Gulf Journal of Mathematics
Vol 6, Issue 3 (2018) 70-77

A NOTE ON MULTIPLIERS AND COMMUTATIVITY OF PRIME RINGS

KYUNG HO KIM

Abstract. In this paper, we investigate the commutativity of prime rings admitting multipliers of R satisfying certain identities and some related results have also been discussed.

1. Introduction and Preliminaries

An additive mapping $d : R \to R$ is called a *derivation* if $d(xy) = d(x)y + yd(x)$ holds for all $x, y \in R$. Over the last few decares, several authors have investigated the relationship between the commutativity of the ring R and certain specific types of derivations of R. The first result in this direction is due to E. C. Posner [9] who proved that if a ring R admits a nonzero derivation d such that $[d(x), x] \in Z(R)$ for all $x \in R$, then R is commutative. This result was subsequently, refined and extended by a number of authors. In [7], Bresar and Vuckman showed that a prime ring must be commutative if it admits a nonzero left derivation. Recently, many authors have obtained commutativity theorems for prime and semiprime rings admitting derivation, generalized derivation. Many considerable works have been done on left (right) multipliers in prime and semiprime rings during the last couple of decades([10-12]). In this paper, we investigate the commutativity of prime rings admitting multipliers of R satisfying certain identities and some related results have also been discussed. Throughout R will represent an associative ring with center $Z(R)$. For all $x, y \in R$, as a usual commutator, we shall write $[x, y] = xy - yx$, and $x \circ y = xy + yx$. Also, we make use of the following two basic identities without any specific mention:

$$x \circ (yz) = (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z$$

$$(xy) \circ z = x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z]$$

$$[xy, z] = x[y, z] + [x, z]y \text{ and } [x, yz] = y[x, z] + [x, y]z$$

for all $x, y, z \in R$. Let R is a ring. An additive mapping $F : R \to R$ is called a *left multiplier* if $F(xy) = F(x)y$ holds for every $x, y \in R$. Similarly, an additive mapping $F : R \to R$ is called a *right multiplier* if $F(xy) = xF(y)$ holds for every $x, y \in R$. If F is both a left and a right multiplier of R, then it is called a *multiplier* of R. An additive mapping $F : R \to R$ is called a *generalized derivation* if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all

Date: Received: Oct 21, 2017; Accepted: Aug 12, 2018.
2010 *Mathematics Subject Classification.* Primary 16W25; Secondary 16N60, 16U80.
Key words and phrases. Derivation, multiplier, prime, commutative.

$x, y \in R$, and d is called the *associated derivation* of F. Obviously, a generalized derivation with $d = 0$ covers the concept of left multipliers. It is easy to see that $F : R \to R$ is a generalized derivation if and only if F is of the form $F = d + H$, where d is a derivation and H is a left multiplier.

Definition 1.1. Let R be a prime ring. If F is a nonzero multiplier of R, then $F(x) \in Z(R)$ for all $x \in Z(R)$.

Proof. Let $z \in Z(R)$. By definition of F, we have

$$F(xz) = xF(z) = F(z)x = F(zx)$$

for every $x \in R$. Hence $F(z) \in Z(R)$.

□

Lemma 1.2. *Let R be a prime ring. If $z \in Z(R) - \{0\}$ and $zx \in Z(R)$, then $x \in Z(R)$.*

2. Multipliers and commutativity of prime rings

Theorem 2.1. *Let R be a prime ring. If $F : R \to R$ is a nonzero multiplier of R and $F(R) \subseteq Z(R)$, then R is commutative.*

Proof. By hypothesis, we have

$$[F(x), r] = 0, \ \forall \ x, r \in R. \tag{2.1}$$

Replacing x by xy in (1), we obtain $[F(x)y, r] = 0$ for all $x, y, r \in R$, which implies that $F(x)[y, r] + [F(x), r]y = 0$ for all $x, y, r \in R$. By the assumption, we get

$$F(x)[y, r] = 0, \ \forall \ x, y, r \in R. \tag{2.2}$$

Substituting yz for y in this relation, we have $F(x)y[z, r] = 0$ for all $x, y, z, r \in R$. This implies that $F(x)R[z, r] = 0$ for all $x, z, r \in R$. Since R is prime, we have $F(x) = 0$ or $[z, r] = 0$ for all $x, z, r \in R$. Let $K = \{x \in R | F(x) = 0\}$ and $L = \{z \in R | [z, r] = 0, \forall \ r \in R\}$. Then K and L are both additive subgroups and $K \cup L = R$, but $(R, +)$ is not union of two its proper subgroups, which implies that either $K = R$ or $L = R$. In the former case, we have $F = 0$, contradiction, and so $L = R$, that is, $[z, r] = 0$ for all $z, r \in R$, which implies that R is commutative.

□

Theorem 2.2. *Let R be a prime ring and $F : R \to R$ be an multiplier of R. If $F(xy) = F(x)F(y)$ for all $x, y \in R$ and $F(x) \neq x$ for all $x \in R$, then $F = 0$.*

Proof. By hypothesis, we have

$$F(xy) = F(x)y = F(x)F(y), \ \forall \ x, y \in R. \tag{2.3}$$

Replacing x by xw in (3) where $w \in R$, we have $F(xw)y = F(xw)F(y)$, that is, $F(x)wy = F(x)wF(y)$ for all $x, y, w \in R$. This implies that $F(x)w(y - F(y)) = 0$ for all $x, y, w \in R$. Hence $F(x)R(y - F(y)) = \{0\}$ $x, y \in R$. Since R is prime, we have $F(x) = 0$ for all $x \in R$ or $y - F(y) = 0$ for all $x \in R$. But $F(x) \neq x$ for all $x \in R$, and so $F(x) = 0$ for all $x \in R$, that is, $F = 0$.

□

Theorem 2.3. *Let R be a prime ring. If F is a nonzero multiplier of R such that $F([x,y]) \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F([x,y]) \in Z(R),\ \forall\ x, y \in R. \tag{2.4}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (4),

$$F(z[x,y]) \in Z(R),\ \forall\ x, y, z \in R, \tag{2.5}$$

which implies that $F(z)[x,y] \in Z(R)$ for all $x, y, z \in R$. Since R is prime and $F(z) \neq 0$, we have $[x,y] \in Z(R)$ for all $x, y \in R$. This implies that

$$[r,[x,y]] = 0,\ \forall\ x, y, r \in R. \tag{2.6}$$

Taking yx instead of y in the relation (6), we have $[y,x][r,x] = 0$ for all $x, y, r \in R$. Again, replacing r by rs where $s \in R$, in the last relation, we have $[y,x]R[s,x] = \{0\}$ for all $x, y, s \in R$. Since R is prime, we have either $[y,x] = 0$ or $[s,x] = 0$ for all $x, y, s \in R$. Let $K = \{x \in R | [y,x] = 0, \text{for all } y \in R\}$ and $L = \{x | [s,x] = 0, \text{for all } s \in R\}$. Then K and L are both additive subgroups and $K \cup L = R$, but $(R,+)$ is not union of two its proper subgroups, which implies that either $K = R$ or $L = R$. That is, in both cases, R is commutative.

□

Theorem 2.4. *Let R be a prime ring. If F is a nonzero multiplier of R such that $F(x \circ y) \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F(x \circ y) \in Z(R),\ \forall\ x, y \in R. \tag{2.7}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (7), we have

$$F(z(x \circ y)) \in Z(R),\ \forall\ x, y, z \in R, \tag{2.8}$$

which implies that $F(z)(x \circ y) \in Z(R)$ for all $x, y, z \in R$. Since R is prime and $F(z) \neq 0$, we have $(x \circ y) \in Z(R)$ for all $x, y \in R$. This implies that

$$[r, x \circ y] = 0$$

for all $x, y, r \in R$. Substituting yx for y in this relation, we obtain $[r, yx \circ y] = [r,y](x \circ y) = 0$ for all $x, y, r \in R$. Taking sr instead of r in the last relation, we have $[s,y]r(x \circ y) = 0$ for all $x, y, r, s \in R$. This implies that $[s,y]R(x \circ y) = \{0\}$ $x, y, s \in R$. Since R is prime, we have either $[s,y] = 0$ or $x \circ y = 0$ for all $x, y, s \in R$. Let $K = \{y \in R | [s,y] = 0\}$ and $L = \{y | x \circ y = 0\}$ for all $x, s \in R$. Then K and L are both additive subgroups and $K \cup L = R$, but $(R,+)$ is not union of two its proper subgroups, which implies that either $K = R$ or $L = R$. In the former case, R is commutative. If $L = R$, we have $x \circ y = 0$ for all $x, y \in R$. Replacing x by xs in the last relation, we obtain $x[s,y] = 0$ for all $x, y, s \in R$. That is, $R[s,y] = \{0\}$. This implies that $[s,y]R[s,y] = \{0\}$ for $y, s \in R$. Since R is prime, we have $[s,y] = 0$ for all $y, s \in R$, which means that R is commutative.
□

Theorem 2.5. *Let R be a prime ring. If F is a nonzero multiplier of R such that $x \circ F(y) \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$x \circ F(y) \in Z(R), \ \forall \ x, y \in R. \tag{2.9}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing y by zy in (9), we have $F(z)(x \circ y) \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) \neq 0$, we have $x \circ y \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.4, we get the required result.

□

Theorem 2.6. *Let R be a prime ring. If F is a nonzero multiplier of R such that $[x, F(y)] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$[x, F(y)] \in Z(R), \ \forall \ x, y \in R. \tag{2.10}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing y by zy in (10), we have $F(z)[x, y] + [x, F(z)]y \in Z(R)$ for all $x, y \in R$, which implies that $F(z)[x, y] \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) \neq 0$, we have $[x, y] \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.3, we get the required result.

□

Theorem 2.7. *Let R be a prime ring. If F is a nonzero multiplier of R such that $[F(x), F(y)] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$[F(x), F(y)] \in Z(R), \ \forall \ x, y \in R. \tag{2.11}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing y by zy in (11), we have $F(z)[F(x), y] + [F(x), F(z)]y \in Z(R)$ for all $x, y \in R$, which implies that $F(z)[F(x), y] \in Z(R)$ for all $x, y, z \in R$. Since R is prime and $F(z) \neq 0$, we have $[F(x), y] \in Z(R)$ for all $x, y \in R$. Hence, by Theorem 3.6, we get the required result.

□

Theorem 2.8. *Let R be a prime ring. If F is a nonzero multiplier of R such that $F(x) \circ F(y) \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F(x) \circ F(y) \in Z(R), \ \forall \ x, y \in R. \tag{2.12}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing y by zy in (12), we have $F(z)(F(x) \circ y) + [F(x), F(z)]y \in Z(R)$ for all $x, y, z \in R$, which implies that $F(z)(F(x) \circ y) \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) \neq 0$, we have $F(x) \circ y \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.5, we get the required result.

□

Theorem 2.9. *Let R be a prime ring. If F is a multiplier of R such that $F(x)F(y) = 0$ for all $x, y \in R$, then $F = 0$.*

Proof. By hypothesis, we have

$$F(x)F(y) = 0, \ \forall \ x, y \in R. \tag{2.13}$$

Replacing x by xz in (13), we get $F(xz)F(y) = 0$ for all $x, y, z \in R$, which implies that $F(x)zF(y) = 0$ for all $x, y, z \in R$. Hence $F(x)RF(y) = 0$ for all $x, y \in R$. Since R is prime, we obtain $F(x) = 0$ or $F(y) = 0$ for all $x, y \in R$. This implies that $F = 0$.

□

Theorem 2.10. *Let R be a prime ring. If F is a multiplier of R such that $F([x, y]) - [x, y] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F([x, y]) - [x, y] \in Z(R), \ \forall \ x, y \in R. \tag{2.14}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (14), we have $F([zx, y]) - [zx, y] \in Z(R)$ for all $x, y \in R$, which implies that $F(z[x, y] + [z, y]x) - z[x, y] - [z, y]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)[x, y] - z[x, y] \in Z(R)$ for all $x, y \in R$. This implies that $(F(z) - z)[x, y] \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) - z \in Z(R)$, we have $[x, y] \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.3, we get the required result.

□

Theorem 2.11. *Let R be a prime ring. If F is a multiplier of R such that $F(x \circ y) - (x \circ y) \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F(x \circ y) - (x \circ y) \in Z(R), \ \forall \ x, y \in R. \tag{2.15}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (15), we have $F(zx \circ y) - (zx \circ y) \in Z(R)$ for all $x, y \in R$, which implies that $F(z(x \circ y) - [z, y]x) - z(x \circ y) + [z, y]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)(x \circ y) - z(x \circ y) \in Z(R)$ for all $x, y \in R$. This implies that $(F(z) - z)(x \circ y) \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) - z \in Z(R)$, we have $x \circ y \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.4, we get the required result.

□

Theorem 2.12. *Let R be a prime ring. If F is a nonzero multiplier of R such that $[F(x), y] - [x, y] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$[F(x), y] - [x, y] \in Z(R),\ \forall\ x, y \in R. \tag{2.16}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (16), we have $[F(zx), y)] - [zx, y] \in Z(R)$ for all $x, y \in R$, which implies that $F(z)[x, y] + [F(z), y]x - z[x, y] - [z, y]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)[x, y] - z[x, y] \in Z(R)$ for all $x, y \in R$, and so $(F(z) - z)[x, y] \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) - z \in Z(R)$, we have $[x, y] \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.3, we get the required result.

□

Theorem 2.13. *Let R be a prime ring. If F is a nonzero multiplier of R such that $(F(x) \circ y) - (x \circ y) \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$(F(x) \circ y) - (x \circ y) \in Z(R),\ \forall\ x, y \in R. \tag{2.17}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (17), we have $(F(zx) \circ y) - (zx \circ y) \in Z(R)$ for all $x, y \in R$, which implies that $F(z)(x \circ y) - [F(z), y]x - z(x \circ y) - [z, y]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)(x \circ y) - z(x \circ y) \in Z(R)$ for all $x, y \in R$, and so $(F(z) - z)(x \circ y) \in Z(R)$ for all $x, y \in R$. Since R is prime and $F(z) - z \in Z(R)$, we have $x \circ y \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 3.4, we get the required result.

□

Theorem 2.14. *Let R be a prime ring. If F is a nonzero multiplier of R such that $[F(x), F(y)] - [x, y] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$[F(x), F(y)] - [x, y] \in Z(R),\ \forall\ x, y \in R. \tag{2.18}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (18), we have $[F(z)x, F(y)] - [zx, y] \in Z(R)$ for all $x, y \in R$, which implies that $F(z)[x, F(y)] + [F(z), F(y)]x - [z, y]x - z[x, y] \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)[x, F(y)] - z[x, y] \in Z(R)$ for all $x, y \in R$. Taking y in place of x in the last relation, we obtain $F(z)[y, F(y)] \in Z(R)$ for all $y \in R$. Since R is prime and $F(z) \neq 0$, we have $[y, F(y)] \in Z(R)$ for all $y \in R$. Hence, by Theorem 3.6, we get the required result.

□

Theorem 2.15. *Let R be a prime ring. If F is a nonzero multiplier of R such that $(F(x) \circ F(y)) - [x, y] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$(F(x) \circ F(y)) - [x, y] \in Z(R), \ \forall \ x, y \in R. \tag{2.19}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (19), we have $([F(z)x \circ F(y)) - [zx, y] \in Z(R)$ for all $x, y \in R$, which implies that $F(z)(x \circ F(y)) + [F(z), F(y)]x - z[x, y] - [z, y]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)[x, F(y)] - z[x, y] \in Z(R)$ for all $x, y \in R$. Taking y in place of x in the last relation, we obtain $F(z)[y, F(y)] \in Z(R)$ for all $y \in R$. Since R is prime and $F(z) \neq 0$, we have $[y, F(y)] \in Z(R)$ for all $y \in R$. Hence, by Theorem 3.6, we get the required result. □

Theorem 2.16. *Let R be a prime ring. If F is a nonzero multiplier of R such that $F([x, y]) - [F(x), F(y)] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F([x, y]) - [F(x), F(y)] \in Z(R), \ \forall \ x, y \in R. \tag{2.20}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (20), we have $F([zx, y]) - [F(zx), F(y)] \in Z(R)$ for all $x, y \in R$, which implies that $F(z)[x, y] + [F(z), F(y)]x - F(z)[x, F(y)] - [F(z), F(y)]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)[x, y] - F(z)[x, F(y)] \in Z(R)$ for all $x, y \in R$. Taking y in place of x in the last relation, we obtain $F(z)[y, F(y)] \in Z(R)$ for all $y \in R$. Since R is prime and $F(z) \neq 0$, we have $[y, F(y)] \in Z(R)$ for all $y \in R$. Hence, by Theorem 3.6, we get the required result. □

Theorem 2.17. *Let R be a prime ring. If F is a nonzero multiplier of R such that $F(x \circ y) - [F(x), F(y)] \in Z(R)$ for all $x, y \in R$ and $F(Z(R)) \neq 0$, then R is commutative.*

Proof. By hypothesis, we have

$$F(x \circ y) - [F(x), F(y)] \in Z(R), \ \forall \ x, y \in R. \tag{2.21}$$

Since $F(Z(R)) \neq 0$, there exists $z \in Z(R)$ such that $F(z) \neq 0$. Thus $F(z) \in Z(R)$ by Lemma 2.1. Replacing x by zx in (21), we have $F(zx \circ y) - [F(zx), F(y)] \in Z(R)$ for all $x, y \in R$, which implies that $F(z)(x \circ y) + [F(z), F(y)]x - F(z)[x, F(y)] - [F(z), F(y)]x \in Z(R)$ for all $x, y \in R$. Hence we get $F(z)(x \circ y) - F(z)[x, F(y)] \in Z(R)$ for all $x, y \in R$. Taking $F(y)$ in place of x in the last relation, we obtain $F(z)(F(y) \circ y) \in Z(R)$ for all $y, z \in Z(R)$. Since R is prime and $F(z) \neq 0$, we have $F(y) \circ y \in Z(R)$ for all $y \in R$. Hence, by Theorem 3.5, we get the required result. □

References

[1] Ashraf and S. Ali, *On (σ, τ)-derivations of prime rings,* Sarajevo J. Math **16** (2008), 23-30.

[2] M. Ashraf and S. Ali, *On left multipliers and the commutativity of rings,* DEMONSTRATIO MATHEMATICA **42 (4)** (2008), 763-771.

[3] H. E. Bell, *On derivations in near-rings II, Near-rings, Near-field and K-loops,* Hamburg, 1995), 191-197 Math. Appl., 426, Kluwer Acad.publ., Dordrecht, 1997.

[4] H. E. Bell and M. N. Daif, *On commutativity and strong commutativity preserving maps,* Canad. Math, Bull **37** (1994), 443-447.

[5] M. Bresar, *On a generalization of the notion of centralizing mappings,* Proc. Amer. Math. Soc, **114** (1992), 641-649.

[6] M. Bresar, *On the distance the composition of two derivation to the generalized derivations,* Glasgow Math. J. **33** (1991), 89-93.

[7] M. Bresar and J. Vukman, *On left derivations and related mappings,* Proc. Amer. Math. Soc, **110** (1990), 7-16.

[8] O. Golbasi, *Some properties of prime near-rings with (σ, τ)-derivation,* Siberian Mathematical Journal, **46** (2005), 270-275.

[9] E. C. Posner, *Derivations in prime rings,* Proc. Amer. Math. Soc, **8** (1957), 1093-1100.

[10] J. Vukman, *Centralizer on semiprime rings,* Comment. Math. Univ. Carolinae, **42** (2001), 237-245.

[11] J. Vukman, *Identity related to centralizer in semiprime rings,* Comment. Math. Univ. Carolinae, **40** (1999), 447-456.

[12] B. Zalar, *On centralizer of semiprime rings,* Comment. Math. Univ. Carolinae, **32** (1991), 609-614.

[1] Department of Mathematics, Korea National University of Transportation, Chungju, 27469, Korea.

Email address: ghkim@ut.ac.kr