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ON DISTANCE k-DOMINATION NUMBER OF GRAPHS UNDER PRODUCT OPERATIONS

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Abstract. For a positive integer k, a subset $S \subseteq V(G)$ is said to be a k-dominating set of G if every vertex $v \in V(G) \setminus S$ is at a distance of at most k from some vertex of S. The cardinality of the minimum k- dominating set of G is the k-domination number, denoted by $\gamma_k(G)$. In this article, we determine the distance k-domination number for two graphs under some product operations in terms of its factor graphs.

1. Introduction and Preliminaries

Graphs considered in this paper are nontrivial, simple, finite, connected and undirected unless otherwise stated. A subset S of a vertex set $V(G)$ is called a dominating set of G if for every $v \in V(G)\backslash S$, there exists a vertex $x \in S$ such that xv is an edge of G. For every vertex $v \in V(G)$, the *open neighbourhood* $N(v)$ is the set of all vertices adjacent to v in G. The *closed neighbourhood* $N[v]$ of v is defined as $N[v] = N(v) \cup \{v\}$. The distance $d(u, v)$ between two vertices u and v is the length of the shortest path between u and v in G. The *open k-neighbourhood* set $N_k(v)$ of a vertex $v \in V(G)$ is the set of all vertices of G which are different from v and at distance at most k from v in G. That is, $N_k(v) = \{u \in V(G) : 1 \leq d(u, v) \leq k\}$. The *closed k-neighbourhood* $N_k[v]$ of v is defined as $N_k[v] = N_k(v) \cup \{v\}$. Note that $N(v) = N_1(v)$. For any $S \subseteq V(G)$ and $u \in S$, $d(S, v) = min\{d(u, v) : u, v \in S\}$ and $N_k[S] = \{v : d(S, v) \leq k\}$. The problem of finding a minimal *distance k-dominating set* was considered by Slater[14] with special reference to communication networks while the distance k-dominating set was defined by Henning et al. [9]. For an integer $k \geq 1$, a set $S \subseteq V(G)$ is a *distance k-dominating set* of G if every vertex in $V(G) \setminus S$ is within distance k from some vertex $v \in S$. That is, $N_k[S] = V(G)$. The *minimum cardinality* among all *distance k-dominating sets* of G is called the *distance k-domination number* of G and is denoted by $\gamma_k(G)$. We note here our awareness of the fact that the symbol $\gamma_k(G)$ which we are using for *distance k-domination number*, has been used by other authors for k-domination number (which is the size of a smallest set of vertices with the property that every vertex not in the set

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is adjacent to at least k that are in the set). This can be seen for example in [2], [3], [5], [6], [12]. In order to avoid confusion, we will throughout refer to $\gamma_k(G)$ explicitly as the *distance k-domination number*.

For any graph G, determining $\gamma_k(G)$ is an NP-hard problem even for bipartite graphs [8]. Many authors have extensively studied the concept of *distance k-dominating set* to analyse the distance parameters in various contexts and graph constructions; for example, [4], [7], [13] . For additional information on *distance k-domination*, readers can refer the survey by Henning [10].

For the three graph products (namely: Cartesian, strong, lexicographic) of G and H, the vertex set is $V(G) \times V(H)$. Their edge sets are defined as follows. In the *Cartesian product* $G\Box H$, two vertices (g_1, h_1) and (g_2, h_2) are adjacent if either (a) $g_1g_2 \in E(G)$ and $h_1 = h_2$ or (b) $g_1 = g_2$ and $h_1h_2 \in E(H)$. On the other hand, two distinct vertices (g_1, h_1) and (g_2, h_2) are adjacent with respect to the *strong product* $G \boxtimes H$ if, (a) $g_1 = g_2$ and $h_1h_2 \in E(H)$, or (b) $h_1 = h_2$ and $g_1g_2 \in E(G)$, or (c) $g_1g_2 \in E(G)$ and $h_1h_2 \in E(H)$. Finally, two vertices (g, h) and (g', h') are adjacent in the *lexicographic product* $G \circ H$ (also $G[H]$) either if $gg' \in E(G)$ or if $g = g'$ and $hh' \in E(H)$. For $* \in \{\Box, \boxtimes, \circ\}$ we call the product $G * H$ *nontrivial*, if both G and H have at least two vertices. For $h \in V(H)$, $g \in V(G)$, and $* \in \{\Box, \boxtimes, \circ\}$, we denote $G^h = \{(g, h) \in G * H : g \in V(G)\}$ a G *layer* in $G * H$, and ${}^gH = \{(g, h) \in G * H : h \in V(H)\}$ an H *layer* in $G * H$. Note that the subgraph of $G * H$ induced on G^h is isomorphic to G and the subgraph of $G * H$ induced on ${}^gH$ is isomorphic to H for $* \in \{\Box, \boxtimes, \circ\}$. The map $p_G : V(G * H) \to V(G)$ defined with $p_G((g, h)) = g$ is called a *projection map* onto G for $* \in \{\Box, \boxtimes, \circ\}$. Similarly we can define the *projection map* onto H.

In the remainder of the article, we investigate the *distance k-domination number* of graphs under various operations. This includes: Cartesian Product, Strong Product, Lexicographic Product and Corona Product.

2. Cartesian Product of Graphs

Following lemma is a direct consequence of the definition of a projection map and the distance k-dominating set of graphs under the *Cartesian product*.

Lemma 2.1. *Let G and H be two connected graphs and S be any distance k-dominating set of $G\Box H$. Then, $p_G(S)$ and $p_H(S)$ are distance k-dominating sets of G and H, respectively.*

Proof. Let S be any distance k-dominating set of $G\Box H$. Then for any $(u, v) \in V(G\Box H)$, there exists $(g, h) \in S$ with $d_{G\Box H}((u, v), (g, H)) \leq k$. By the distance formula of Cartesian product of graphs, we have $d_{G\Box H}((u, v), (g, h)) = d_G(u, g) + d_H(v, h) \leq k$. This implies that $d_G(u, g) \leq k$ and $d_H(v, h) \leq k$, where $g \in p_G(S), h \in p_H(S)$, $u \in V(G)$ and $v \in V(H)$. And this is true for any $u \in V(G)$ and $v \in V(H)$. i. e., for any $u \in V(G)$ and $v \in V(H)$, we have $d_G(u, p_G(S) \leq k$ and $d_H(v, p_H(S) \leq k$. Therefore $p_G(S)$ and $p_H(S)$ are distance k-dominating sets of G and H, respectively. $\Box$

Theorem 2.2. *Let G and H be two connected graphs and let S_1 and S_2 be two distance k-dominating sets of G and H respectively. Then, $S_1 \times S_2$ is a distance $2k$-dominating set of $G\Box H$.*

Proof. Let S_1 and S_2 be the distance k-dominating sets of G and H respectively. Then, our goal is to prove $S_1 \times S_2$ is a distance $2k$-dominating set of $G\Box H$. It is worth noting that $\max\{d_G(S_1, g) : g \in V(G)\} \leq k$ and $\max\{d_H(S_2, h) : h \in V(G)\} \leq k$. In addition, the distance formula in the Cartesian product of graphs says that for any two vertices $(g_1, h_1), (g_2, h_2) \in V(G\Box H)$, we have $d_{G\Box H}((g_1, h_1), (g_2, h_2)) = d_G(g_1, g_2) + d_H(h_1, h_2)$. In that case, we can conclude that $\max\{d_{G\Box H}((S_1 \times S_2, (g, h)) : g \in V(G), h \in V(H)\} \leq 2k$. Therefore, $S_1 \times S_2$ is a distance $2k$-dominating set of $G\Box H$. $\square$

Corollary 2.3. *If S_1 and S_2 are distance k-dominating sets of G and H respectively, then $S_1 \times S_2$ is a distance k-dominating set of $G\Box H$ if and only if $max\{d_G(S_1, g) : g \in V(G)\} \leq \lfloor k/2 \rfloor$ and $max\{d_H(S_2, h) : h \in V(H)\} \leq \lfloor k/2 \rfloor$.*

Lemma 2.4. *If S_1 and S_2 are distance k-dominating sets of G and H, respectively, then $S_1 \times V(H)$ and $V(G) \times S_2$ are distance k-dominating set of $G\Box H$.*

Proof. Assume that S_1 and S_2 are distance k-dominating sets of G and H, respectively. Consider $S_1 \times V(H)$. Then, we have to prove for any $(g, h) \in V(G\Box H)$, $d_{G\Box H}(S_1 \times V(H), (g, h)) \leq k$. Let (g, h) be any vertex of $G\Box H$. Since S_1 is a distance k-dominating set of G, in the G-layer G^h, the distance between $S_1 \times \{h\}$ and (g, h) will be less than or equal to k. However, $S_1 \times \{h\} \subseteq S_1 \times V(H)$, it follows that $d_{G\Box H}(S_1 \times V(H), (g, h)) \leq k$. By similar argument, we can prove that $d_{G\Box H}(V(G) \times S_2, (g, h)) \leq k$. So, $S_1 \times V(H)$ and $V(G) \times S_2$ are distance k-dominating sets of $G\Box H$. $\square$

Theorem 2.5. *Let G and H be two connected graphs. Then, $\gamma_k(G)\gamma_k(H) \leq \gamma_k(G\Box H) \leq min\{\gamma_k(G)m, n\gamma_k(H)\}$, where n and m are the orders of G and H, respectively .*

Proof. First, we have to prove $\gamma_k(G)\gamma_k(H) \leq \gamma_k(G\Box H)$. Assume for a moment that S is a minimum distance k-dominating set with $|S| < \gamma_k(G)\gamma_k(H)$. Then $|p_G(S)| < \gamma_k(G)$ or $|p_H(S)| < \gamma_k(H)$. Assume $|p_G(S)| < \gamma_k(G)$. Then there is a vertex $g \in V(G)$ with $d(p_G(S), g) > k$. Then by the distance formula for Cartesian product of graphs, for any $h \in V(H)$, $d((g, h), S) > k$. This contradicts the fact that S is a minimum distance k-dominating set of $G\Box H$. A similar contradiction will get when $|p_H(S)| < \gamma_k(H)$. Therefore $\gamma_k(G)\gamma_k(H) \leq \gamma_k(G\Box H)$.

Previous Lemma says that if S_1 and S_2 are distance k-dominating sets of G and H respectively, then $S_1 \times V(H)$ and $V(G) \times S_2$ are distance k-dominating sets of $G\Box H$. Now, it is worth noting that S_1 and S_2 are minimum distance k-dominating sets of G and H respectively, and suffices to say that $S_1 \times V(H)$ and $V(G) \times S_2$ are distance k-dominating sets of $G\Box H$. Thus, we conclude that $\gamma_k(G\Box H) \leq min\{\gamma_k(G)m, n\gamma_k(H)\}$. $\square$

The lower bound of Theorem 2.5 is sharp. For any non-zero integer r with $\gamma_r(G) = 1 = \gamma_r(H)$, let $k = 2r$. Then $\gamma_k(G) = 1 = \gamma_k(H)$ and by the distance formula of Cartesian product of graphs $\gamma_k(G\Box H) = 1$. In that case $\gamma_k(G)\gamma_k(H) = 1 = \gamma_k(G\Box H)$. For the sharpness of the upper bound, take G and H as paths, each with radius k.

3. Strong Product of Graphs

Theorem 3.1. *Let G and H be two connected graphs and let S be any distance k-dominating set of $G \boxtimes H$. Then, $p_G(S)$ and $p_H(S)$ are distance k-dominating sets of G and H respectively.*

Proof. Assume for a moment that $p_G(S)$ is not a distance k-dominating set of G. Then there exists some $g \in V(G)$ with $d_G(p_G(S), g) > k$. Consequently, we can find a vertex $g' \in V(p_G(S))$ with $min\{(d_G(g^\star, g) : g^\star \in V(p_G(S))\} = d_G(g', g) > k$. Now, $g' \in V(p_G(S))$ implies that there exists some $h \in V(H)$ with $(g', h) \in S$. In this case, $k < d_{G\boxtimes H}((g', h), (g, h)) = max\{d_G(g', g), d_H(h, h)\} = d_G(g', g)$ implies $k < d_G(g', g) = d_{G\boxtimes H}(S, (g, h))$. This contradicts the fact that S is a distance k-dominating set of $G \boxtimes H$. Therefore, $d_G(p_G(S), g) \leq k$ for any $g \in V(G)$, and the result follows. $\square$

Theorem 3.2. *Let G and H be two connected graphs. Then $S_1 \times S_2$ is a distance k-dominating set of $G \boxtimes H$ if and only if S_1 and S_2 are distance k-dominating sets of G and H respectively.*

Proof. ($\Rightarrow$) Assume that $S_1 \times S_2$ is a distance k-dominating set of $G\boxtimes H$. Then, by virtue of Theorem 6, we can conclude that $p_G(S_1 \times S_2) = S_1$ and $p_H(S_1 \times S_2) = S_2$ are k-dominating sets of G and H, respectively.

($\Leftarrow$) Let S_1 and S_2 be the two distance k-dominating sets of G and H, respectively. Seeking a contradiction, assume that $S_1 \times S_2$ is not a distance k-dominating set of $G \boxtimes H$. Then, there exists a vertex $(g, h) \in V(G \boxtimes H)$ with $d_{G\boxtimes H}(S_1 \times S_2, (g, h)) > k$. Now, $d_{G\boxtimes H}(S_1 \times S_2, (g, h)) > k$ implies $\min\{d_{G\boxtimes H}((g', h'), (g, h)) : (g', h') \in S_1 \times S_2\} > k$. Assume that $\min\{d_{G\boxtimes H}((g', h'), (g, h))/(g', h') \in S_1 \times S_2\} = d_{G\boxtimes H}((g^\star, h^\star), (g, h))$. Then, it follows that $d_{G\boxtimes H}((g^\star, h^\star), (g, h)) > k$ implies that $\max\{d_G(g^\star, g), d_H(h^\star, h)\} > k$. In that case, it follows that $d_G(g^\star, g) > k$ or $d_H(h^\star, h) > k$ which means that $d_G(S_1, g) > k$ or $d_H(S_2, h) > k$. This contradicts the fact that S_1 and S_2 are distance k-dominating sets of G and H, respectively. Hence, $S_1 \times S_2$ is a distance k-dominating set of $G \boxtimes H$. $\square$

Theorem 3.3. *Let G and H be two nontrivial connected graphs. Then,$\gamma_k(G \boxtimes H) = \gamma_k(G)\gamma_k(H)$.*

Proof. Let S_1 and S_2 be the minimum distance k-dominating sets of G and H, respectively. By Theorem 2.7, it directly follows that $S_1 \times S_2$ is a distance k-dominating set of $G \boxtimes H$. Seeking a contradiction, assume that S is a minimum distance k-dominating set of $G \boxtimes H$ with $|S| < S_1 \times S_2$. Then either $|p_G(S)| < |S_1|$ or $|p_H(S)| < |S_2|$. Assume for a moment that $|p_G(S)| < |S_1|$, then $p_G(S)$ will not be a distance k- dominating set of G. Then there exists some $g \in V(G)$ with $d(p_G(S), g) > k$. Now, for any $h \in V(H)$, we have $d(S, (g, h)) = max\{d(p_G(S), g), d(p_H(S), h)\} > k$. This contradicts the fact that S is a distance k-dominating set of $G \boxtimes H$. Hence the result. $\square$

Theorem 3.4. *Let G and H be two nontrivial connected graphs. If $k = min\{diam(G), diam(H)\}$, then*

$$\gamma_k(G \boxtimes H) = \begin{cases} \gamma_k(G), & \text{if } k{=}diam(H) \\ \gamma_k(H), & \text{if } k{=}diam(G). \end{cases}$$

Proof. Assume for a moment that $k = min\{diam(G), diam(H)\} = diam(H)$. From the definition of distance k-dominating set, if k is the diameter of H, then $\gamma_k(H) = 1$. Let S_1 be the smallest distance k-dominating set of G. i.e., $\gamma_k(G) = |S_1|$. If v is a centre vertex of H, then $\{v\}$ will be a distance k-dominating set of H.

Claim: $S_1 \times \{v\}$ is the smallest distance k-dominating set of $G \boxtimes H$.

Now we have to prove $S_1 \times \{v\}$ is a distance k-dominating set of $G \boxtimes H$. Let $(x, y) \in V(G \boxtimes H)$. Since $x \in V(G)$ and S_1 is the distance k-dominating set of G, $d(S_1, x) \leq k$. Since $y \in V(H)$ and v is a distance k-dominating set of H, then $d(y, v) \leq k$ and $d(S_1 \times \{v\}), (x, y)) = max\{d(S_1, x), d(v, y)\} \leq k$. Since (x, y) is an arbitrary vertex of $G \boxtimes H$, $S_1 \times \{v\}$ is a distance k-dominating set of $G \boxtimes H$. Therefore, $\gamma_k(G) \leq \gamma_k(G \boxtimes H)$. Now to prove $S_1 \times \{v\}$ is the smallest distance k-dominating set of $G \boxtimes H$. Assume for a moment that S is the smallest distance k-dominating set of $G \boxtimes H$ with $|S| < \gamma_k(G)$. Then $|p_G(S)| < \gamma_k(G)$ and hence $p_G(S)$ will not be a distance k-dominating set of G. i.e., there exists a vertex $g \in V(G)$ with $d(g, p_G(S)) > k$. Then, by the distance formula for strong product of graphs, we can say that $d(S, (g, h)) > k$ for any $h \in V(H)$, which contradicts the fact that S is a distance k-dominating set of $G \boxtimes H$. Therefore $\gamma_k(G \boxtimes H) \geq \gamma_k(G)$. Using similar argument, we can prove that $\gamma_k(G \boxtimes H) = \gamma_k(H)$ if $k = diam(G)$. □

We know that $diam(G \boxtimes H) = max\{diam(G), diam(H)\}$. So, if we consider k as $max\{diam(G), diam(H)\}$, then each centre vertex will be a distance k-dominating set for $G \boxtimes H$. Hence the following remark.

Remark 3.5. If $k = max\{diam(G), diam(H)\}$, then $\gamma_k(G \boxtimes H) = 1$.

4. Lexicographic Product of Graphs

Proposition 4.1. *Let G and H be two connected graphs and let S be a distance k-dominating set of G, then for any $h \in V(H)$, $S \times \{h\}$ is a distance k-dominating set of $G \circ H$.*

Proof. Let S be a distance k-dominating set of G. Then, for any $g \in V(G)$, $d(S, g) \leq k$. For any $(g_1, h_1), (g_2, h_2) \in V(G \circ H)$, we have the following distance formula from [8].

$$d((g_1, h_1), (g_2, h_2)) = \begin{cases} d_G(g_1, g_2) & \text{if } g_1 \neq g_2 \\ min\{d_H(h_1, h_2), 2\} & g_1 = g_2 \text{ and } deg_G(g_1) > 0 \\ d_H(h_1, h_2) & \text{otherwise} \end{cases}$$

From this formula we can say that $d_{G \circ H}(S \times \{h\}, (g, h)) \leq k$, for any $(g, h) \in V(G \circ H)$. □

Lemma 4.2. *If S is a distance k-dominating set of a lexicographic product of two non trivial connected graphs G and H, then $p_G(S)$ will be a distance k-dominating set of G.*

Proof. Seeking for contradiction. Assume that S is a distance k-dominating set of $G \circ H$ and $p_G(S)$ is not a k-dominating set of G. Then, there exists a vertex $g \in V(G)$ with $d_G(p_G(S), g) > k$. More precisely there exists some vertex $g' \in p_G(S))$ with $d_G(g', g) > k$. Because $g' \in p_G(S))$, there exists a vertex $h \in V(H)$ with $(g', h) \in S$. Then, by the distance formula of lexicographic product of graphs, we can say that $d_{G \circ H}((g', h), (g, h)) > k$. i.e., $d_{G \circ H}(S, (g, h)) > k$. This contradicts the fact that S is a distance k-dominating set of $G \circ H$. Therefore, $p_G(S)$ is a k-dominating set of G. □

Theorem 4.3. *Let G and H be two connected graphs. Then, $\gamma_k(G \circ H) = \gamma_k(G)$*

Proof. From Proposition 4.1, it is evident that $\gamma_k(G \circ H) \leq \gamma_k(G)$. Seeking for contradiction. Assume for a moment that S is the smallest distance k-dominating set of $G \circ H$ with $|S| < \gamma_k(G)$. Then $|p_G(S)| < \gamma_k(G)$. In view of Lemma 2.1, $p_G(S)$ is a distance k-dominating set of G and its cardinality is less than the k-domination number $\gamma_k(G)$, a contradiction. Therefore $\gamma_k(G \circ H) \geq \gamma_k(G)$ and the result follows. □

5. Corona Product of Graphs

Let G_1 and G_2 be two nontrivial connected graphs with orders n_1 and n_2 respectively. The corona product $G_1 \odot G_2$ of G_1 and G_2 is obtained by taking one copy of G_1 and n_1 copies of G_2 and by joining each vertex of the i-th copy of G_2 to the i-th vertex of G_1, where $1 \leq i \leq n_1$. The corona product of G_1 and G_2 has total $(n_1 n_2 + n_1)$ number of vertices. The degree of a vertex v of $G_1 \odot G_2$ is given by

$$deg_{G_1 \odot G_2}(v) = \begin{cases} deg_{G_1}(v) + n_2, & v \in V(G_1); \\ deg_{G_2}(v) + 1, & v \in V(G_{2,i}), i = 1, 2, ..., n_1. \end{cases},$$

where $G_{2,i}$ is the i-th copy of the graph G_2.

Theorem 5.1. *For any two non-trivial connected graphs G_1 and G_2, the distance k-domination number of the corona product $G_1 \odot G_2$ is at most the distance $(k-1)-$domination of G_1.*

Proof. Let S be a distance k-dominating set of G_1. Then, we have to prove that for any vertex $v \in V(G_1 \odot G_2) \setminus S$, $d_{G_1 \odot G_2}(S, v) \leq k+1$. Note that the distance between two vertices in G_1 is same as that in the corona product $G_1 \odot G_2$. Since S is a distance k-dominating set of G_1, we get $d_{G_1 \odot G_2}(S, u) \leq k < k+1$. Now, let r_i be a vertex in the i^{th} G_2-layer, and r be the vertex in G_1 to which the i^{th} G_2-layer is attached. Then, $d_{G_1 \odot G_2}(S, r) \leq k$ and $d_{G_1 \odot G_2}(r, r_i) = 1$. Therefore, $d_{G_1 \odot G_2}(S, r_i) \leq k+1$ and S is a distance $(k+1)$-dominating set of $G_1 \odot G_2$. □

The following corollaries are the immediate consequence of Theorem 5.1.

Corollary 5.2. *Let G be a graph and K_n be the complete graph of order $n \geq 1$. Then, $\gamma_k(G + K_n) = 1$.*

The following definition is needed for our next result. Let $[N_k(S)] = \{v \in V(G) : d_G(S, v) = k\}$

Corollary 5.3. *If G_1 and G_2 are two nontrivial connected graphs, then $\gamma_k(G_1 \odot G_2) \leq \gamma_k(G_1) + |[N_k(S)]|$.*

Proof. Assume that G_1 and G_2 are two nontrivial connected graphs and let S be a minimum distance k-dominating set of G_1. Then, for any $v \in V(G_1)$, $d_{G_1}(S, v) = d_{G_1 \odot G_2}(S, v) \leq k$. Consider a vertex u_i in the i^{th} G_2-layer and let u be the vertex of G_1 to which the i^{th} G_2-layer is attached. If $d_{G_1}(S, u) < k$, then $d_{G_1 \odot G_2}(S, u_i) \leq k$. The other possibility is $d_{G_1}(S, u) = k$. In this case $d_{G_1 \odot G_2}(S, u_i) = k + 1$. So, $d_{G_1 \odot G_2}(S \cup [N_k(S)], u_i) \leq k$ and $S \cup [N_k(S)]$ is a distance k-dominating set of $G_1 \odot G_2$. Hence it follows that $\gamma_k(G_1 \odot G_2) \leq \gamma_k(G_1) + |[N_k(S)]|$. $\square$

6. Conclusion

This paper characterizes the distance k-dominating set of graphs under various graph operations, including Cartesian, Strong, Lexicographic and Corona products. Further studies will explore additional graph operations, such as Tensor product. In future, we would like to extend the definition of the distance k-dominating set by incorporating the concepts of outer-connected and outer-convex dominating sets in graphs and to characterize these new domination parameters under various graph operations.

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