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A COMMON FIXED POINT THEOREM FOR FOUR MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACE

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Abstract. In this paper, we prove a common fixed point theorem in intuitionistic fuzzy metric space by combining the ideas of pointwise R- weak commutativity and reciprocal continuity of mappings satisfying contractive condition.

1. Introduction

Atanassove [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park[7] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t- conorms as a generalization of fuzzy metric space due to Kramosil et al.[4]. In 2006, Turkoglu [9] proved Jungck's [3] common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings. In this paper, we prove a common fixed point theorem in intuitionistic fuzzy metric space by combining the ideas of pointwise R- weak commutativity and reciprocal continuity of mappings satisfying contractive condition.

2. Preliminaries

Definition 2.1. [8] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if * satisfies the following conditions:
(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2. [8] A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:
(i) $\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous;

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(iii) $a \diamond 0 = a$ for all $a \in [0, 1]$;
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Alaca et al.[1] defined the notion of intuitionistic fuzzy metric space as follows:

Definition 2.3. [1] A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:
(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all$x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y \in X$, $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous;
(vii) $lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) for all $x, y \in X$, $N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous;
(xiii) $lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.
Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.4. [1] Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t-norm * and t-conorm $\diamond$ are associated as $x \diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Lemma 2.5. [1] *In an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.*

Definition 2.6. [1] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be
(i) convergent to a point $x \in X$ if, for all $t > 0$, $lim_{n\to\infty} M(x_n, x, t) = 1$ and $lim_{n\to\infty} N(x_n, x, t) = 0$;
(ii) Cauchy sequence if, for all $t > 0$ and $p > 0$, $lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.

Definition 2.7. [1] An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.8. [10] A pair of self mappings (A, S) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be commuting if
$M(ASx, SAx, t) = 1$ and $N(ASx, SAx, t) = 0$
for all $x \in X$ and $t > 0$.

Definition 2.9. [10] A pair of self mappings (A, S) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be weakly commuting if
$M(ASx, SAx, t) \geq M(Ax, Sx, t)$ and $N(ASx, SAx, t) \leq N(Ax, Sx, t)$
for all $x \in X$ and $t > 0$.

Definition 2.10. [10] A pair of self mappings (A, S) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be compatible if
$lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$
for all $t > 0$,
whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = u$ for some $u \in X$.

Definition 2.11. [9] A pair of self mappings (A, S) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be pointwise R-weakly commuting, if given $x \in X$, there exist $R > 0$ such that for all $t > 0$
$M(ASx, SAx, t) \geq M(Ax, Sx, \frac{t}{R})$
and
$N(ASx, SAx, t) \leq N(Ax, Sx, \frac{t}{R})$.
Clearly, every pair of weakly commuting mappings is pointwise R-weakly commuting with $R = 1$.

Definition 2.12. [6] Two mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be reciprocally continuous if $ASu_n \to Az$, $SAu_n \to Sz$, whenever $\{u_n\}$ is a sequence such that $Au_n \to z$, $Su_n \to z$ for some $z \in X$.

If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true.

Lemma 2.13. [11] *let $\{u_n\}$ is a sequence in an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. If there exists a constant $k \in (0, 1)$ such that*
$M(u_n, u_{n+1}, kt) \geq M(u_{n-1}, u_n, t)$ and $N(u_n, u_{n+1}, kt) \leq N(u_{n-1}, u_n, t)$ for all n = 1, 2, 3,....
Then $\{u_n\}$ is a Cauchy sequence in X.

Lemma 2.14. [11] *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1)$*
$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$.
Then $x = y$.

3. Main results

Lemma 3.1. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. Further, let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self mappings of X satisfying*
(3.1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$;
(3.2) there exists a constant $k \in (0, 1)$ such that

$$\begin{aligned}[1 + aM(Sx, Ty, kt)] * M(Ax, By, kt) \geq\ & a[M(Ax, Sx, kt) * M(By, Ty, kt) * \\ & M(By, Sx, kt) * M(Ax, Ty, kt)] \\ & + M(Ty, Sx, t) * M(Ax, Sx, t) * M(By, Ty, t) \\ & * M(By, Sx, \alpha t) * M(Ax, Ty, (2 - \alpha)t)\end{aligned}$$

and

$$\begin{aligned}[1+aN(Sx,Ty,kt)]\diamond N(Ax,By,kt) \leq\ & a[N(Ax,Sx,kt)\diamond N(By,Ty,kt)\diamond \\ & N(By,Sx,kt)\diamond N(Ax,Ty,kt)] \\ & +N(Ty,Sx,t)\diamond N(Ax,Sx,t)\diamond N(By,Ty,t) \\ & \diamond N(By,Sx,\alpha t)\diamond N(Ax,Ty,(2-\alpha)t)\end{aligned}$$

for all $x,y \in X, a \geq 0, \alpha \in (0,2), t > 0$. Then, the continuity of one of the mappings in compatible pair (A,S) and (B,T) on $(X,M,N,*,\diamond)$ implies their reciprocal continuity.

Proof. First, assume that A and S are compatible and S is continuous. We show that A and S are reciprocally continuous. Let $\{u_n\}$ be a sequence such that $Au_n \to z$ and $Su_n \to z$ for some $z \in X$ as $n \to \infty$. Since S is continuous, we have $SAu_n \to Sz$ and $SSu_n \to Sz$ as $n \to \infty$ and since (A,S) is compatible, we have
$lim_{n\to\infty}M(ASu_n,SAu_n,t) = 1$, $lim_{n\to\infty}N(ASu_n,SAu_n,t) = 0$.
This gives,
$lim_{n\to\infty}M(ASu_n,Sz,t) = 1$, $lim_{n\to\infty}N(ASu_n,Sz,t) = 0$.
That is $ASu_n \to Sz$ as $n \to \infty$. By (3.1), for each n, there exists $v_n \in X$ such that $ASu_n = Tv_n$. Thus, we have $SSu_n \to Sz, SAu_n \to Sz, ASu_n \to Sz$ and $Tv_n \to Sz$ as $n \to \infty$ whenever $ASu_n = Tv_n$.
Now we claim that $Bv_n \to Sz$ as $n \to \infty$. Suppose not, then by (3.2), take $\alpha = 1$,

$$\begin{aligned}[1+aM(SSu_n,Tv_n,kt)]*M(ASu_n,Bv_n,kt) \geq\ & a[M(ASu_n,SSu_n,kt)*M(Bv_n,Tv_n,kt) \\ & *M(Bv_n,SSu_n,kt)*M(ASu_n,Tv_n,kt)] \\ & +M(Tv_n,SSu_n,t)*M(ASu_n,SSu_n,t) \\ & *M(Bv_n,Tv_n,t) \\ & *M(Bv_n,SSu_n,t) \\ & *M(ASu_n,Tv_n,t)\end{aligned}$$

and

$$\begin{aligned}[1+aN(SSu_n,Tv_n,kt)]\diamond N(ASu_n,Bv_n,kt) \leq\ & a[N(ASu_n,SSu_n,kt)\diamond N(Bv_n,Tv_n,kt) \\ & \diamond N(Bv_n,SSu_n,kt)\diamond N(ASu_n,Tv_n,kt)] \\ & +N(Tv_n,SSu_n,t)\diamond N(ASu_n,SSu_n,t) \\ & \diamond N(Bv_n,Tv_n,t)\diamond N(Bv_n,SSu_n,t) \\ & \diamond N(ASu_n,Tv_n,t).\end{aligned}$$

Taking $n \to \infty$,

$$\begin{aligned}[1+aM(Sz,Sz,kt)]*M(Sz,Bv_n,kt) \geq\ & a[M(Sz,Sz,kt)*M(Bv_n,Sz,kt) \\ & *M(Bv_n,Sz,kt)*M(Sz,Sz,kt)] \\ & +M(Sz,Sz,t)*M(Sz,Sz,t)*M(Bv_n,Sz,t) \\ & *M(Bv_n,Sz,t)*M(Sz,Sz,t)\end{aligned}$$

and

$$\begin{aligned}[1+aN(Sz,Sz,kt)]\diamond N(Sz,Bv_n,kt) \leq a[&N(Sz,Sz,kt)\diamond N(Bv_n,Sz,kt)\\ &\diamond N(Bv_n,Sz,kt)\diamond N(Sz,Sz,kt)]\\ &+N(Sz,Sz,t)\diamond N(Sz,Sz,t)\diamond N(Bv_n,Sz,t)\\ &\diamond N(Bv_n,Sz,t)\diamond N(Sz,Sz,t)\end{aligned}$$

This implies
$M(Sz,Bv_n,kt) \geq M(Sz,Bv_n,t)$ and $N(Sz,Bv_n,kt) \leq N(Sz,Bv_n,t)$.
By Lemma 2.14, we have $Bv_n \to Sz$ as $n \to \infty$.
Again by (3.2), take $\alpha = 1$,

$$\begin{aligned}[1+aM(Sz,Tv_n,kt)]*M(Az,Bv_n,kt) \geq a[&M(Az,Sz,kt)*M(Bv_n,Tv_n,kt)\\ &*M(Bv_n,Sz,kt)*M(Az,Tv_n,kt)]\\ &+M(Tv_n,Sz,t)*M(Az,Sz,t)*M(Bv_n,Tv_n,t)\\ &*M(Bv_n,Sz,t)*M(Az,Tv_n,t)\end{aligned}$$

and

$$\begin{aligned}[1+aN(Sz,Tv_n,kt)]\diamond N(Az,Bv_n,kt) \leq a[&N(Az,Sz,kt)\diamond N(Bv_n,Tv_n,kt)\\ &\diamond N(Bv_n,Sz,kt)\diamond N(Az,Tv_n,kt)]\\ &+N(Tv_n,Sz,t)\diamond N(Az,Sz,t)\diamond N(Bv_n,Tv_n,t)\\ &\diamond N(Bv_n,Sz,t)\diamond N(Az,Tv_n,t).\end{aligned}$$

Taking $n \to \infty$,

$$\begin{aligned}[1+aM(Sz,Sz,kt)]*M(Az,Sz,kt) \geq a[&M(Az,Sz,kt)*M(Sz,Sz,kt)\\ &*M(Sz,Sz,kt)*M(Az,Sz,kt)]\\ &+M(Sz,Sz,t)*M(Az,Sz,t)*M(Sz,Sz,t)\\ &*M(Sz,Sz,t)*M(Az,Sz,t)\end{aligned}$$

and

$$\begin{aligned}[1+aN(Sz,Sz,kt)]\diamond N(Az,Sz,kt) \leq a[&N(Az,Sz,kt)\diamond N(Sz,Sz,kt)\\ &\diamond N(Sz,Sz,kt)\diamond N(Az,Sz,kt)]\\ &+N(Sz,Sz,t)\diamond N(Az,Sz,t)\diamond N(Sz,Sz,t)\\ &\diamond N(,Sz,t)\diamond N(Az,Sz,t).\end{aligned}$$

This implies
$M(Az,Sz,kt) \geq M(Az,Sz,t)$ and $N(Az,Sz,kt) \leq N(Az,Sz,t)$.
By Lemma 2.14, $Az = Sz$. Therefore, $SAu_n \to Sz$, $ASu_n \to Sz = Az$ as $n \to \infty$. Hence, A and S are reciprocally continuous on X. If the pair (B,T) is assumed to be compatible and T is continuous, the proof is similar. □

Theorem 3.2. *Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric space. Further, let (A,S) and (B,T) be pointwise R- weakly commuting pairs of self mappings of X satisfying (3.1), (3.2).*

If one of the mappings in compatible pair (A, S) or (B, T) is continuous, then A, B, S and T have a unique common fixed point.

Proof. By (3.1), since $A(X) \subseteq T(X)$ for any point $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since, $B(X) \subseteq S(X)$, for this point $x_1 \in X$, we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that for $n = 0, 1, 2,...$
$y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$.
By (3.2), for all $t > 0$ and $\alpha = (1 - q)$ with $q \in (0, 1)$, we have

$$\begin{aligned}[1 + aM(y_{2n}, y_{2n+1}, kt)] * M(y_{2n+1}, y_{2n+1}, kt) \geq\ & a[M(y_{2n+2}, y_{2n+1}, kt) * M(y_{2n+1}, y_{2n}, kt) * \\ & M(y_{2n+1}, y_{2n+1}, kt) * M(y_{2n+2}, y_{2n}, kt)] \\ & + M(y_{2n}, y_{2n+1}, t) * M(y_{2n+2}, y_{2n+1}, t) \\ & * M(y_{2n+1}, y_{2n}, t) * M(y_{2n+1}, y_{2n+1}, (1 - q)t) \\ & * M(y_{2n+2}, y_{2n}, (1 + q)t) \\ & \geq a[M(y_{2n+1}, y_{2n}, kt) * M(y_{2n+2}, y_{2n}, kt)] \\ & * M(y_{2n}, y_{2n+1}, t) * M(y_{2n+2}, y_{2n+1}, t) \\ & * M(y_{2n}, y_{2n+2}, qt)\end{aligned}$$

and

$$\begin{aligned}[1 + aN(y_{2n}, y_{2n+1}, kt)] \diamond N(y_{2n+1}, y_{2n+1}, kt) \leq\ & a[N(y_{2n+2}, y_{2n+1}, kt) \diamond N(y_{2n+1}, y_{2n}, kt) \diamond \\ & N(y_{2n+1}, y_{2n+1}, kt) \diamond N(y_{2n+2}, y_{2n}, kt)] \\ & + N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+2}, y_{2n+1}, t) \\ & \diamond N(y_{2n+1}, y_{2n}, t) \\ & \diamond N(y_{2n+1}, y_{2n+1}, (1 - q)t) \\ & \diamond N(y_{2n+2}, y_{2n}, (1 + q)t) \\ & \leq a[N(y_{2n+1}, y_{2n}, kt) \diamond N(y_{2n+2}, y_{2n}, kt)] \\ & \diamond N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+2}, y_{2n+1}, t) \\ & \diamond N(y_{2n}, y_{2n+2}, qt)\end{aligned}$$

Thus it follows that
$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n}, y_{2n+1}, qt)$ ....(3a)
and
$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t) \diamond N(y_{2n}, y_{2n+1}, qt)$.....(3b)
Since the t- norm and the t- conorm are continuous and $M(x, y, .)$ is left continuous and $N(x, y, .)$ is right continuous.
Letting $q \to 1$ in (3a) and (3b), we have
$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t)$ ....(3c)
and
$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t)$. ....(3d)
Similarly, we also have
$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t) * M(y_{2n+2}, y_{2n+3}, t)$ ....(3e)
and

$N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1}, y_{2n+2}, t) \diamond N(y_{2n+2}, y_{2n+3}, t)$. ....(3f)
In general, for m = 1,2,3,...
$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t) * M(y_{m+1}, y_{m+2}, t)$
and
$N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t) \diamond N(y_{m+1}, y_{m+2}, t)$.
Consequently, it follows that for m = 1,2,3,, p = 1,2,3,...
$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t) * M(y_{m+1}, y_{m+2}, \frac{t}{k^p})$
and
$N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t) \diamond N(y_{m+1}, y_{m+2}, \frac{t}{k^p})$.
As $p \to \infty$,
$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t)$
and
$N(y_{m+1}, y_{m+2}, kt) \leq N(y_m, y_{m+1}, t)$.
Hence by Lemma 2.13, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, $\{y_n\}$ converges to $z \in X$. Its subsequences $\{Ax_{2n}\}$, $\{Tx_{2n+1}\}$, $\{Bx_{2n+1}\}$ and $\{Sx_{2n+2}\}$ also converges to z.
Now, suppose that (A, S) is a compatible pair and S is continuous. Then by Lemma 3.1, A and S are reciprocally continuous, then $SAx_n \to Sz, ASx_n \to Az$ as $n \to \infty$. As, (A, S) is a compatible pair. This implies
$lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$;
i.e. $M(Az, Sz, t) = 1$ and $N(Az, Sz, t) = 0$ as $n \to \infty$.
Hence, $Az = Sz$.
Since $A(X) \subseteq T(X$, there exists a point $p \in X$ such that $Az = Tp = Sz$.
By (3.2), take $\alpha = 1$,

$$\begin{aligned}[1 + aM(Sz, Tp, kt)] * M(Az, Bp, kt) \geq a[&M(Az, Sz, kt) * M(Bp, Tp, kt) * \\ &M(Bp, Sz, kt) * M(Az, Tp, kt)] \\ &+ M(Tp, Sz, t) * M(Az, Sz, t) * M(Bp, Tp, t) \\ &* M(Bp, Sz, t) * M(Az, Tp, t)\end{aligned}$$

and

$$\begin{aligned}[1 + aN(Sz, Tp, kt)] \diamond N(Az, Bp, kt) \leq a[&N(Az, Sz, kt) \diamond N(Bp, Tp, kt) \diamond \\ &N(Bp, Sz, kt) \diamond N(Az, Tp, kt)] \\ &+ N(Tp, Sz, t) \diamond N(Az, Sz, t) \diamond N(Bp, Tp, t) \\ &\diamond N(Bp, Sz, t) \diamond N(Az, Tp, t)\end{aligned}$$

This implies $M(Az, Bp, kt) \geq M(Az, Bp, t)$ and $N(Az, Bp, kt) \leq N(Az, Bp, t)$.
By Lemma 2.14, we have $Az = Bp$.
Thus, $Az = Bp = Sz = Tp$.
Since, A and S are pointwise R - weakly commuting mappings, there exists $R > 0$, such that
$M(ASz, SAz, t) \geq M(Az, Sz, t) = 1$ and $N(ASz, SAz, t) \leq N(Az, Sz,) = 0$.
Therefore, $ASz = SAz$ and $AAz = ASz = SAz = SSz$.
Similarly, B and T are pointwise R-weakly commuting mappings, we have $BBp =$

$BTp = TBp = TTp$. Again by (3.2), take $\alpha = 1$,

$$\begin{aligned}[1 + aM(SAz, Tp, kt)] * M(AAz, Bp, kt) \geq\ & a[M(AAz, SAz, kt) * M(Bp, Tp, kt) * \\ & M(Bp, SAz, kt) * M(AAz, Tp, kt)] \\ & + M(Tp, SAz, t) * M(AAz, SAz, t) \\ & * M(Bp, Tp, t) * M(Bp, SAz, t) \\ & * M(AAz, Tp, t)\end{aligned}$$

and

$$\begin{aligned}[1 + aN(SAz, Tp, kt)] \diamond N(AAz, Bp, kt) \leq\ & a[N(AAz, SAz, kt) \diamond N(Bp, Tp, kt) \diamond \\ & N(Bp, SAz, kt) \diamond N(AAz, Tp, kt)] \\ & + N(Tp, SAz, t) \diamond N(AAz, SAz, t) \\ & \diamond N(Bp, Tp, t) \diamond N(Bp, SAz, t) \\ & \diamond N(AAz, Tp, t)\end{aligned}$$

This implies, for all $t > 0$
$M(AAz, Bp, kt) \geq M(AAz, Bp, t)$ and $N(AAz, Bp, kt) \leq N(AAz, Bp, t)$.
Hence by Lemma 2.14, $AAz = Az = SAz$. Hence Az is common fixed point of A and S. Similarly by (3.2), $Bp = Az$ is a common fixed point of B and T. Hence, Az is a common fixed point of A, B, S and T.
For Uniqueness: Suppose that $Ap(\neq Az)$ is another common fixed point of A, B, S and T. Again by (3.2), take $\alpha = 1$,

$$\begin{aligned}[1 + aM(SAz, TAp, kt)] * M(AAz, BAp, kt) \geq\ & a[M(AAz, SAz, kt) \\ & * M(BAp, TAp, kt) * M(BAp, SAz, kt) \\ & * M(AAz, TAp, kt)] + M(TAp, SAz, t) \\ & * M(AAz, SAz, t) * M(BAp, TAp, t) \\ & * M(BAp, SAz, t) * M(AAz, TAp, t)\end{aligned}$$

and

$$\begin{aligned}[1 + aN(SAz, TAp, kt)] \diamond N(AAz, BAp, kt) \leq\ & a[N(AAz, SAz, kt) \\ & \diamond N(BAp, TAp, kt) \diamond N(BAp, SAz, kt) \diamond \\ & N(AAz, TAp, kt)] + N(TAp, SAz, t) \\ & \diamond N(AAz, SAz, t) \diamond N(BAp, TAp, t) \\ & \diamond N(BAp, SAz, t) \diamond N(AAz, TAp, t)\end{aligned}$$

This implies, for all $t > 0$
$M(Az, Ap, kt) \geq M(Az, Ap, t)$ and $N(Az, Ap, kt) \leq N(Az, Ap, t)$.
Hence by using Lemma 2.14, $Az = Ap$. Thus, uniqueness follows. □

Taking $S = T = I_X$ in above theorem, we get following result:

Corollary 3.3. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. Further, let A and B are reciprocally continuous mappings on X satisfying*

(3.3) there exists a constant $k \in (0,1)$ such that

$$\begin{aligned}[1+aM(x,y,kt)] * M(Ax,By,kt) \geq a[&M(Ax,x,kt) * M(By,y,kt)* \\ &M(By,x,kt) * M(Ax,y,kt)] \\ &+M(y,x,t) * M(Ax,x,t) * M(By,y,t) \\ &* M(By,x,\alpha t) * M(Ax,y,(2-\alpha)t)\end{aligned}$$

and

$$\begin{aligned}[1+aN(x,y,kt)] \diamond N(Ax,By,kt) \leq a[&N(Ax,x,kt) \diamond N(By,y,kt)\diamond \\ &N(By,x,kt) \diamond N(Ax,y,kt)] \\ &+N(y,x,t) \diamond N(Ax,x,t) \diamond N(By,y,t) \\ &\diamond N(By,x,\alpha t) \diamond N(Ax,y,(2-\alpha)t)\end{aligned}$$

for all $x, y \in X, a \geq 0, \alpha \in (0,2), t > 0$.
Then A and B has a unique common fixed point.

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