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# ON SYMMETRIC BI-DERIVATIONS AND COMMUTATIVITY OF PRIME RINGS

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ABSTRACT. In this paper, we study the commutativity of prime rings satisfying certain identities involving symmetric bi-derivations on rings and some related results have also been discussed.

## 1. Introduction and Preliminaries

Let $S$ be a nonempty subset of $R$. A function $f : R \to R$ is said to be *centralizing* on $S$ if $[f(x), x] \in Z(R)$ for all $x \in R$. In the special case, when $[f(x), x] = 0$, $f$ is said to be commuting on $S$. the study of such mappings were initiated by Posner. In [8], Posner prove that if a prime ring has a nonzero commuting derivation, then $R$ is commutative. Over the last five decades, many authors [1, 9] have proved commutativity theorems for prime and semiprime rings admitting various types of additive maps like automorphisms, derivations, bi-derivations and generalized derivations which are centralizing or commuting on certain appropriate subsets of $R$. In this paper, we study the commutativity of prime rings satisfying certain identities involving symmetric bi-derivations on rings and some related results have also been discussed.

Throughout, $R$ represents an associative ring with center $Z(R)$. Let $R$ be a ring. Then $R$ is said to be *prime* if $aRb = 0$ implies $a = 0$, or $b = 0$ for all $a, b \in R$. For any $x, y \in R$, $[x, y] = xy - yx, x \circ y = xy + yx$. Also, we make use of the following two basic identities without any specific mention:

$$x \circ (yz) = (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z$$

$$(xy) \circ z = x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z]$$

$$[xy, z] = x[y, z] + [x, z]y \text{ and } [x, yz] = y[x, z] + [x, y]z.$$

An additive mapping $d : R \to R$ is called a *derivation* if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. A mapping $D : R \times R \to R$ is said to be *symmetric* if $D(x, y) = D(y, x)$ for all $x, y \in R$. A mapping $d : R \to R$ defined by $d(x) = D(x, x)$ is called a *trace* of $F$, where $D : R \times R \to R$ is a symmetric mapping. It is obvious that if $D : R \times R \to R$ is a symmetric mapping, which is also bi-additive (i.e., additive in both arguments), then the trace $d$ of $D$ satisfies the relation

$$d(x + y) = d(x) + 2D(x, y) + d(y)$$

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for all $x, y \in R$. Let $D$ be a symmetric bi-additive mapping of $R$. Then $D(0, y) = 0$ for all $y \in R$. and so $D(-x, y) = -D(x, y)$ for all $x, y \in R$. Therefore the mapping $d : R \to R$ defined by $d(x) = D(x, x)$ is an even function of $R$. As usual, an element $c \in R$ for which $d(c) = D(c, c) = 0$ is called a *constant* . A symmetric bi-additive (i.e., additive in both arguments) mapping $D : R \times R \to R$ is called a *symmetric bi-derivation* if $D(xy, z) = D(x, z)y + xD(y, z)$ is fulfilled for all $x, y, z \in R$.

Let $R$ be a ring and let $F$ be a symmetric map. A function $F : R \times R \to R$ is called a *symmetric bi-multiplier* on $R$ if it satisfies the following condition

$$F(xz, y) = F(x, y)z$$

for all $x, y, z \in R$.

**Lemma 1.1.** ([3]) *Let $R$ be a prime ring. If $z \in Z(R) - \{0\}$ and $zx, xz \in Z(R)$, then $x \in Z(R)$.*

## 2. Symmetric bi-derivations and commutativity of prime rings

**Lemma 2.1.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$, then $D(x, z) \in Z(R)$ for all $x \in Z(R)$ and $z \in R$.*

*Proof.* Let $x \in Z(R)$. By definition of $D$, we have

$$D(xy, t) = D(x, t)y + xD(y, t) \tag{2.1}$$

for all $x, y, t \in R$. Also, we have

$$\begin{aligned} D(yx, t) &= D(y, t)x + yD(x, t) \\ &= xD(y, t) + yD(x, t) \end{aligned} \tag{2.2}$$

for all $y, t \in R$. Combining (2.1) with (2.2), we have $D(x, t)y = yD(x, t)$ for all $y \in R$, which implies $D(x, t) \in Z(R)$.

□

**Theorem 2.2.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ and $D(R, R) \subseteq Z(R)$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$[D(x, t), r] = 0, \ \forall\ x, t, r \in R. \tag{2.3}$$

Replacing $x$ by $xy$ in (2.3), we obtain

$$\begin{aligned} 0 = [D(xy, t), r] &= [D(x, t)y + xD(y, t), r] \\ &= [D(x, t)y, r] + [xD(y, t), r] \\ &= [D(x, t), r]y + D(x, t)[y, r] + [x, r]D(y, t) + x[D(y, t), r] \\ &= D(x, t)[y, r] + [x, r]D(y, t) \end{aligned} \tag{2.4}$$

for all $x, y, t, r \in R$. Substituting $r$ for $x$ in (2.4), we have

$$D(r, t)[y, r] = 0, \ \forall\ y, t, r \in R. \tag{2.5}$$

Substituting $yz$ for $y$ in (2.5), we have $D(r, t)y[z, r] = 0$ for all $t, y, z, r \in R$. This implies that $D(r, t)R[z, r] = \{0\}$ for all $z, t, r \in R$. Since $R$ is prime, we have $D(r,t) = 0$ or $[z,r] = 0$ for all $z,t,r \in R$. Let $K = \{r \in R | D(r,t) = 0\}$ and $L = \{r \in R | [z,r] = 0, \forall\ z,r,t \in R\}$. Then $K$ and $L$ are both additive subgroups and $K \cup L = R$, but $(R,+)$ is not union of two its proper subgroups, which implies that either $K = R$ or $L = R$. In the former case, we have $D = 0$, contradiction, and so $L = R$, that is, $[z,r] = 0$ for all $z,r \in R$, which implies that $R$ is commutative.

□

**Theorem 2.3.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ such that $D([x,y],t) \in Z(R)$ for all $x,y,t \in R$ and $D(Z(R),R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$D([x,y],t) \in Z(R),\ \forall\ x,y,t \in R. \tag{2.6}$$

Since $D(Z(R),R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.6), we have

$$D([zx,y],t) = D([z,y]x + z[x,y],t) = D([z,y]x,t) + D(z[x,y],t) \in Z(R) \tag{2.7}$$

for all $x,y,z,t \in R$. which implies that $D(z,t)[x,y] \in Z(R)$ for all $x,y,z,t \in R$. Since $R$ is prime and $D(z,t) \neq 0$, we have $[x,y] \in Z(R)$ for all $x,y \in R$. This implies that

$$[r,[x,y]] = 0,\ \forall\ x,y,r \in R. \tag{2.8}$$

Taking $yx$ instead of $y$ in the relation (2.8), we have $[y,x][r,x] = 0$ for all $x,y,r \in R$. Again, replacing $r$ by $rs$ in the last relation, we have $[y,x]R[r,x] = \{0\}$ for all $x,y,r \in R$. Since $R$ is prime, we have either $[y,x] = 0$ or $[R,x] = 0$ for all $x,y \in R$. Let $K = \{x \in R | [y,x] = 0\}$and $L = \{x | [r,x] = 0\}$. Then $K$ and $L$ are both additive subgroups and $K \cup L = R$, but $(R,+)$ is not union of two its proper subgroups, which implies that either $K = R$ or $L = R$. That is, In both cases, $R$ is commutative.

□

**Theorem 2.4.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ such that $D(x \circ y,t) \in Z(R)$ for all $x,y,t \in R$ and $D(Z(R),R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$D(x \circ y,t) \in Z(R),\ \forall\ x,y,t \in R. \tag{2.9}$$

Since $D(Z(R),R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.9), we have

$$D(z(x \circ y),t) \in Z(R),\ \forall\ x,y,z,t \in R, \tag{2.10}$$

which implies that $D(z,t)(x \circ y) \in Z(R)$ for all $x,y,z,t \in R$. Since $R$ is prime and $D(z,t) \neq 0$, we have $x \circ y \in Z(R)$ for all $x,y,t \in R$. Replacing $x$ by $xz$ in the last relation and using the fact that $yx = -xy$, we obtain $x[z,y] = 0$ for all $x,y,z \in R$. That is, $R[z,y] = \{0\}$. This implies that $[z,y]R[z,y] = \{0\}$ for $y,z \in R$. Since $R$ is prime, we have $[z,y] = 0$ for all $y,z \in R$, which means that $R$ is commutative.

□

**Theorem 2.5.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-multiplier of $R$ such that $F(x,t)F(y,t)=0$ for all $x,y,t \in R$, $F=0$.*

*Proof.* By hypothesis, we have

$$F(x,t)F(y,t)=0,\ \forall\ x,y,t \in R. \tag{2.11}$$

Replacing $x$ by $xz$ in (2.11), we get $F(xz,t)F(y,t)=0$ for all $x,y,z,t \in R$, which implies that $F(x,t)zF(y,t)=0$ for all $x,y,z,t \in R$. Hence $F(x,t)RF(y,t)=\{0\}$ for all $x,y,t \in R$. Since $R$ is prime, we obtain $F(x,t)=0$ or $F(y,t)=0$ for all $x,y,t \in R$. This implies that $F=0$.

□

**Theorem 2.6.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ such that $D([x,y],t)-[x,y] \in Z(R)$ for all $x,y,t \in R$ and $D(Z(R),R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$D([x,y],t)-[x,y] \in Z(R),\ \forall\ x,y,t \in R. \tag{2.12}$$

Since $D(Z(R),R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.12), we have

$$\begin{aligned} D([zx,y],t)-[zx,y] &= D(z[x,y]+[z,y]x,t)-z[x,y]-[z,y]x \\ &= D(z,t)[x,y]+zD([x,y],t)-z[x,y] \\ &= D(z,t)[x,y]-z(D([x,y],t)-[x,y]) \in Z(R) \end{aligned} \tag{2.13}$$

for all $x,y,t \in R$. Hence we have $D(z,t)[x,y] \in Z(R)$ for all $x,y,t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $[x,y] \in Z(R)$ for all $x,y \in R$.

Using the same argument of the last part of proof of Theorem 2.3, we get the required result.

□

**Theorem 2.7.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ such that $D(x \circ y,t)-(x \circ y) \in Z(R)$ for all $x,y,t \in R$ and $D(Z(R),R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$D(x \circ y,t)-(x \circ y) \in Z(R),\ \forall\ x,y,t \in R. \tag{2.14}$$

Since $D(Z(R),R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.14), we have

$$\begin{aligned} D(zx \circ y,t)-(zx \circ y) &= D(z(x \circ y)-[z,y]x,t)-z(x \circ y)+[z,y]x \\ &= D(z,t)(x \circ y)+zD((x \circ y),t)-z(x \circ y) \\ &= D(z,t)(x \circ y)-z(D(x \circ y,t)-(x \circ y)) \in Z(R) \end{aligned} \tag{2.15}$$

for all $x,y,z,t \in R$. Hence we have $D(z,t)(x \circ y) \in Z(R)$ for all $x,y,z,t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $x \circ y \in Z(R)$ for all $x,y \in R$.

Using the same argument of the last part of proof of Theorem 2.4, we get the required result.

□

**Theorem 2.8.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ such that $[D(x,t),y]-[x,y]\in Z(R)$ for all $x,y,t\in R$ and $D(Z(R),R)\neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$[D(x,t),y]-[x,y]\in Z(R),\ \forall\ x,y,t\in R. \tag{2.16}$$

Since $D(Z(R),R)\neq 0$, there exists $z\in Z(R)$ such that $D(z,t)\neq 0$. Thus $D(z,t)\in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.16), we have

$$\begin{aligned}
&[D(zx,t),y]-[zx,y]=[D(z,t)x+zD(x,t),y]-z[x,y]-[z,y]\\
&=[D(z,t)x,y]+[zD(x,t),y]-z[x,y]\\
&=D(z,t)[x,y]+[D(z,t),y]x+z[D(x,t),y]+[z,y]D(x,t)-z[x,y]\\
&=D(z,t)[x,y]+z([D(x,t),y]-[x,y])\in Z(R) \qquad (2.17)
\end{aligned}$$

for all $x,y,z,t\in R$. By hypothesis, we have $D(z,t)[x,y]\in Z(R)$ for all $x,y,z,t\in R$. Since $R$ is prime and $D(z,t)\in Z(R)$, we have $[x,y]\in Z(R)$ for all $x,y\in R$.

Using the same argument of the last part of proof of Theorem 2.3, we get the required result.

□

**Theorem 2.9.** *Let $R$ be a prime ring. If $D$ is a nonzero symmetric bi-derivation of $R$ such that $(D(x,t)\circ y)-(x\circ y)\in Z(R)$ for all $x,y,t\in R$ and $D(Z(R),R)\neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$(D(x,t)\circ y)-(x\circ y)\in Z(R),\ \forall\ x,y,t\in R. \tag{2.18}$$

Since $D(Z(R),R)\neq 0$, there exists $z\in Z(R)$ such that $D(z,t)\neq 0$. Thus $D(z,t)\in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.18), we have

$$\begin{aligned}
&(D(zx,t)\circ y)-(zx\circ y)=((D(z,t)x+zD(x,t))\circ y)-(zx\circ y)\\
&=D(z,t)x\circ y+zD(x,t)\circ y-(zx\circ y)\\
&=D(z,t)(x\circ y)-[D(z,t),y]x+z(D(x,t)\circ y)-[z,y]D(x,t)-z(x\circ y)-[z,y]x\\
&=D(z,t)(x\circ y)+z((D(x,t)\circ y)-(x\circ y))\in Z(R) \qquad (2.19)
\end{aligned}$$

for all $x,y,z,t\in R$. By hypothesis, we have $D(z,t)(x\circ y)\in Z(R)$ for all $x,y,z,t\in R$. Since $R$ is prime and $D(z,t)\in Z(R)$, we have $x\circ y\in Z(R)$ for all $x,y\in R$.

Using the same argument of the last part of proof of Theorem 2.4, we get the required result.

□

**Theorem 2.10.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-derivation of $R$ such that $x\circ D(y,t)\in Z(R)$ for all $x,y,t\in R$ and $D(Z(R),R)\neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$x \circ D(y,t) \in Z(R),\ \forall\ x,y,t \in R. \tag{2.20}$$

Since $D(Z(R),R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $y$ by $zy$ in (2.20), we have

$$\begin{aligned} x \circ D(zy,t) &= x \circ (D(z,t)y + zD(y,t)) \\ &== x \circ zD(y,t) + x \circ D(z,t)y \\ &= z(x \circ D(y,t)) + [x,z]D(y,t) + D(z,t)(x \circ y) + [x,D(z,t)]y \\ &= D(z,t)(x \circ y) + z(x \circ D(y,t)) \in Z(R) \end{aligned} \tag{2.21}$$

for all $x,y,z,t \in R$. By hypothesis, we have $D(z,t)(x \circ y) \in Z(R)$ for all $x,y,z,t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $x \circ y \in Z(R)$ for all $x,y \in R$.

Using the same argument of the last part of proof of Theorem 2.4, we get the required result.

□

**Theorem 2.11.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-derivation of $R$ such that $[x,D(y,t)] \in Z(R)$ for all $x,y,t \in R$ and $D(Z(R),R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$[x,D(y,t)] \in Z(R),\ \forall\ x,y,t \in R. \tag{2.22}$$

Since $D(Z(R),R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $y$ by $zy$ in (2.22), we have

$$\begin{aligned} &[x,D(zy,t)] = [x,(D(z,t)y + zD(y,t))] = [x,(D(z,t)y] + [x,zD(y,t)] \\ &= D(z,t)[x,y] + [x,D(z,t)]y + z[x,D(y,t)] + [x,z]D(y,t) \\ &= D(z,t)[x,y] + z[x,D(y,t)] \in Z(R) \end{aligned} \tag{2.23}$$

for all $x,y,z,t \in R$. By hypothesis, we have $D(z,t)[x,y] \in Z(R)$ for all $x,y,z,t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $[x,y] \in Z(R)$ for all $x,y \in R$.

Using the same argument of the last part of proof of Theorem 2.3, we get the required result.

□

**Theorem 2.12.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-derivation of $R$ such that $[D(x,t),D(y,t)]-[x,y] \in Z(R)$ for all $x,y,t \in R$ and $D(Z(R),R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$[D(x,t),D(y,t)] - [x,y] \in Z(R),\ \forall\ x,y,t \in R. \tag{2.24}$$

Since $D(Z(R), R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.24), we have

$$\begin{aligned}
&[D(zx,t), D(y,t)] - [zx, y] = [D(z,t)x + zD(x,t), D(y,t)] - z[x,y] + [z,y]x \\
&= [D(z,t)x, D(y,t)] + [zD(x,t), D(y,t)] - z[x,y] \\
&= D(z,t)[x, D(y,t)] + [D(z,t), D(y,t)]x + z[D(x,t), D(y,t)] + [z, D(y,t)]D(x,t) - z[x,y] \\
&= D(z,t)[x, D(y,t)] + z([D(x,t), D(y,t)] - [x,y]) \in Z(R) \qquad (2.25)
\end{aligned}$$

for all $x, y, z, t \in R$. By hypothesis, we have $D(z,t)[x, D(y,t)] \in Z(R)$ for all $x, y, z, t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $[x, D(y,t)] \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 2.11, we get the required result.

□

**Theorem 2.13.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-derivation of $R$ such that $(D(x,t) \circ D(y,t)) - [x,y] \in Z(R)$ for all $x, y, t \in R$ and $D(Z(R), R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$(D(x,t) \circ D(y,t)) - [x,y] \in Z(R), \ \forall\ x, y, t \in R. \qquad (2.26)$$

Since $D(Z(R), R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.26), we have

$$\begin{aligned}
&(D(zx,t) \circ D(y,t)) - [zx, y] = (D(z,t)x + zD(x,t) \circ D(y,t)) - z[x,y] + [z,y]x \\
&= (D(z,t)x \circ D(y,t)] + (zD(x,t) \circ D(y,t)) - z[x,y] \\
&= D(z,t)(x \circ D(y,t)) + z(D(x,t) \circ D(y,t)) - z[x,y] \\
&= D(z,t)[x, D(y,t)] + z(D(x,t) \circ D(y,t) - [x,y]) \in Z(R) \qquad (2.27)
\end{aligned}$$

for all $x, y, z, t \in R$. By hypothesis, we have $D(z,t)[x, D(y,t)] \in Z(R)$ for all $x, y, z, t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $[x, D(y,t)] \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 2.11, we get the required result.

□

**Theorem 2.14.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-derivation of $R$ such that $(D(x,t) \circ D(y,t)) - (x \circ y) \in Z(R)$ for all $x, y, t \in R$ and $D(Z(R), R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$(D(x,t) \circ D(y,t)) - (x \circ y) \in Z(R), \ \forall\ x, y, t \in R. \qquad (2.28)$$

Since $D(Z(R), R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.28), we have

$$
\begin{aligned}
&(D(zx,t) \circ D(y,t)) - (zx \circ y) = ((D(z,t)x + zD(x,t)) \circ D(y,t)) - z(x \circ y) - [z,y]x \\
&= (D(z,t)x \circ D(y,t)] + (zD(x,t) \circ D(y,t)) - z(x \circ y) \\
&= D(z,t)(x \circ D(y,t)) + z(D(x,t) \circ D(y,t)) - z(x \circ y) \\
&= D(z,t)(x \circ D(y,t)) + z(D(x,t) \circ D(y,t) - (x \circ y)) \in Z(R) \qquad (2.29)
\end{aligned}
$$

for all $x, y, z, t \in R$. By hypothesis, we have $D(z,t)(x \circ D(y,t)) \in Z(R)$ for all $x, y, z, t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $x \circ D(y,t) \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 2.10, we get the required result.

□

**Theorem 2.15.** *Let $R$ be a prime ring. If $F$ is a nonzero symmetric bi-derivation of $R$ such that $[D(x,t), D(y,t)] - (x \circ y) \in Z(R)$ for all $x, y, t \in R$ and $D(Z(R), R) \neq 0$, then $R$ is commutative.*

*Proof.* By hypothesis, we have

$$[D(x,t), D(y,t)] - (x \circ y) \in Z(R), \ \forall\, x, y, t \in R. \qquad (2.30)$$

Since $D(Z(R), R) \neq 0$, there exists $z \in Z(R)$ such that $D(z,t) \neq 0$. Thus $D(z,t) \in Z(R)$ by Lemma 2.1. Replacing $x$ by $zx$ in (2.30), we have

$$
\begin{aligned}
&[D(zx,t), D(y,t)] - (zx \circ y) = [D(z,t)x + zD(x,t), D(y,t)] - z(x \circ y) + [z,y]x \\
&= [D(z,t)x, D(y,t)] + [zD(x,t), D(y,t)] - z(x \circ y) \\
&= D(z,t)[x, D(y,t)] + [D(z,t), D(y,t)]x + z[D(x,t), D(y,t)] + [z, D(y,t)]D(x,t) - z(x \circ y) \\
&= D(z,t)[x, D(y,t)] + z([D(x,t), D(y,t)] - (x \circ y)) \in Z(R) \qquad (2.31)
\end{aligned}
$$

for all $x, y, z, t \in R$. By hypothesis, we have $D(z,t)[x, D(y,t)] \in Z(R)$ for all $x, y, z, t \in R$. Since $R$ is prime and $D(z,t) \in Z(R)$, we have $[x, D(y,t)] \in Z(R)$ for all $x, y \in R$.

Using the same argument of the last part of proof of Theorem 2.11, we get the required result.

□

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