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SECURE EQUITABLE SUBDIVISION NUMBER OF GRAPHS

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ABSTRACT. In this paper, we initiate the study of a new domination parameter: the secure equitable subdivision number of graphs. Moreover, we define the secure equitable subdivision critical graphs and the secure equitable subdivision stable graphs. Furthermore, we investigate how to construct a secure equitable subdivision critical graph.

1. INTRODUCTION

All the graphs that are taken into consideration in this article, if not indicated, are simple connected graphs G having a vertex set $V(G)$ and an edge set $E(G)$. Any set of vertices that are close to a vertex v in G is called its *neighborhood* $N(v)$ and the *closed neiborhood* $N[v] = N(v) \cup \{v\}$. An *equitable neighborhood* $N_e(v)$ of a vertex v is the collection of all vertices $x \in N(v)$ with $|deg(v) - deg(x)| \leq 1$. A vertex v is said to be an *equitable isolate* if for any $x \in N(v)$ we have $x \notin N_e(v)$. The minimum and maximum degrees in a graph G are represented by the symbols $\delta(G)$ and $\Delta(G)$, respectively. A wheel graph of order $n+1$ is denoted by $W_{1,n}$.

A set $S \subset V(G)$ is called a *dominating set* of G, if for every vertex $v \in V(G) \setminus S$ there exists at least one vertex $u \in S$ such that $u \in N(v)$. The minimum number of vertices that can dominate a graph G is called the *domination number* of G. The dominating vertices are considered to be the guards that protect the entire vertices of a graph. For any dominating set S of a graph G, $x \in S$ and $y \in V(G) \setminus S$, the vertex y is said to be S-*external private equitable neighbor* of x if $N(y) \cap S = \{x\}$ and $|deg(y) - deg(x)| \leq 1$. The set of all S-external private neighbors of the vertex x is denoted by $pen(x, S)$. If x protects y, then $pen(x, S) \subseteq N[y]$. The union of all S-external private equitable neighborhoods in G is denoted by $Z = \cup_{i=1}^{n} pen(x_i, S)$, where $S = \{x_i : 1 \leq i \leq n\}$. Several domination parameters have been studied (see [1],[3],[9]). For any basic definitions and concepts in graph domination, we refer to [7].

An edge uv in a graph G is said to be subdivided, if uv is split into two edges up and pv by adjoining a new vertex p common to both the new edges. The introduction of the new vertex may or may not alter the domination number of a graph. This notion was studied by Arumugam in the year 2000. He initiated the study of

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the parameter domination subdivision number in graphs. The minimum number of edges that must be subdivided (once) to increase the domination number is called the *domination subdivision number*, denoted by $sd_{\gamma}(G)$. The parameter is further studied in [6] and [4]. The concept of domination subdivision number is widely studied for varied domination parameters. Divya Rashmi, Somasundaram and Arumugam studied the secure domination subdivision number of graphs in [5]. Sumathi and Alarmelumangai established the equitable domination subdivision number in [11]. The study of total domination subdivision number was initiated by haynes, Henning and Hopkins in [8]. Atapour, Sheikholeslami and Khodkar instituted and investigated the roman domination subdivision number in [2]. Many more parameters were grounded on the idea of domination subdivision number.

The secure equitable domination is an evolved strategy of domination in graphs, which is abstracted from the idea of the secure domination and the equitable domination in graphs. A dominating set S of a graph G is called a *secure equitable dominating set* if for each vertex $x \in V(G) \setminus S$ there exists at least one vertex $y \in S$ such that y belongs to the equitable neighborhood of x and the vertex y can be replaced by a vertex x without losing domination property[10]. Whenever we say a vertex u defends v, it denotes that the vertex v is secure equitably protected by the vertex u. Also, swapping the vertex u with v indicates that we are replacing one vertex with the other. We use the notation SEDS to denote a secure equitable dominating set.

In this paper, we study a graph's secure equitable domination subdivision number. Moreover, in detail, we discuss how an edge's subdivision can alter a graph's secure equitable domination number. We also initiate the study of the secure equitable subdivision critical graphs and the secure equitable subdivision safe graphs.

2. Secure Equitable Domination Subdivision Number

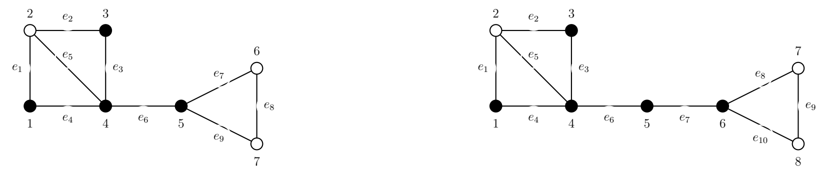
This section defines the secure equitable subdivision number and the secure equitable subdivision set of a graph.

Definition 2.1. The minimum number of edges that must be subdivided once to increase the secure equitable domination number of a graph G is called the secure equitable domination subdivision number (se-subdivision number) of G and is denoted by $sd_{\gamma_{se}}(G)$.

Definition 2.2. The set of all edges that are subdivided to obtain the se-subdivision number of a graph G is called secure equitable domination subdivision set (se-subdivision set) of G and is denoted by $sd_{S_{se}}(G)$.

Example 2.3. In Figure 1(A), the secure equitable domination in a graph G is demonstrated. The graph H (Figure 1(B)) is obtained by subdividing the edge $e_6 \in E(G)$. We can see that $\gamma^e_{sec}(H) = \gamma^e_{sec}(G) + 1$. Hence, $sd_{\gamma_{se}}(G) = 1$ and $sd_{S_{se}}(G) = \{e_6\}$

We now investigate how edge subdivision affects a graph's secure equitable domination number.



(A) $\gamma^e_{sec}(G) = 4$ (B) $\gamma^e_{sec}(H) = 5$

FIGURE 1

Lemma 2.4. *Let H be a graph obtained from a graph G by subdividing an edge $uv \in E(G)$ and adding a vertex p. If S is a SEDS of G and $u, v \in S$, then $\gamma^e_{sec}(H) = \gamma^e_{sec}(G)$ if and only if any of the following condition holds:*

- (i) *uv is a stem with a support vertex of degree greater than 3.*
- (ii) *the newly added vertex p belongs to the external private equitable neighborhood of at least one of the incident vertices, say v, and for each $w \in N(v)$, $w \notin S$ we have $N(w) \cap (S - \{v\}) \neq \phi$.*
- (iii) *the newly added vertex does not belong to the external private equitable neighborhood of any of the incident vertices of the subdivided edge but at least for one incident vertex, say u, $p \in N_e(u)$ and for each $w \in N(u)$, $w \notin S$ we have $N(w) \cap (S - \{u\}) \neq \phi$.*

Proof. Let p be a vertex that divides the edge uv in G. Let the vertices u and v be in SEDS such as $uv \in E(G)$. Let S and S' stand for SEDS of G and H, respectively.

Suppose u is a leaf of G and v is the support vertex of u with $deg(v) > 3$. Then, u is the only vertex in S which defends p, since $|deg(u) - deg(p)| \leq 1$ and $p \in N(u)$. Moreover, $p \in pen(u, S)$. Hence, $p \notin S'$ and $\gamma^e_{sec}(H) = \gamma^e_{sec}(G)$. Thus, (i) s proved.

Suppose $p \in pen(v, S)$. Then, $p \notin N_e(v)$. If we swap the vertices p and v, every vertex $w \in N(v)$ outside the SEDS will be protected since $N(w) \cap (S - \{v\}) = \phi$. Moreover, the new set of dominating vertices forms a SEDS of H and hence, $\gamma^e_{sec}(H) = \gamma^e_{sec}(G)$. Furthermore, if $p \in pen(v, S)$, $p \in pen(u, S)$ and $|pen(u, S)| > 1$, then p can be only swapped with v. Suppose, on the contrary, p is swapped with u. Then, at least one vertex $z \in N(u)$ is left unprotected, since $z \in pen(u, S)$. Hence, (ii) is proved.

If $p \in N_e(u)$ but $p \notin N_e(v)$ then, p can be swapped with u and the swapping will not affect the secure equitability of the graph, since every vertex $z \in N(u)$ has a dominating neighbor vertex. In the other case, if $p \in N_e(u)$ and $p \in N_e(v)$, then p can be swapped with either u or v. Hence, (iii) is proved.

Conversely, assume that (iii) holds. Then, the vertex p can be defended by $u \in S$ and the safe swapping of p and u is also possible. Consequently, the vertex p does not belong to the SEDS of H. As a result, $\gamma^e_{sec}(H) = \gamma^e_{sec}(G)$.

Now, (ii) implies that every vertex $w \in N_e(v)$ where $w \notin S$ has a neighbor in S other than the vertex v. Therefore, if we swap p and v also, the secure equitability of the graph is not affected. Hence, the result.

Finally, if u is a leaf and its support vertex has a degree greater than 3, then u defends p since p is a member of $N_e(u)$ and $(S \setminus \{u\}) \cup \{p\}$ is a dominating set of G. The stem $uv \notin sd_{S_{se}}(G)$ as a result. Hence, $\gamma_{sec}^e(H) = \gamma_{sec}^e(G)$. □

Lemma 2.5. *Let H be a graph obtained from a graph G by subdividing an edge $uv \in E(G)$ by adding a vertex p. If $u, v \in S$, a SEDS of G then, $\gamma_{sec}^e(H) = \gamma_{sec}^e(G) + 1$ if and only if any of the following conditions holds:*

(i) *$p \notin N_e(u)$ and $p \notin N_e(v)$.*
(ii) *$p \in pen(u, S)$ with $|pen(u, S)| \geq 2$ but $p \notin N_e(v)$.*
(iii) *$p \in N_e(u)$ and $p \in N_e(v)$ but for any $z \in N(u)$, $z \notin S$ we have $N(z) \cap (S - \{u\}) = \phi$ and for any $w \in N(v)$, $w \notin S$ we have $N(w) \cap (S - \{v\}) = \phi$.*

Proof. Assume that $p \notin N_e(u)$ and $p \notin N_e(v)$. Then, the vertex p is an equitable isolate. Hence, p is not defended by any vertex of G. Thus, p is added to the set S so that the new set of dominating vertices is a SEDS of H. Therefore, $\gamma_{sec}^e(H) = \gamma_{sec}^e(G) + 1$.

Suppose (ii) is true. If $p \notin N_e(v)$, then p can not be defended by the vertex v. Suppose $p \in pen(u, S)$ and $|pen(u, S)| \geq 2$. Then, whenever u is swapped with p, a vertex $w \in pen(u, S)$ exists, $w \neq p$ and w is not defended by any vertices of G. Hence, the swapping of u with p is not possible. Consequently, p is added to the SEDS of H.

Suppose (iii) holds. If we swap p with u, then z will not be defended by any dominating vertex of G, since $N_e(z) \cap (S - \{u\}) = \phi$. Similarly, swapping p with v is also impossible, since w will become undefended. Hence, p is a secure equitable dominating vertex of H.

Conversely, assume that $\gamma_{sec}^e(H) = \gamma_{sec}^e(G) + 1$. Then, $uv \in sd_{S_{se}}(G)$ and p is an element in the SEDS of H. Thus, p is not defended by any vertex of the SEDS of G. Therefore, (i), (ii) and (iii) are true, since in all these cases the vertices u and v do not defend the vertex p added. □

Lemma 2.6. *Let H be a graph obtained from a graph G by subdividing an edge $uv \in E(G)$ by adding a vertex p. If $u \in S$ and $v \notin S$ where S is a SEDS of G. Then, $\gamma_{sec}^e(H) = \gamma_{sec}^e(G)$ if and only if any of the following conditions holds:*

(i) *u is an equitably isolated leaf of G.*
(ii) *$p \in N_e(u)$ and for every $z \in N(u)$, $z \notin S$ we have $N(z) \cap (S - \{u\}) \neq \phi$.*

Proof. Suppose $u \in S$ is an equitably isolated leaf of G. Then, the vertex p is defended by u, since $p \in N_e(u)$. Hence, $\gamma_{sec}^e(H) = \gamma_{sec}^e(G)$. Suppose (ii) holds. If we replace the vertex p with the vertex u, then any vertex $z \in N(u)$ remains protected, since $N(z) \cap (S - \{u\}) \neq \phi$. Thus, we have $\gamma_{sec}^e(H) = \gamma_{sec}^e(G)$. Conversely, suppose the conditions (i) and (ii) holds. The vertex p can be defended by the vertex u, since $p \in N_e(u)$ and $N(z) \cap (S - \{u\}) \neq \phi$ for every $z \in N(u)$. Hence, the proof of the converse part is obvious. □

Lemma 2.7. *Let H be a graph obtained from a graph G by subdividing an edge $uv \in E(G)$ by adding a vertex p. If $u \in S$ and $v \notin S$ where S is a SEDS of G. Then, $\gamma_{sec}^e(H) = \gamma_{sec}^e(G) + 1$ if and only if any of the following conditions holds:*

(i) *$p \notin N_e(u)$.*

(ii) $p \in pen(u, S)$ *and* $|pen(u, S)| \geq 2$.
(iii) $p \in N_e(u)$ *and there exists* $z \in N_e(u)$ *such that* $z \notin S$ *and* $N(z) \cap (S - \{u\}) = \phi$.

Proof. Suppose $p \notin N_e(u)$. Then, u does not defend p. Also, v can not defend p, since $v \notin S$. Then, we add the vertex p to the set S so that the new set of dominating vertices is a SEDS of the graph H. Hence, $\gamma^e_{sec}(H) = \gamma^e_{sec}(G) + 1$.

Now, let $p \in pen(u, S)$ and $|pen(u, S)| \geq 2$. Since $|pen(u, S)| \geq 2$, at least one vertex $z \in pen(u)$ and $z \neq p$ exists. If we replace p with u, then the vertex z will not be defended by any vertex of S. Hence, the vertex p belongs to the SEDS of H and the result follows.

Suppose (iii) holds. If we swap the vertex p with the vertex u, then the vertex z will not be defended by any vertex of S since $N(z) \cap (S - \{u\}) = \phi$. Thus, swapping the vertices u and p is impossible. Consequently, the vertex p is a secure equitable dominating vertex of H. Hence the result is true. The converse part can also be proved using similar arguments. □

Lemma 2.8. *Let H be a graph obtained from a graph G by subdividing an edge $uv \in E(G)$ by adding a vertex p. If $u, v \notin S$ where S is a SEDS of G, then $\gamma^e_{sec}(H) = \gamma^e_{sec}(G) + 1$.*

Proof. The proof of Lemma 2.8 is obvious since $u, v \notin S$, the vertex p is not defended by any vertex of H. Then, p is added to the set S so that $\gamma^e_{sec}(H) = \gamma^e_{sec}(G) + 1$. □

Theorem 2.9. *Let G be a simple connected graph and let H be a graph created by splitting an edge (once) in G. Then, any one of the following will be true.*

(i) $\gamma^e_{sec}(H) = \gamma^e_{sec}(G)$,
(ii) $\gamma^e_{sec}(H) = \gamma^e_{sec}(G) + 1$.

Proof. Lemma 2.4, Lemma 2.5, Lemma 2.6, Lemma 2.7 and Lemma 2.8 lead to the proof of Theorem 2.9. □

Theorem 2.10. *An edge $uv \in sd_{S_{se}}(G)$ if u and v are of degree at least 3.*

Proof. Suppose if $uv \in E(G)$ is a stem where u is the leaf and v is the support vertex, then the vertex added by the subdivision of uv, say w, is secure equitably defended by the vertex u, since $|deg(u) - deg(w)| = 1$ and if we replace u with w, then also the equitable domination property persists.

Suppose $uv \in E(G)$ with $deg(u) = 2$ or $deg(v) = 2$. Then, $p \in N_e(u)$ if $deg(u) = 2$ or $p \in N_e(v)$ if $deg(v) = 2$. Then, by Theorem 2.9, either $uv \in sd_{S_{se}}(G)$ or $uv \notin sd_{S_{se}}(G)$.

Now, assume that $deg(u) \geq 3$ and $deg(v) \geq 3$. Then, $p \in S$ if $u, v \notin S$ by Lemma 2.8. If $u, v \in S$ or either u or v is an element of S, then $p \in S$ by Lemma 2.5 and Lemma 2.7. Thus, $uv \in sd_{S_{se}}(G)$ if $deg(u) \geq 3$ and $deg(v) \geq 3$. Thus,$uv \in sd_{S_{se}}(G)$ if both $deg(u)$ and $deg(v)$ are at least 3. □

Theorem 2.10 immediately leads to the Corollary 2.11.

Corollary 2.11. *The set $sd_{S_{se}}(G)$ does not contain a stem of a graph G with a support vertex v and $deg(v) \geq 2$.*

Corollary 2.12. *For any graph G on n vertices with at least one leaf and a support vertex v of $deg(v) \geq 3$ we have $|sd_{S_{se}}(G)| < n$.*

Proof. The result is clear by Corollary 2.11 and Theorem 2.10, since any leaf of the subdivided stem defends the newly added vertex. □

Definition 2.13. The class of graphs for which subdivision of any edge results in the increment of secure equitable domination number are called secure equitable subdivision critical graphs (se-subdivision critical graphs).

Example 2.14. The complete graphs are se-subdivision critical graphs. The subdivision of any edge of a complete graph increases the se-domination number by 1.

Example 2.15. Any wheel graph and wheel related graph classes like double wheel and closed helm are se-subdivision critical graphs.

Theorem 2.16. *If G is a se-subdivision critical graph, then $\delta(G) \geq 2$.*

Proof. Suppose $\delta(G) = 1$. If we subdivide any stem of the graph, then the leaf will defend the new vertex. Hence, the graph G can not be a se-subdivision critical graph. Now, suppose $\delta(G) = 2$. Assume that we subdivide an edge $xy \in E(G)$, and a new vertex p is added. Then, either p belongs to the equitable neighborhood of the vertices x and y, or it belongs to the equitable neighborhood of both. Consequently, p may or may not belong to the SEDS by Theorem 2.9. The result is clear for $n \geq 3$, since the vertex p does not belong to the equitable neighborhood of x or y. □

Corollary 2.17. *Any k-reglar or $(k, k+1)$-biregular graphs are se-subdivision critical graphs where $k \geq 3$.*

Proof. The proof is obvious by Theorem 2.16 □

Theorem 2.18. *A graph G where every vertex is an equitable isolate is a se-subdivision critical graph if and only if $\delta(G) \geq 3$.*

Proof. Suppose G is a graph in which every vertex is an equitable isolate and let $\delta(G) < 3$. If G has a leaf, then G can not be a se-subdivision critical graph by Corollary 2.11. Now, let $v \in V(G)$ and $\deg(v) = 2$. Suppose we subdivide an edge $uv \in E(G)$. Since they are equitable isolates, every vertex of the graph G belongs to the SEDS of G. As a result, the vertex v can defend the newly added vertex which subdivides the edge uv since it belongs to the equitable neighborhood of v. Now, let $\delta(G) \geq 3$. If we subdivide any edge $uv \in E(G)$, then neither u nor v can defend the newly added vertex since it does not belong to $N_e(v)$ or $N_e(u)$. Hence, the subdivision of any edge increases the se-domination number of the graph G whenever $\delta(G) \geq 3$. Thus, G is a se-subdivision critical graph if $\delta(G) \geq 3$.

To the contrary, we assume that G is a se-subdivision critical graph with every vertex as an equitable isolate. Then, $\delta(G) \geq 2$ by Theorem 2.16. If v is a vertex of degree 2, then $\deg(u) \geq 4$ for every $u \in N(v)$. Consequently, if we divide an edge $uv \in E(G)$, then the newly added vertex belongs to $N_e(v)$ and it is defended by v. Thus, the $\gamma^e_{sec}(G)$ remains the same, which is a contradiction to our assumption. Thus, $\delta(G) \geq 3$. □

Construction of a se-subdivision critical graph in which every vertex is an equitable isolate.

Let $a, b \in \mathbf{N}$. Then, there exists a se-subdivision critical graph $G_1(a,b)$ of order $a + ab + 3$ vertices and size $a + 3ab$ where $a \geq 3$ and $b \geq a+1$. The following steps carry out the construction of the graph:

Step1 Take any a vertices say, v_1, v_2,...,v_a. Every vertex v_i is adjoined with a common vertex v by adding an edge between each v_i, $1 \leq i \leq a$ and the vertex v.

Step2 Take a copies of $\overline{K_b}$ and every vertex of each copy of $\overline{K_b}$ is adjoined to each v_i where $1 \leq i \leq a$.

Step3 Add two more vertices to the graph and add edges between every vertex of a copies of $\overline{K_b}$ to the newly added vertices.

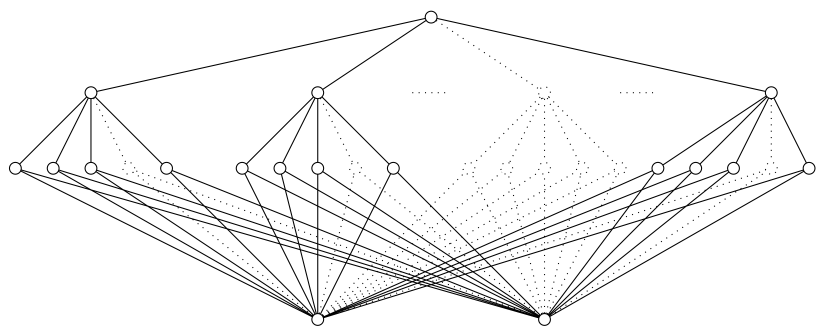


FIGURE 2. Structure of a se-subdivision critical graph with every vertex is an equitable isolate.

Thus, the resultant graph $G_1(a,b)$ is a graph with radius = diameter = 3 and have 4 levels of vertices. The first level consists of a single vertex v, the second level consists of the vertices v_1, v_2,...,v_a, the third level is constructed by taking ab number of vertices and the fourth level has 2 vertices. Also, none of the vertices of $G_1(a,b)$ can be defended by any other vertex of the graph as $N_e(v) = \phi$ for every $v \in V(G_1(a,b))$. Therefore, every vertex $v \in V(G_1(a,b))$ is an equitable isolate and the subdivision of any edge will result in the increment of secure equitable domination number of $G_1(a,b)$. Hence, $G_1(a,b)$ is a se-subdivision critical graph. Figure 2 shows the general construction of $G_1(a,b)$.

Construction of a se-subdivision critical graph $\mathbf{G_2(n)}$ of order $\mathbf{4n+1}$ with $\delta(\mathbf{G_2(n)}) \geq \mathbf{2}$ and $\gamma^{\mathbf{e}}_{\mathbf{sec}}(\mathbf{G_2(n)}) = \mathbf{2n+1}$ where $\mathbf{n \geq 3}$.

The construction of a se-subdivision critical graph of order $4n+1$ where $n \geq 3$ can be done as follows:

Step1 Take a flower graph Fl_n on $2n+1$ vertices obtained by attaching a vertex to each vertex on the perimeter of $W_{1,n}$ by adding edges between those vertices attached and the universal vertex of $W_{1,n}$. Let the vertices on the perimeter of the wheel $W_{1,n}$ be u_1, $u_2 \ldots u_n$, the vertices attached to each u_i be v_1, $v_2 \ldots v_n$ where $1 \leq i \leq n$ and u_0 be the universal vertex of Fl_n.

Step2 Take n copies of P_2 and attach each copy of P_2 to every vertex v_i where $1 \leq i \leq n$ by adding edges between every vertex of P_2 and the corresponding v_i.

Let S denote the SEDS of the constructed graph $G_2(n)$. Then, $u_0 \in S$ as $|deg(u_0) - deg(u)| > 1$ for every $u \in V(G_2(n))$ and $u_0u \in E(G_2(n))$. Moreover, no vertex on the copies of P_2 are defended by any other vertex of $G_2(n)$. Thus, one vertex from each copy of P_2 belongs to the set S. Further, each v_i belongs to the equitable neighborhood of the corresponding u_i. Therefore, n more vertices must be in S as S is a SEDS of the graph $G_2(n)$. Hence, $\gamma^e_{sec}(G_2(n)) = 2n + 1$.

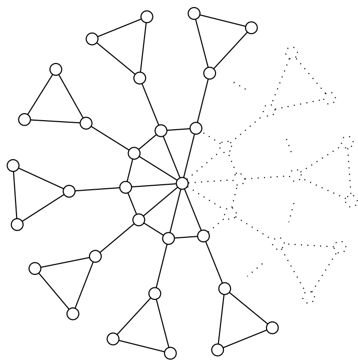


FIGURE 3. Structure of a se-subdivision critical graph $G_2(n)$ of order $4n + 1$ with $\delta(G_2(n)) \geq 2$ and $\gamma^e_{sec}(G_2(n)) = 2n + 1$ where $n \geq 3$.

Suppose we subdivide any edge e of the graph $G_2(n)$ and w is the vertex added to $V(J)$. If $e \in V(Fl_n)$, then w is not defended by any of the incident vertices of the edge e. On the other hand, if $e \in \{uv : u \in V(Fl_n), v \in V(P_2) \text{ or } e \in E(P_2)\}$, then either w is not defended by the dominating vertex of the corresponding copy of P_2 or the swapping of the vertex w with a dominating vertex in $N_e(w)$, the remaining vertex of P_2 will become undefended. Hence, the subdivision of any edge of the graph $G_2(n)$ results in the increment of the se-domination number by 1 and $G_2(n)$ is a se-domination critical graph. The construction of a se-subdivision critical graph $G_2(n)$ is illustrated in Figure 3.

Definition 2.19. The graphs in which the edge subdivision fails to increase the secure equitable domination number are called se-subdivision stable graphs.

Example 2.20. A k-star graph is a se-subdivision stable graph. The node added by subdividing each stem in a k-star is secure equitably defended by the corresponding leaf.

Example 2.21. The tensor product $G = C_3 \times S_4$ is a se-subdivision stable graph where every vertex is an equitable isolate and $\delta(G) = 1$.

Remark 2.22. By Example 2.20, it is clear that for any graph $G \cong K_{1,n-1}$, $\gamma^e_{sec}(G) = \gamma^e_{sec}(H)$, where H is a graph obtained from G by subdividing every edge $uv \in E(G)$.

Remark 2.23. A se-subdivision critical graph does not contain an equitably isolated leaf.

3. Conclusion

In this paper, we began the study of a new domination parameter called the se-subdivision number of a graph. Further, we defined the se-domination critical graphs and the se-domination stable graphs. In addition, we discussed the construction of the se-subdivision critical graphs. We characterized the graphs with $sd_{\gamma_{se}}(G) = 1$.

The revamping of a network is a common phenomenon due to the requirement of security reinforcement and client extension or deletion. One such case is the addition of new processors to the network. The introduction of a new processor (or processors) may change the existing topology of the network. We consider a network where the servers are selected based on the se-domination strategy. The minimum number of processors that can be introduced between the interconnections in a network (one new processor be- tween a pair of processors) so that the number of servers needed for the data transmission increase is given by the se-subdivision number of the network.

References

1. Alwardi, A., & Nayaka, S. R. (2018). Transversal domination in graphs. Gulf Journal of Mathematics, 6(2). https://doi.org/10.56947/gjom.v6i2.132
2. Atapour, M., Sheikholeslami, S. M., & Khodkar, A. (2009). Roman domination subdivision number of graphs. Aequationes mathematicae, 78(3), 237-245. http://dx.doi.org/10.1007/s00010-009-2981-y
3. Caay, M., & Durog, M. (2021). On some independent equitable domination of graphs. Gulf Journal of Mathematics, 11(1), 57-64. https://doi.org/10.56947/gjom.v11i1.668
4. Dettlaff, M., Raczek, J., & Topp, J. (2019). Domination subdivision and domination multisubdivision numbers of graphs. Discussiones Mathematicae Graph Theory, 39, 829-839. http://dx.doi.org/10.7151/dmgt.2103
5. Divya Rashmi, S. V., Somasundaram, A., & Arumugam, S. (2019). Secure domination subdivision number of a graph. Discrete Mathematics, Algorithms and Applications, 11, 1950036. http://dx.doi.org/10.1142/S1793830919500368
6. Haynes, T., Hedetniemi, S., Hedetniemi, S., Jacobs, D., Knisely, J., & Van der Merwe, L. (2001). Domination subdivision numbers. Discussiones Mathematicae Graph Theory, 21, 239-253. http://dx.doi.org/10.7151/dmgt.1147
7. Haynes, T. W., Hedetniemi, S. T., & Henning, M. A. (2020). Topics in domination in graphs. Springer. http://dx.doi.org/10.1007/978-3-030-51117-3
8. Haynes, T., Henning, M., & Hopkins, L. (2004). Total domination subdivision numbers of graphs. Discussiones Mathematicae Graph Theory, 24, 457-467. https://doi.org/10.7151/dmgt.1244
9. Jayasekaran, C., & Binoja, L. G. (2024). Relatively prime restrained detour domination number of a graph. Gulf Journal of Mathematics, 16(2), 291-297. https://doi.org/10.56947/gjom.v16i2.1844
10. Muthusubramanian, L., Sundareswaran, R., & Swaminathan, V. (2023). Secure equitability in graphs. Discrete Mathematics, Algorithms and Applications, 15(2), 2250081. http://dx.doi.org/10.1142/S1793830922500811
11. Sumathi, P., & Alarmelumangai, G. (2014). Locating equitable domination and independence subdivision numbers of graphs. Bulletin of Mathematical Sciences and Applications, 9, 27-32. http://dx.doi.org/10.18052/www.scipress.com/BMSA.9.27

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