

THE USE OF MANIPULATIVES IN EXPERIENTIAL LEARNING IN SOLVING THREE-DIMENSIONAL TASKS IN TRIGONOMETRY

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ABSTRACT. This article employs Kolb's Experiential Learning Theory (ELT) to address weaknesses in South Africa's Mathematics education, focusing on the use of concrete mathematical manipulatives for enhancing conceptual understanding in grade 12 learners. A Mathematical manipulative is defined as any material or object from the real world that children can move around to show a mathematical concept. Kolb's Experiential Theory links learning style to academic achievement. Kolb's ELT involves four learning modes: concrete experience, reflective observation, abstract conceptualization, and active experimentation. The use of manipulatives completes Kolb's learning cycle by providing a concrete experience, interaction and reflection with peers, abstract conceptualisation of triangles in the various planes and finally active experimentation. The study aims at improving the application of the sine rule, cosine rule, and area rule in solving 3-dimensional trigonometric problems. This qualitative study investigated the influence of mathematical models on teaching 3D trigonometric problems based on a sample of sixteen Grade 12 learners. Data was collected through observation, interviews, and written responses. The findings in this paper show that learners using manipulatives displayed heightened interests and improved understanding.

1. INTRODUCTION

Mathematics performance in South Africa reveals persistent weaknesses, with trigonometry being a notable examinable section in grade 12. In the researcher's experience and conversations with fellow educators, the application of the sine rule, cosine rule, and area rule proves to be intricate and perplexing for learners, particularly when solving three-dimensional problems. The challenges persist in teaching grade 12 learners about solving three-dimensional trigonometric problems, as many struggle with breaking down problems and choosing appropriate methods and formulas. The rationale of this study is to focus on how to improve Mathematical learning through the use of 3-dimensional models. The education in Mathematics in South Africa continues to show an area of weakness. Trigonometry is currently an examinable subject and is assessed in grade 12 examinations.

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According to the authors in the paper [1], manipulatives are tools used in mathematics instruction that, when used effectively, can positively assist pupils to grasp mathematical concepts taught in secondary schools. Research has been carried out and the learning model most applicable to learning mathematics is Kolb's model of experiential learning which has been utilized extensively to enhance and evaluate teaching [11] and [4].

In this study, Kolb's theory informs educators that a variety of teaching methods can be used to improve teaching practices that ensure links between theory and application. In addition, Kolb's theory emphasises the importance of encouraging learners to reflect on their work and ideas and give immediate feedback to reinforce their learning. In this study, the learners interacted and engaged with the manipulative. They observed the scenario and could better visualize the horizontal plane, vertical plane and slant plane. Thereafter, they discussed and exchanged ideas. Learners then planned and solved the problem and became aware that alternative solutions existed.

In Kolb's model, the learners' learning style is established by two factors: does a learner favour the concrete to abstract or the active experimentation to reflective observation. These preferences form a classification scheme consisting of four learning styles [7], (1) Concrete, reflective- learners who construct on previous experiences; (2) Concrete, active- learners who learn by trial and error; (3) Abstract, reflective- learners who learn from detailed explanations and (4) Abstract, active- Learners who learn by developing individual strategies. The aims were sorted to foster a rationale for the use of manipulatives as a sustainable learning tool in the mathematics learning process. With this aim in mind, we formulated the critical research question: How did manipulatives help learners to adopt more active approaches towards learning of three-dimensional trigonometric problems amongst Grade 12 learners?

2. BACKGROUND

This qualitative study investigated the influence of mathematical models on teaching 3D trigonometric problems based on a sample of sixteen Grade 12 learners. Data was collected through observation, interviews, and written responses. The authors in the paper [17] claim that manipulatives have several benefits which include (1) being an additional resource in learning mathematics, aid learners to form a link with the real-world knowledge and (3) helping in understanding and memorizing the related materials in Mathematics.

Studies conducted by the authors in papers [16], [7] and [20] carried out with teachers, discovered that they were keen to use manipulatives in their lessons as they believed that the manipulatives made their teaching easier and that learners were better able to concretize abstract mathematical concepts. Whilst these studies supported the use of manipulatives, there exist studies by authors in the paper [2] and [14] claim that teachers did not believe that concrete manipulatives were necessary for their lessons and that they could not be applied to all Mathematics topics.

This study focuses on three specific learning styles namely visual, auditory and kinesthetic. Learning style is an important modality in the learning process. The authors in the paper [6] claim that modalities are often linked with learning styles. It is an association with how learners make use of their senses in the process of learning. Teaching in a way that combines different learning styles is probably the most appropriate and beneficial option in South African schools. The combination of the kinesthetic, visual and auditory learning styles would work well in teaching 3D trigonometry Mathematics. One of the most suitable methods of efficient teaching in Mathematics education is via manipulative supported teaching. Since manipulative-supported education targets more than one sense organ, it is persistent, quick and meaningful and requires little time to ascertain what is required by the learners to plan and prepare the lesson [19]. The use of manipulatives allows for information to be received visually and kinaesthetically [10]. The authors in the paper [18] manipulative teaching material is considered to be any object from the real world that children can rotate, play or build a model of to demonstrate a scientific concept. They are concrete and hands-on models that gravitate to all senses and learners can touch. These manipulative teaching materials must be linked to a learner's world. Individuals have a specific learning style preference depending on the situation. Most people will therefore have a combination of learning styles. The author in the paper [13] claims that better and permanent learning is more likely to be achieved with the inclusion of more sense organs in the learning process. With the provision of manipulatives with the activity sheets the learners were provided the opportunity to interact and converse with each other. The manipulatives provided an opportunity to engage learners' senses. Learners were able to see, touch and rotate the manipulative, hear each other's contributions to the solution of the problem and add and exchange ideas. The auditory, visual and kinesthetic learning styles are most relevant to this study and figure 1 below provides an understanding of learning styles and overlapping areas that occur when manipulatives are used.

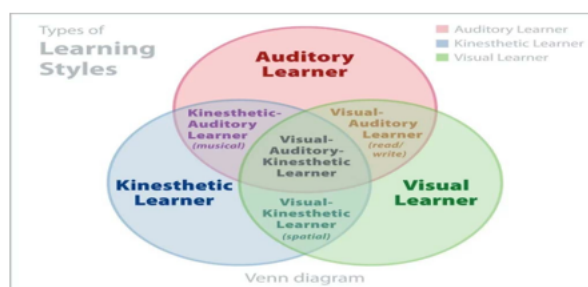


Figure 1. shows the 3 learning styles that are most relevant to this study and the overlapping of the learning styles.

Kolb's learning cycle is built on John Dewy's claim which states that learning must be a grounded experience, Kurt Lewis's ideas of the significance of active learning and Jean Piaget's emphasis on the interaction between a person and

the environment on intelligence. Figure 2 represents the four stages, which Kolb states must be completed in order for learning to occur. The cycle begins with the concrete experience.



Figure 2.: Kolb's' Learning Cycle Source [5]

The concrete experience stage starts with doing something in which the individual has a task to do. Aiming to learn requires active involvement. In Kolb's model, one cannot learn by merely reading or watching. For effective learning to take place, the individual must actively do and get involved. In this study learners are exposed to a concrete experience, this being the interaction with the manipulative, namely a 3D mathematics model. The reflective observation stage is the second stage of the cycle, which involves reflective observation. This denotes taking time out from "doing and stepping back from the task in order to review what has been completed and experienced". Several questions are asked at this stage and other members or participants of a team or group communicate with each other. Important discussion takes place, which requires rich vocabulary to communicate with others in the team or group. In this study the learners engage with the manipulative and discuss the different planes in the manipulatives and analyze the given information. They further discuss which formulae will be selected based on the data given. For several learners, this is where the metamorphosis from seeing and doing to reflecting can embed the learning into real-time absorption of materials and mathematics. It could possibly be where learners are shown how to fulfill goals and how to apply them in various circumstances.

The abstract conceptualization stage of the cycle is the process of making sense of what has occurred and involves interpreting events and making sense of relationships that exist. The learner makes comparisons between what then have completed and what they know. Learners may draw upon theory from textbooks, models, and ideas from others, previous observations or other knowledge they have acquired. In this study, at this stage, the grade 12 learners made comparisons between the various triangles in the horizontal, vertical and slant planes. They reflected on what they knew about trigonometry namely ratios in a right angled triangles, the sine rule, cosine rule and area rule. They drew from their existing knowledge, experience, models that they familiar with, observations and ideas from their classmates. In this study learners formed new ideas or revised current abstract ideas based on the reflections that emerge from the reflective observation stage. The learners now have the opportunity to see how the ideas and concepts learned previously can be utilized in the real world. The last stage, being the active experimentation stage of the learning cycle is where learners think about

how they are going to apply their newly acquired knowledge. Planning allows taking the new understanding and translates it into predictions or if refined or revised actions should be carried out. If the learning is to be useful, the learner needs to place it in a relevant context else the learning is likely to be forgotten. This is where the learner applies new ideas to the surrounding to determine if any modifications are required in the next appearance of the experience. In this study learners apply their knowledge to determine if there are alternative solutions or if there is a shorter and quicker way of solving the problem. Through the learners' discussions and interactions with each other they were able to plan and draw flow charts to enable them to correctly select the formulae that used all the given information in order to solve for the unknown. They realised that they should look for right angled triangles and one could apply the theorem of Pythagoras or if two sides and an included angle were given then, they could use the cosine rule or if two sides and an angle were given they needed to apply the sin rule. Learners realised that planning played a crucial role in understanding. The learners were able to see the usefulness of learning in one's life.

3. RESULTS AND FINDINGS

The researcher walked around and took notes of learners' conversations and interactions with each other and with the manipulatives. The learners touched and rotated the physical concrete manipulative which showed the first stage of Kolb's learning Cycle. Concrete Experience (CE) is a combination of divergent and accommodation of styles. The learners learned through feelings and emphasizing their concrete experiences with the manipulative. Peer collaboration was encouraged by allowing learners to work on the activity sheet with the 3-D trigonometric problem and with the concrete manipulative. There was a great learner-learner interaction promoted as learners were encouraged to verbalise what they saw and thought about the 3D trigonometric problem. The Reflection Observation (RO) is a combination of divergence with assimilation types. The learners learn through observation. The emphasis is observed before assessing, listening to a scenario from various perspectives and listening to the meaning of things observed. In this learning process, the grade 12 learners used their thoughts and feelings to form opinions about the possible solution to the 3D problem. In Abstract Conceptualisation (AC) of Kolb's Cycle, learners learn through thought and focus more on logical analysis of ideas, systematic planning and intellectual understanding of the problem. The learning process therefore relies on systematic planning and developing ideas and theories to solve problems. The learners in the study were motivated to explain and justify their written answers. It was through the learners' comments, questions and answers that their mental construction regarding Abstract Conceptualization (AC) were accessed. In Kolb's Cycle the Active Experimentation is where learners learn through action, tend to be confident in terms of carrying out tasks, are willing to take risks and influence others through their actions. The learning process appreciates success in completing work, achievements and influence on others. Therefore, this study aims to investigate the use of manipulatives in experiential learning in terms of

Kolb's learning cycle in solving three-dimensional tasks in trigonometry. The researcher walked around the classroom and made notes on learners' conversations, discussions and observations of learners' interactions. It was observed that learner-learner interaction had increased. Learner one said to her group "the angle looks like 90° ". Learner 2 argued, "don't assume it to be 90° , even though it looks like 90° ". Teacher always says don't assume things unless it is given or stated". Learners touched the manipulatives and some even rotated the manipulative. Learner two used his pen to draw in the air around the triangles that he could see in the vertical plane and horizontal plane. Learner three shouted out "I can see a triangle in the slant plane as well".

Detailed analyses of question four and question five are provided to determine the central role that experience plays in the learning experience QUESTION 4(NSC PAPER 2-Nov 2011) The figure 3 below represents a triangular right prism with $BA = BC = 5$ units, $\angle ABC = 50^\circ$ and $\angle FAC = 25^\circ$

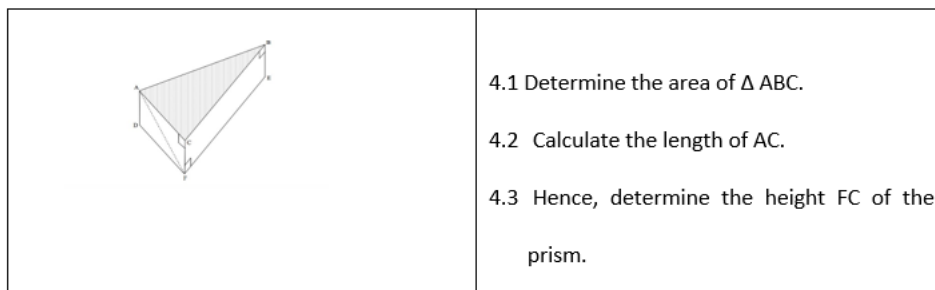


Figure 3: Question 4 from the National Senior Certificate, Paper 2, November 2011 examination

Question 4 was a level one and level two-type question and was answered well by most of the learners. Question 4.1 was a knowledge-type question, a mere recall of the area formula with no making a specific variable the subject of the formula. The level of the question can be classified as easy. Question 4.2 requires the identification of the cosine rule and the mere substitution of values. Question 4.3 can be classified as a routine procedure that needs conceptual understanding. Learners need to extract triangle ACF, and use answer obtained in question 4.2 to calculate the length of AC. Finally, use a trig ratio to solve for FC or alternately use the sine rule.

4.1) $\Delta BAC = \frac{1}{2} ac \sin A$
 $= \frac{1}{2} (5)(5) \sin 150^\circ$
 $= 9,58 \text{ cm}$

4.2) $\frac{a}{\sin A} = \frac{b}{\sin B}$
 $\frac{5}{\sin 150^\circ} = \frac{AC}{\sin 50^\circ}$
 $AC = 4,23$

4.3) $\tan \theta = \frac{CF}{AC}$
 $\tan 45 = \frac{CF}{4,23}$
 $\tan 45 \times 4,23 = CF$
 $CF = 4,23$

Figure 4 Written response of L11 to question 4

In Figure 4, learner eleven for question 4.1 above was able to select the correct formula, substitute correctly and arrive at the correct answer. In question 4.2, Learner eleven executed the problem correctly by applying the sine rule. However, in question 4.3, Learner eleven used an incorrect angle of 450 instead of 250, which resulted in the incorrect height of FC of the prism. Learner eleven on the diagram had indicated 250 but may have made a careless mistake.

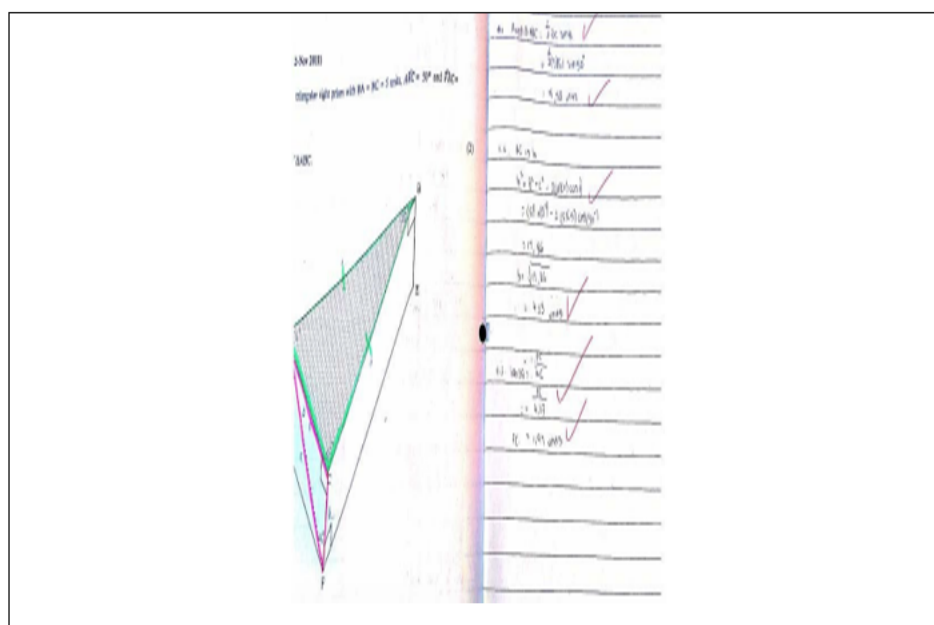


Figure 5 Written response of L6 to question 4.

In figure 5, learner six was able to visualize better with the 3D manipulative and then proceeded to highlight triangle ABC in green and triangle ACF in pink. L6 was better able to see the triangles. She executed question 4.1 and 4.2 well by applying the area rule and cosine respectively. In question 4.3 she selected

the correct trigonometric ratio, substituted correctly and was able to successfully manipulate the equation to calculate the height FC of the prism.

$CDB = 2x$ and $CB D = 90^\circ - x$. The distance between C and D is k metres.

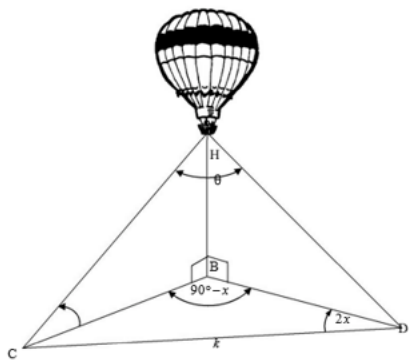
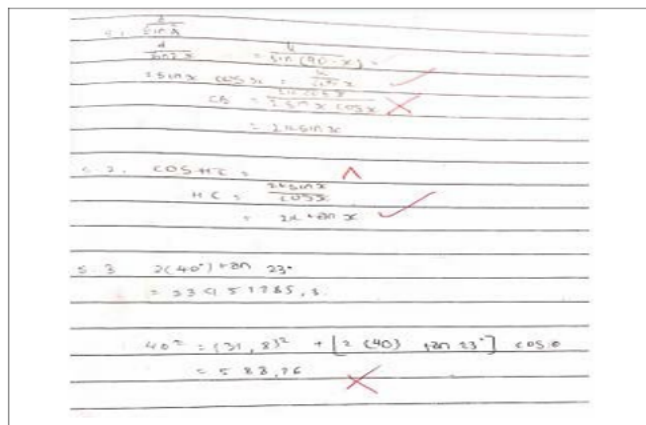
$CDB = 2x$ and $CB D = 90^\circ - x$. The distance between C and D is k metres. 	5.1 Show that $CB = 2k \sin x$. (5) 5.2 Hence, show that the length of rope HC is $2k \tan x$. (3) 5.3 If $k = 40$ m, $x = 23^\circ$ and $HD = 31,8$ m, calculate θ, the angle between the two ropes. (4)
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Figure 6: Question 5 from the National Senior Certificate, Paper 2, November 2012 examination

QUESTION 5 (NSC PAPER 2-NOV 2012) This problem in figure 6 involves complex calculations and higher-order reasoning. In question 5.1 triangle CBD on the horizontal plane would have to be used. Given two angles and a side and required to find a side would imply the use of the sine rule. Use of the reduction formula and knowledge of double angles are required. In question 5.2 triangle CHB on the vertical plane needs to be extracted, then use of trig ratio, followed by the application of the quotient identity. In question 5.3 triangle CHD on the slant plane needs to be extracted. Side HC had to be calculated. Finally given 3 sides the cosine rule could be used to calculate the angle. Learners struggled with concepts that needed deeper conceptual understanding. Questions, which required interpretation of information or justification, posed the greatest chal-



lenges.

Figure 7. Written Learner response of L6 to question 5.

Figure 7. Written

Learner response of L6 to question 5. In figure 7, learner six choose the sine rule and substituted correctly. However, Learner six was not able to manipulate the equation in the third line. Learner six omitted replacing d with CB and incorrectly cross-multiplied the equation. Learner six was unable to logically calculate the distance between C and D in terms of k metres and merely wrote $2k \sin x$, which shows no logical link from the second last step to the last step.

Question 5.2 Learner six did not write the trig ratio applied in solving the question. The solution contained six steps but Learner six merely wrote the last two steps. A possible reason could be that Learner six copied the answer from the group without understanding. Question 5.3 Learner six was unable to use the calculator correctly to calculate HC , in addition the cosine rule was incorrect. It is evident that Learner six struggled with concepts that needed deeper conceptual understanding. Questions, which required interpretation of information or justification, posed, as the greatest challenges to Learner six.

5.1) $\frac{CB}{\sin 30^\circ} = \frac{d}{\sin 40^\circ}$ ✓
 $\frac{CB}{\frac{1}{2}} = \frac{d}{\frac{2}{5}}$ ✓
 $2CB = \frac{2d}{5}$ ✓
 $CB = \frac{d}{5}$ ✓
 $CB = \frac{2k \sin 40^\circ}{5}$ ✓
 $CB = \frac{2k \sin 40^\circ}{5}$ ✓

5.2) $\cos 30^\circ = \frac{CB}{CH}$ ✓
 $\cos 30^\circ = \frac{CB}{CH}$ ✓
 $\cos 30^\circ = \frac{CB}{CH}$ ✓
 $2k \sin 40^\circ = (HC) (\cos 30^\circ)$ ✓
 $HC = \frac{2k \sin 40^\circ}{\cos 30^\circ}$ ✓
 $HC = 2k \tan 40^\circ$ ✓

5.3) $HC = 2(40) \tan 23^\circ$ ✓
 $HC = 33.96 \text{ m}$ ✓

$h^2 = d^2 + p^2 - 2(d)(p) \cos \theta$ ✓
 $(40)^2 = (33.96)^2 + (33.96)^2 - 2(33.96)(33.96) \cos \theta$ ✓
 $-544.92 = -2159.86 \cos \theta$ ✓
 $\cos \theta = \frac{2159.86}{544.92}$ ✓
 $\theta = \cos^{-1} \left(\frac{2159.86}{544.92} \right)$ ✓
 $\theta = 74.35^\circ$ ✓

Figure 8. Written learner response of L12 to question 5.

In figure 8, learner twelve displayed skills and the ability to solve this problem that involved complex calculations and higher order reasoning. Learner twelve was able to use triangle CBD on the horizontal plane. Learner twelve applied the sine rule to arrive at the correct answer in question 5.1. In question 5.2, Learner twelve extracted triangle CHB on the vertical plane and executed the $\cos x = CB/CH$ ratio. Learner twelve was able to apply algebraic skills and manipulate the equation to prove that the magnitude of $HC = 2k \tan x$. In question 5.3 Learner twelve was once again able to extract triangle CHD on the slant plane, Learner twelve successfully calculated the value of side HC using $HC = 2k \tan x$. Given 3 sides Learner twelve then applied the cosine rule to calculate the angle.

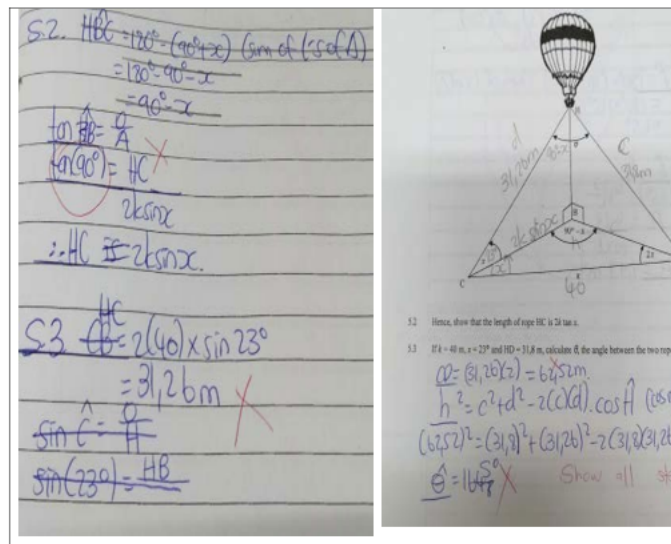


Figure 9. Written Learner response of L8 to question 5.

In figure 9, learner eight displayed poor skills and the inability to solve this problem that involved complex calculations and higher order reasoning. Learner eight was unable to use triangle CBD on the horizontal plane. Learner eight applied the sine rule to arrive at the correct answer in question 5.1. From the diagram it was given that angle HBC was 90° but Learner eight used the sum of angles of a triangle and discovered angle HBC was $90^\circ - x$. Learner eight realized that this was incorrect and cancelled the solution. Learner eight made a second attempt. In question 5.2 Learner eight used an incorrect angle and an incorrect trigonometric ratio. Learner eight used $\tan \theta$ instead of $\cos \theta$. Learner eight failed to realize that $\tan 90^\circ$ is undefined and should have immediately seen the error. L8 continued to cross-multiply and obtained an incorrect answer. In the solution it would appear that $\tan 90^\circ$ reduced to 1, which is incorrect, $\sin 90^\circ$ reduces to 1. Learner eight merely cross-multiplied and wrote an incorrect answer which contained $\sin x$ instead of $\tan x$ as asked in question 5.2 to prove $HC = 2k \tan x$. It was observed that this learner opted to work alone and did not enjoy group work. In question 5.3 Learner eight continued to carry incorrect answer from 5.2. $HC = 2(40) \times \tan 230^\circ$. Learner eight used $\sin 230^\circ$ which resulted in an incorrect answer. $CD = k$ was given as 40 cm . Learner eight indicated this measurement on the diagram but proceeded to calculate CD doubling CH . Learner eight was able to identify the cosine rule needed to be applied, given three sides of a triangle and required to calculate an angle. The incorrect substitution of h resulted in the incorrect answer. Further more the learner did not show all steps of manipulation of the equation and rushed to obtain a final answer. It was also observed that the learner put $\angle BCD = 2x$.

4. CONCLUSION & FUTURE SCOPE

This article is based on research in which we studied sixteen participants' written responses to questionnaires, activity worksheets and semi-structured interviews. In the discussion exemplars for question four and question five were presented. These questions highlighted the learner's mental construction and learning preference and relevance to experiential learning along with Kolb's learning cycle. In this study the use of manipulatives catered for learners with different learning styles. Some learners gravitated to one specific style while others had a combination of learning styles. When learners engaged with the manipulatives, they were able to interact with the teaching aid. Learners create meaning through negotiating, diagnosing and challenging misconceptions and beliefs. The authors in the paper [15] add that cognitive processes are assisted by manipulatives and another advantage that exists is that manipulatives engage students and increase both interest and enjoyment in Mathematics. In his study learners who engaged with manipulatives stated that they were more interested in Mathematics. Thus, manipulative usage encouraged a learning environment that promoted engagement and understanding.

The authors in the paper [9] maintain that the use of physical manipulatives has proved to be effective in classrooms as they are multi-sensory and can represent ideas and concepts in many ways. In addition, the authors in the papers [3] and [12] claim that manipulatives not only contribute to the learners' cognitive aspect but also develops the psychomotor skills by attending to the learners' sense of sight, touch and hearing. The authors in the paper [8] point out that Kolb's theory contains several strengths. The theory gives ready pointers to application; directs educators to ensure that a variety of teaching methods are used; provides a theoretical rationale for what many teachers already do and then informs teachers on how to improve on teaching practice that ensures links between theory and application; makes explicit the importance of encouraging learners to reflect on their work and ideas and gives immediate feedback to reinforce their learning; can be applied to different subjects/disciplines; can be used by groups or individuals; supports educators in developing awareness to students from diverse backgrounds in the classroom, and makes educators feel conscious of how the different learning styles have to be combined for effective learning to take place.

When learners interacted with the manipulatives in this study, it was observed that the Kolb's cycle was applied. The flow chart figure 10 depicts the process that ensued. Learners interacted and engaged with the manipulative. They observed the scenario and could better visualize the horizontal plane, vertical plane and slant plane. Thereafter, they discussed and exchanged ideas. Learners then planned and solved the problem and became aware that alternative solutions existed.



Figure 10 The flow chart depicts the process that learners used.

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