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PREŠIĆ-MAIA TYPE THEOREMS IN ORDERED METRIC SPACES

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Abstract. The purpose of this paper is to prove some fixed point theorems in ordered metric spaces with two comparable metrics. Our results generalize and unify the fixed point theorems of Prešić, Maia and some recent results from ordered metric spaces, into product spaces when underlying space is ordered. An iterative method for constructing the fixed points is also provided.

1. Introduction and preliminaries

In 1922, Banach [5] proved following theorem, known as Banach contraction mapping theorem.

Theorem 1.1. *Let (X, d) be a complete metric space and $f : X \to X$ be a contractive mapping, that is, there exists $\lambda \in [0, 1)$ such that*

$$d(fx, fy) \leq \lambda d(x, y), \text{ for all } x, y \in X.$$

Then f has a unique fixed point, that is, there exists $x^* \in X$ such that $x^* = fx^*$. Moreover, for any $x_0 \in X$ the iterative sequence $x_{n+1} = fx_n$ converges to x^*.

Banach contraction mapping theorem ensure the existence and uniqueness of fixed point of a self map on a complete metric space. Due to simplicity and various applications, several authors generalized the Banach contraction mapping theorem. In 1965 S.B. Prešić [21, 22] extended Banach contraction mapping principle to mappings defined on product spaces and proved following theorem.

Theorem 1.2. *Let (X, d) be a complete metric space, k a positive integer and $f : X^k \to X$ a mapping satisfying the following contractive type condition:*

$$d(f(x_1, x_2, \ldots, x_k), f(x_2, x_3, \ldots, x_{k+1})) \leq \sum_{i=1}^{k} q_i d(x_i, x_{i+1}), \tag{1.1}$$

for every $x_1, x_2, \ldots, x_{k+1} \in X$, where $q_1, q_2, \ldots, q_k$ are nonnegative constants such that $q_1 + q_2 + \cdots + q_k < 1$. Then there exists a unique point $x \in X$ such that $f(x, x, \ldots, x) = x$. Moreover if $x_1, x_2, \ldots, x_k$ are arbitrary points in X and for $n \in \mathbb{N}$,

$$x_{n+k} = f(x_n, x_{n+1}, \ldots, x_{n+k-1}), \tag{1.2}$$

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then the sequence $\{x_n\}$ *is convergent and* $\lim x_n = f(\lim x_n, \lim x_n, \ldots, \lim x_n)$.

Note that condition (1.1) in the case $k = 1$ reduces to the well-known Banach contraction mapping principle. So, Theorem 1.2 is a generalization of the Banach fixed point theorem. Some generalizations and applications of Prešić theorem can be seen in [6, 8, 9, 10, 19, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35].

On the other hand, In 1968 Maia [13] generalized the Banach contraction mapping theorem for spaces with two comparable metrics and proved following theorem

Theorem 1.3. *Let* X *be a metric space with two metrics* d *and* δ *such that* $d(x, y) \leq \delta(x, y)$ *for all* $x, y \in X$. *Furthermore, if* X *is complete with respect to* d, $T : X \to X$ *be a mapping continuous with respect to* d *and a contraction with respect to* δ, *then there exists one and only one fixed point for* T *in* X.

Note that if we replace δ by d in above theorem, we get the Banach contraction mapping theorem. So Theorem 1.3 is a generalization of Banach fixed point theorem. Some generalizations of Maia's theorem can be seen in [12, 15, 36].

The existence of fixed point in partially ordered sets was investigated by Ran and Reurings [24] and then by Nieto and Lopez [16, 17]. Some applications of fixed point theorems in ordered metric spaces to differential equations can be seen in [16, 17]. Several authors generalized the results of these papers in different directions (see, e.g. [1, 2, 3, 7, 11, 18, 23, 26]). The following version of the fixed point theorem was proved, among others, in these papers.

Theorem 1.4. *Let* $(X, \preceq)$ *be a partially ordered set and let* d *be a metric on* X *such that* (X, d) *is a complete metric space. Let* $f : X \to X$ *be a nondecreasing map with respect to* $\preceq$. *Suppose that the following conditions hold:*

(i) *there exists* $k \in (0, 1)$ *such that* $d(fx, fy) \leq kd(x, y)$ *for all* $x, y \in X$ *with* $y \preceq x$;
(ii) *there exists* $x_0 \in X$ *such that* $x_0 \preceq fx_0$;
(iii) f *is continuous.*

Then f *has a fixed point* $x^* \in X$.

The k-step iterative sequence given by (1.2) represents a nonlinear difference equation and the solution of this equation can be assumed to be a fixed point of f, i.e. solution of (1.2) is a point $x^* \in X$ such that $x^* = f(x^*, x^*, \ldots, x^*)$. The Prešić theorem insures the convergence of the sequence $\{x_n\}$ defined by (1.2) and provides a sufficient condition for the existence of solution of (1.2). In the present paper we generalize and unify the results of Prešić, Maia and Theorem 1.4, in product spaces when the underlying space is ordered and we find the solution of equation (1.2) when mapping f satisfies Prešić type contractive condition on ordered space with two comparable metrics. Also we approximate the solution of equation (1.2) with an iterative sequence and obtain the rate of convergence.

Following definitions will be needed in sequel.

Definition 1.5. Let X be any nonempty set, k a positive integer and $f : X^k \to X$ be a mapping. An element $x \in X$ is called a fixed point of f if $f(x, x \ldots, x) = x$.

Definition 1.6. For any mapping $f : X^k \to X$, k a positive integer, we define its associate operator $F : X \to X$ by $Fx = f(x, x, \ldots, x)$ for all $x \in X$. Clearly, $x \in X$ is a fixed point of f if and only if it is a fixed point of F, in the sense of the classical definition. For details see [20] and references therein.

Definition 1.7. Let X be any nonempty set equipped with a partial order relation " $\preceq$ ", k a positive integer and $f : X^k \to X$ be a mapping. A sequence $\{x_n\}$ in X is said to be nondecreasing with respect to " $\preceq$ ", if $x_n \preceq x_{n+1}$ for all $n \in \mathbb{N}$. The mapping f is said to be nondecreasing with respect to " $\preceq$ " if for any finite nondecreasing sequence $\{x_n\}_{n=1}^{k+1}$ we have $f(x_1, x_2, \ldots, x_k) \preceq f(x_2, x_3, \ldots, x_{k+1})$.

Remark 1.8. Note that, if f if nondecreasing, then its associate operator F is nondecreasing. Indeed if $x, y \in X$ and $x \preceq y$ then as f is nondecreasing we obtain

$$F(x) = f(x, \ldots, x) \preceq f(x, \ldots, x, y) \preceq \cdots \preceq f(x, y, \ldots, y) \preceq f(y, \ldots, y) = F(y).$$

Therefore F is nondecreasing.

Definition 1.9. Let X be any nonempty set equipped with a partial order relation " $\preceq$ " and $T : X \to X$ be a mapping. Elements $x, y \in X$ are called comparable if, $x \preceq y$ or $y \preceq x$. A nonempty subset $\mathcal{A}$ of X is said to be well ordered if, for all $a, b \in \mathcal{A}$ we have, $a \preceq b$ or $b \preceq a$ i.e. all elements of $\mathcal{A}$ are comparable.

Definition 1.10. Let X be any nonempty set equipped with a partial order relation " $\preceq$ " such that (X, d) is a metric space. Let k be a positive integer and $f : X^k \to X$ be a mapping. f is said to be continuous if its associate operator F is continuous, i.e. if for every sequence $\{x_n\}$ in X with $\lim_{n\to\infty} x_n = x$, we have $\lim_{n\to\infty} Fx_n = Fx$.

Now we can state our main results.

2. Main results

Theorem 2.1. *Let X be any nonempty set equipped with partial order relation " $\preceq$ ", d and δ are two metrics on X such that (X, d) is complete metric space and $d(x, y) \leq \delta(x, y)$ for all $x, y \in X$ with $x \preceq y$. Suppose $f : X^k \to X$ be a mapping such that the following conditions hold:*

(I)

$$\delta(f(x_1, x_2, \ldots, x_k), f(x_2, x_3, \ldots, x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i \delta(x_i, x_{i+1}), \tag{2.1}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$ with $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq x_{k+1}$, where α_i are nonnegative constants such that $\sum_{i=1}^{k} \alpha_1 < 1$;

(II) *there exists $x_1, x_2, \ldots, x_k \in X$ such that $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq f(x_1, x_2, \ldots, x_k)$;*

(III) *f is continuous in (X,d) and nondecreasing with respect to " $\preceq$ ".*

Then the sequence $\{x_n\}$ defined by (1.2) converges to fixed point u of f. Also the set of fixed points of f is well ordered if and only if f has a unique fixed point. Moreover, if there exists $z_0 \in X$ such that $z_0 \preceq u$ then the iterative sequence $\{z_n\}$ defined by $z_{n+1} = Fz_n$ for all $n \geq 0$, where F is associate operator of f, converges to u as well, with a rate estimated by $d(z_n, u) \leq L\theta^{nk}$, where θ, L are nonnegative constants and $\theta \in [0,1)$.

Proof. Starting with given $x_1, x_2, \ldots, x_k \in X$ we define a sequence $\{x_n\}$ as follows: let $x_{k+1} = f(x_1, x_2, \ldots, x_k)$, then $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq f(x_1, x_2, \ldots, x_k) = x_{k+1}$ i.e. $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq x_{k+1}$, and f is nondecreasing with respect to " $\preceq$ ", we have $f(x_1, x_2, \ldots, x_k) \preceq f(x_2, x_3, \ldots, x_{k+1})$. Let $x_{k+2} = f(x_2, x_3, \ldots, x_{k+1})$, then we have, $x_{k+1} \preceq x_{k+2}$. Continuing this process we obtain sequence $\{x_n\}$ such that

$$x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq x_{k+1} \preceq \cdots \preceq x_n \preceq x_{n+1} \preceq \cdots$$

and

$$x_{n+k} = f(x_n, x_{n+1}, \ldots, x_{n+k-1}), \quad n \in \mathbb{N}.$$

Thus $\{x_n\}$ is a nondecreasing sequence with respect to " $\preceq$ ".

For simplicity set $\delta_n = \delta(x_n, x_{n+1})$, $\mu = max\{\frac{\delta_1}{\theta}, \frac{\delta_2}{\theta^2}, \ldots, \frac{\delta_k}{\theta^k}\}$, where $\theta = [\sum_{i=1}^{k} \alpha_i]^{\frac{1}{k}}$.

By mathematical induction we shall show that

$$\delta_n \leq \mu\theta^n \text{ for all } n \in \mathbb{N}. \tag{2.2}$$

According to the definition of μ it is clear that (2.2) is true for $n = 1, 2, \ldots, k$. Let the following k inequalities

$$\delta_n \leq \mu\theta^n, \delta_{n+1} \leq \mu\theta^{n+1}, \ldots, \delta_{n+k-1} \leq \mu\theta^{n+k-1}$$

be the induction hypothesis.
As $x_n \preceq x_{n+1} \preceq \cdots \preceq x_{n+k}$, so using (2.1) we obtain

$$\begin{aligned}
\delta_{n+k} &= \delta(x_{n+k}, x_{n+k+1}) \\
&= \delta(f(x_n, \ldots, x_{n+k-1}), f(x_{n+1}, \ldots, x_{n+k})) \\
&\leq \alpha_1\delta(x_n, x_{n+1}) + \alpha_2\delta(x_{n+1}, x_{n+2}) + \cdots + \alpha_k\delta(x_{n+k-1}, x_{n+k}) \\
&= \alpha_1\delta_n + \alpha_2\delta_{n+1} + \cdots + \alpha_k\delta_{n+k-1} \\
&\leq \alpha_1\mu\theta^n + \alpha_2\mu\theta^{n+1} + \cdots + \alpha_k\mu\theta^{n+k-1} \\
&\leq \alpha_1\mu\theta^n + \alpha_2\mu\theta^n + \cdots + \alpha_k\mu\theta^n \ (\text{as } \theta = [\sum_{i=1}^{k} \alpha_i]^{\frac{1}{k}} < 1) \\
&= [\sum_{i=1}^{k} \alpha_i]\mu\theta^n \\
&= \mu\theta^{n+k},
\end{aligned}$$

and the inductive proof of (2.2) is complete.
Let $n, m \in \mathbb{N}$ with $m > n$, then

$$\begin{aligned}\delta(x_n, x_m) &\leq \delta(x_n, x_{n+1}) + \delta(x_{n+1}, x_{n+2}) + \cdots + \delta(x_{m-1}, x_m)\\ &= \delta_n + \delta_{n+1} + \cdots + \delta_{m-1}\\ &\leq \mu\theta^n + \mu\theta^{n+1} + \cdots + \mu\theta^{m-1}\\ &\leq [1 + \theta + \theta^2 + \cdots]\mu\theta^n\\ &= \frac{\mu\theta^n}{1-\theta}.\end{aligned}$$

i.e.

$$\delta(x_n, x_m) \leq \frac{\mu\theta^n}{1-\theta} \quad \text{for all} \quad n \in \mathbb{N}.$$

As $d(x, y) \leq \delta(x, y)$ for all $x, y \in X$ with $x \preceq y$ and sequence $\{x_n\}$ is nondecreasing, it follows from above inequality that

$$d(x_n, x_m) \leq \frac{\mu\theta^n}{1-\theta} \quad \text{for all} \quad n \in \mathbb{N}.$$

As $\theta < 1$, it follows from above inequality that $\{x_n\}$ is a Cauchy sequence and (X, d) is complete so there exists $u \in X$ such that

$$\lim_{n\to\infty} d(x_n, u) = 0. \tag{2.3}$$

By continuity of f and (2.3) it follows that

$$\begin{aligned}d(u, Fu) &= d(u, f(u, u, \ldots, u)) = d(u, f(\lim_{n\to\infty} x_n, \lim_{n\to\infty} x_{n+1}, \ldots, \lim_{n\to\infty} x_{n+k-1}))\\ &= \lim_{n\to\infty} d(u, f(x_n, x_{n+1}, \ldots, x_{n+k-1}))\\ &= \lim_{n\to\infty} d(u, x_{n+k})\\ &= 0.\end{aligned}$$

Therefore $Fu = f(u, u, \ldots, u) = u$, i.e. u is a fixed point of f.
Suppose the set of fixed points $\mathcal{A}$ (say) of f is well ordered and $u, v \in \mathcal{A}$. As $\mathcal{A}$ is well ordered, let e.g. $u \preceq v$. Then $f(u, u, \ldots, u) = u, f(v, v, \ldots, v) = v$, from (2.1) it follows that

$$\begin{aligned}\delta(u, v) &= \delta(f(u, u, \ldots, u), f(v, v, \ldots, v))\\ &\leq \delta(f(u, u, \ldots, u), f(u, \ldots, u, v)) + \delta(f(u, \ldots, u, v), f(u, \ldots, u, v, v))\\ &\quad + \cdots + \delta(f(u, v, \ldots, v), f(v, v, \ldots, v))\\ &\leq \alpha_k\delta(u, v) + \alpha_{k-1}\delta(u, v) + \cdots + \alpha_1\delta(u, v)\\ &= [\sum_{i=1}^{k} \alpha_i]\delta(u, v).\end{aligned}$$

As $[\sum_{i=1}^{k} \alpha_i] < 1$ we obtain $\delta(u, v) = 0$ i.e. $u = v$. Thus fixed point is unique.
For converse, if fixed point of f is unique then the set of fixed points of f being singleton therefore well ordered.
Now we approximate the fixed point $u = \lim_{n\to\infty} x_n$. Suppose there exists $z_0 \in X$

such that $z_0 \preceq u = \lim_{n\to\infty} x_n$. As $z_0 \preceq u$ and f is nondecreasing so by Remark 1.8, associate operator F is nondecreasing and

$$z_1 = Fz_0 \preceq Fu = u$$

i.e. $z_1 \preceq u$. Similarly it can be shown that $z_n \preceq u$ for all $n \geq 0$.
As $d(x, y) \leq \delta(x, y)$ for all $x, y \in X$, using (2.1) we obtain

$$\begin{aligned} d(z_n, u) &= d(f(z_{n-1}, \ldots, z_{n-1}), f(u, \ldots, u)) \\ &\leq \delta(f(z_{n-1}, \ldots, z_{n-1}), f(u, \ldots, u)) \\ &\leq \delta(f(z_{n-1}, \ldots, z_{n-1}), f(z_{n-1}, \ldots, z_{n-1}, u)) \\ &\quad + \cdots + \delta(f(z_{n-1}, u, \ldots, u), f(u, \ldots, u)) \\ &\leq [\sum_{i=1}^{k} \alpha_i]\delta(z_{n-1}, u) \\ &= \theta^k \delta(z_{n-1}, u). \end{aligned}$$

Repeating this process we obtain

$$d(z_n, u) \leq L\theta^{nk},$$

where $L = \delta(z_0, u) \geq 0$.
As $\theta < 1$ above inequality shows that $\lim_{n\to\infty} z_n = u$. □

Remark 2.2. For $k = 1$ Theorem 2.1 is an extension and of Maia's theorem [13] in ordered metric spaces, and a generalization of Theorem 1.4. Also Theorem 2.1 is an extension of Prešić's theorem [22] in ordered metric spaces.

Corollary 2.3. *Let X be any nonempty set equipped with partial order relation "$\preceq$", d and δ are two metrics on X such that (X, d) is complete metric space. Suppose $f : X^k \to X$ and $T : X \to X$ be mappings such that $d(x, y) \leq \delta(Tx, Ty)$ for all $x, y \in X$ with $x \preceq y$ and the following conditions hold:*

(I)

$$\delta(Tf(x_1, x_2, \ldots, x_k), Tf(x_2, x_3, \ldots, x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i \delta(Tx_i, Tx_{i+1}), \tag{2.4}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$ with $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq x_{k+1}$, where α_i are nonnegative constants such that $\sum_{i=1}^{k} \alpha_1 < 1$;

(II) *there exists $x_1, x_2, \ldots, x_k \in X$ such that $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq f(x_1, x_2, \ldots, x_k)$;*
(III) *f is continuous in (X, d) and nondecreasing with respect to "$\preceq$";*
(IV) *T is continuous, injective and sequentially convergent.*

Then the sequence $\{x_n\}$ defined by (1.2) converges to fixed point u of f. Also the set of fixed points of f is well ordered if and only if f has a unique fixed point. Moreover, if there exists $z_0 \in X$ such that $z_0 \preceq u$ then the iterative sequence $\{z_n\}$ defined by $z_{n+1} = Fz_n$ for all $n \geq 0$, where F is associate operator of f, converges to u as well, with a rate estimated by $d(z_n, u) \leq L\theta^{nk}$, where θ, L are nonnegative constants and $\theta \in [0, 1)$.

Proof. Define a mapping $D : X \times X \to [0, \infty)$ by

$$D(x, y) = \delta(Tx, Ty) \quad \text{for all } x, y \in X.$$

Then (X, D) is a metric space (see [4]). Note that condition (2.4) reduces to

$$D(f(x_1, x_2, \ldots, x_k), f(x_2, x_3, \ldots, x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i D(x_i, x_{i+1}),$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$ with $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq x_{k+1}$. Also $d(x, y) \leq \delta(Tx, Ty)$ reduces to $d(x, y) \leq D(x, y)$ for all $x, y \in X$ with $x \preceq y$. Therefore the rest part of proof follows by Theorem 2.1. □

Following theorem is a generalization of theorems of Maia and Prešić in metric spaces.

Theorem 2.4. *Let X be any nonempty set, d and δ are two metrics on X such that (X, d) is complete metric space and $d(x, y) \leq \delta(x, y)$ for all $x, y \in X$. Suppose $f : X^k \to X$ be a mapping such that f is continuous in (X, d) and*

$$\delta(f(x_1, x_2, \ldots, x_k), f(x_2, x_3, \ldots, x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i \delta(x_i, x_{i+1}), \tag{2.5}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$, where α_i are nonnegative constants such that $\sum_{i=1}^{k} \alpha_1 < 1$. Then the sequence $\{x_n\}$ defined by (1.2) converges to unique fixed point u of f. Moreover, the iterative sequence $\{z_n\}$ defined by $z_{n+1} = Fz_n$ for all $n \geq 0$, $z_0 \in X$, where F is associate operator of f, converges to u as well, with a rate estimated by $d(z_n, u) \leq L\theta^{nk}$, where θ, L are nonnegative constants and $\theta \in [0, 1)$.

Proof. Note that (2.5) holds for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$, therefore proof of this theorem follows from a similar process as used in the proof of Theorem 2.1. □

In the following theorem condition on f "nondecreasing", continuous and the completeness of (X, d) is replaced by another condition.

Theorem 2.5. *Let X be any nonempty set equipped with partial order relation " $\preceq$ ", d and δ are two metrics on X. Suppose $f : X^k \to X$ be a mapping such that the following conditions hold:*

(I)

$$\delta(f(x_1, x_2, \ldots, x_k), f(x_2, x_3, \ldots, x_{k+1})) \leq \sum_{i=1}^{k} \alpha_i d(x_i, x_{i+1}), \tag{2.6}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$ with $x_1 \preceq x_2 \preceq \cdots \preceq x_k \preceq x_{k+1}$, where α_i are nonnegative constants such that $\sum_{i=1}^{k} \alpha_1 < 1$;

(II) *there exists $u \in X$ such that $d(u, f(u, u, \ldots, u)) \leq \delta(x, f(x, x, \ldots, x))$ for all $x \in X$ and $u \preceq f(u, u, \ldots, u)$.*

Then f has a fixed point.

Proof. Let u be the given point with property (II). Let $z = f(u, u, \ldots, u)$, then by (II) $u \preceq z$. If $z = u$ then u is a fixed point of f. If not then $d(u, z) > 0$ and it follows from (2.6) that

$$\begin{aligned}
\delta(z, f(z, z, \ldots, z)) &= \delta(f(u, u \ldots, u), f(z, z, \ldots, z)) \\
&\leq \delta(f(u, u \ldots, u), f(u, \ldots, u, z)) + \delta(f(u, \ldots, u, z), f(u, \ldots, z, u, u)) \\
&\quad + \cdots + \delta(f(u, z, \ldots, z), f(z, z, \ldots)) \\
&\leq \alpha_k d(u, z) + \alpha_{k-1} d(u, z) + \cdots + \alpha_1 d(u, z) \\
&= [\sum_{i=1}^{k} \alpha_k] d(u, z) \\
&= [\sum_{i=1}^{k} \alpha_k] d(u, f(u, u, \ldots, u)) \\
&< d(u, f(u, u, \ldots, u)) \quad (as \sum_{i=1}^{k} \alpha_k < 1)
\end{aligned}$$

a contradiction. Therefore we must have $d(u, z) = 0$ i.e. $z = f(u, u, \ldots, u) = u$. Thus u is a fixed point of f.

□

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