tag{2.28}$$

Now, applying n to equation (2.28), we have

$$n^2h(X, Y - \eta(Y)\xi) = nh(X,TY).$$

On the other hand, by interchang rales of Y and TY in (2.27), we have

$$nh(X,TY) = h(X,T^2Y).$$

Hence,

$$n^2h(X, Y - \eta(Y)\xi) = nh(X,TY) = h(X,T^2Y) = -\cos^2\theta h(X, Y - \eta(Y)\xi).$$

This implies that h either vanishes on D_θ or h is an eigenvector of n^2 with eigenvalue $-\cos^2\theta$. □

Example 2.13. Let M be a submanifold of $\mathbb{R}^9$ defined by

$$(u, -\sqrt{2}v, v\sin\theta, v\cos\theta, s\cos t, -\cos t, s\sin t, -\sin t, z).$$

We can easily to see that the tangent bundle of M is spanned by the tangent vectors

$$\begin{aligned}
e_1 &= \frac{\partial}{\partial x_1},\\
e_2 &= -\sqrt{2}\frac{\partial}{\partial y_1} + \sin\theta\frac{\partial}{\partial x_2} + \cos\theta\frac{\partial}{\partial y_2},\\
e_3 &= \cos t\frac{\partial}{\partial x_3} + \sin t\frac{\partial}{\partial x_4},\\
e_4 &= -s\sin t\frac{\partial}{\partial x_3} + \sin t\frac{\partial}{\partial y_3} + s\cos t\frac{\partial}{\partial x_4} - \cos t\frac{\partial}{\partial y_4},\\
e_5 &= \xi = \frac{\partial}{\partial z}.
\end{aligned}$$

For the almost contact structure ϕ of $\mathbb{R}^9$, choosing

$$\phi(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial y_i}, \quad \phi(\frac{\partial}{\partial y_j}) = -\frac{\partial}{\partial x_j}, \quad \phi(\frac{\partial}{\partial z}) = 0 \quad, 1 \leq i, j \leq 4.$$

For any vector field $X = \lambda_i \frac{\partial}{\partial x_i} + \mu_j \frac{\partial}{\partial y_i} + \nu \frac{\partial}{\partial z} \in \Gamma(T\mathbb{R}^9)$, then we have

$$g(X, X) = \lambda_i^2 + \mu_j^2 + \nu^2, \quad g(\phi X, \phi X) = \lambda_i^2 + \mu_j^2$$

and

$$\phi^2 X = -\lambda_i \frac{\partial}{\partial x_i} - \mu_j \frac{\partial}{\partial y_i} = -X + \eta(X)\xi,$$

for any $i, j = 1, 2, 3, 4$. It follows that $g(\phi X, \phi X) = g(X, X) - \eta^2(X)$. Thus (ϕ, η, ξ, g) is an almost contact metric structure on $\mathbb{R}^9$. Thus we have

$$\begin{aligned}
\phi e_1 &= \frac{\partial}{\partial y_1}\\
\phi e_2 &= \sqrt{2}\frac{\partial}{\partial x_1} + \sin\theta\frac{\partial}{\partial y_2} - \cos\theta\frac{\partial}{\partial x_2}\\
\phi e_3 &= \cos t\frac{\partial}{\partial y_3} + \sin t\frac{\partial}{\partial y_4}\\
\phi e_4 &= -s\sin t\frac{\partial}{\partial y_3} - \sin t\frac{\partial}{\partial x_3} + s\cos t\frac{\partial}{\partial y_4} + \cos t\frac{\partial}{\partial x_4}.
\end{aligned}$$

By direct calculations, we infer $D_\theta = span\{e_1, e_2\}$ is a slant distribution with slant angle $\alpha = \cos^{-1}(\frac{\sqrt{6}}{3})$. Since ϕe_3 and ϕe_4 are orthogonal to M, $D^\perp = span\{e_3, e_4\}$ is an anti-invariant distribution. Thus M is a 5-dimensional proper pseudo-slant submanifold of $\mathbb{R}^9$ with its usual almost contact metric structure.

Let ∇ be the Levi-Civita connection on $\mathbb{R}^9$. Then we have

$$[e_1, e_1] = [e_2, e_2] = [e_3, e_3] = [e_4, e_4] = [e_5, e_5] = 0,$$

$$[e_1, e_2] = [e_1, e_3] = [e_1, e_4] = [e_1, e_5] = [e_2, e_3] = 0,$$

$$[e_2, e_4] = s\cos t\frac{\sin^2\theta}{\cos\theta}\frac{\partial}{\partial y_2},$$

$$\begin{aligned}[e_3, e_4] &= \left(\frac{-\cos^2 t}{\sin^2 t} + 2\sin t - \sin^2 t\right)\frac{\partial}{\partial x_3} + \left(\frac{\cos^2 t}{s\sin t}\right)\frac{\partial}{\partial y_3}\\ &+ \left(\sin t - \frac{\cos 2t}{\cos t} - \cos t\right)\frac{\partial}{\partial x_4}\\ &+ \left(-\frac{\cos t}{s} + \frac{\sin t}{s} - \frac{\sin^2 t}{\cot s}\right)\frac{\partial}{\partial y_4},\end{aligned}$$

$$[e_2, e_5] = [e_3, e_5] = [e_4, e_5] = 0$$

and

$$\begin{aligned} g(e_1, e_1) &= g(e_3, e_3) = 1, \quad g(e_2, e_2) = 3,\\ g(e_4, e_4) &= s^2 + 1, \quad g(e_5, e_5) = 1,\end{aligned}$$

$$\begin{aligned} g(e_1, e_2) &= g(e_1, e_3) = g(e_1, e_4) = g(e_1, e_5) = 0,\\ g(e_2, e_3) &= g(e_2, e_4) = g(e_2, e_5) = 0,\\ g(e_3, e_4) &= g(e_3, e_5) = g(e_4, e_5) = 0.\end{aligned}$$

By using Koszul formula for the Riemannian metric g, we can find

$$\begin{aligned}\nabla_{e_1}e_1 &= \nabla_{e_1}e_2 = \nabla_{e_1}e_3 = \nabla_{e_1}e_4 = \nabla_{e_1}e_5 = 0,\\ \nabla_{e_2}e_2 &= s\frac{\sin^2\theta\cos t}{2(s^2+1)}e_4,\\ \nabla_{e_2}e_4 &= \left(-\frac{1}{2s}\cos^2 t + \frac{s\sin^2 t}{s^2+1} - \frac{s\sin^3 t}{2(s^2+1)}\right)e_4,\\ \nabla_{e_2}e_5 &= \nabla_{e_3}e_1 = \nabla_{e_3}e_2 = \nabla_{e_3}e_3 = 0,\\ \nabla_{e_3}e_4 &= \left(\sin^2 t - \frac{\cos^3 t}{\sin t} - \frac{\sin t\cos 2t}{\cos t} - \frac{\sin 2t\sin t}{2} + \frac{\sin 2t}{2}\right)e_3,\\ &+ \frac{1}{s^2+1}\left((1-2s)\sin^2 t + s\sin^3 t - s\cos t + \frac{2\cos^2 t}{s} + \frac{s\sin 2t}{2} - \frac{\sin 2t}{2s}\right)e_4,\\ \nabla_{e_3}e_5 &= \nabla_{e_4}e_5 = \nabla_{e_5}e_5 = 0, \quad \nabla_{e_4}e_4 = -se_3.\end{aligned}$$

Thus we can say that M is D_θ-geodesic. But it is not totally geodesic.

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