

SUBMANIFOLDS OF SOME INDEFINITE CONTACT AND PARACONTACT MANIFOLDS

BARNALI LAHA¹, BANDANA DAS² AND ARINDAM BHATTACHARYYA³

ABSTRACT. The purpose of the present paper is to study some types of submanifolds of indefinite contact and paracontact manifolds and a few properties of contact CR-submanifolds of an indefinite Sasakian manifold, indefinite Kenmotsu manifold, indefinite trans-Sasakian manifold and ϵ -paracontact Sasakian manifold.

1. INTRODUCTION AND PRELIMINARIES

Submanifolds of cosymplectic manifolds were discussed by G.D.Ludden in 1970 [9]. After him, B.Y. Chen studied geometry of submanifolds in [5]. B.Y. Chen and K. Ogiue in 1974 have given the notion of totally real submanifolds and differential geometry of Kaehler submanifolds (see [6],[11]). Blair and Shower introduced an almost contact manifold ([3],[4]) M. Kon and K. Yano investigated invariant and anti-invariant submanifolds in [8] and [13]. Later on, A. Bejancu in 1978 and 1979 introduced the notion of CR-submanifolds of a Kaehlerian manifold (see [1], [2]). M. Kobayashi discussed CR-submanifolds of a Sasakian manifold in 1981 [7]. Topology of 3-cosymplectic manifolds was discussed by B.C. Montano in [10]. Recently, in 2010 M.Tarafdar et.al. studied contact CR-submanifolds of an indefinite Sasakian manifold [12].

In this paper we study some types of submanifolds of some indefinite contact and paracontact manifolds and further make an analysis of the properties of contact CR-submanifolds of an indefinite Sasakian manifold, indefinite Kenmotsu manifold, indefinite trans-Sasakian manifold and ϵ -paracontact Sasakian manifold.

Definition 1.1. An $(2n + 1)$ -dimensional semi-Riemannian manifold $(\tilde{M}, \tilde{g})$ is said to be an indefinite almost contact manifold if it admits an indefinite almost contact structure (ϕ, ξ, η) , where ϕ is a tensor field of type $(1, 1)$, ξ is a vector field and η is a 1-form, satisfying

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta \circ \phi = 0, \quad \phi\xi = 0, \quad \eta(\xi) = 1, \quad (1.1)$$

Date: Received: Jul 11, 2013; Accepted: Sep 2, 2013.

2010 Mathematics Subject Classification. 53C12, 53C15, 22E25, 53C40, 53C50.

Key words and phrases. Indefinite cosymplectic manifold, indefinite Sasakian manifold, indefinite Kenmotsu manifold, ϵ -paracontact Sasakian manifold, anti invariant distribution $D^\perp$.

$$\tilde{g}(\phi X, \phi Y) = \tilde{g}(X, Y) - \epsilon \eta(X) \eta(Y), \quad (1.2)$$

$$\tilde{g}(X, \xi) = \epsilon \eta(X), \quad (1.3)$$

for all vector fields X, Y on $\tilde{M}$ [3] and where $\epsilon = \tilde{g}(\xi, \xi) = \pm 1$ and $\tilde{\nabla}$ is the Levi-Civita (L-C) connection for a semi-Riemannian metric $\tilde{g}$.

Definition 1.2. An almost contact structure (ϕ, ξ, η) is said to be normal if the almost complex structure J on the product manifold $\tilde{M} \times R$ is given by

$$J(X, f \frac{d}{dt}) = (\phi X - f \xi, \eta(X) \frac{d}{dt}), \quad (1.4)$$

where f is a C^∞ -function on $\tilde{M} \times R$ having no torsion, i.e., J is integrable, the condition for normality in terms of ϕ, ξ and η is $[\phi, \phi] + 2d\eta \otimes \xi = 0$ on $\tilde{M}$, where $[\phi, \phi]$ is the Nijenhuis tensor of ϕ .

Lastly, the fundamental 2-form Φ is defined by

$$\Phi(X, Y) = \tilde{g}(X, \phi Y). \quad (1.5)$$

Definition 1.3. An indefinite almost contact metric structure $(\phi, \xi, \eta, \tilde{g})$ is said to be indefinite cosymplectic structure, if it is normal and both Φ and η are closed which is characterized by

$$\tilde{\nabla}_X \phi = 0, \quad \tilde{\nabla}_X \eta = 0. \quad (1.6)$$

Now we define contact CR-submanifold of an indefinite cosymplectic manifold.

Definition 1.4. An m -dimensional Riemannian submanifold M of an indefinite cosymplectic manifold $\tilde{M}$ is called a contact CR-submanifold if

- (1) ξ is tangent to M ,
- (2) there exists on M a differentiable distribution $D : x \longrightarrow D_x \subset T_x(M)$, such that D_x is invariant under ϕ ; i.e., $\phi D_x \subset D_x$, for each $x \in M$ and the orthogonal complementary distribution $D^\perp : x \longrightarrow D_x^\perp \subset T_x(M)$ of the distribution D on M is totally real; i.e., $\phi D_x^\perp \subset T_x^\perp(M)$, where $T_x(M)$ and $T_x^\perp(M)$ are the tangent space and the normal space of M at x .

We call D (resp. $D^\perp$) the horizontal (resp. vertical) distribution. Also the contact CR-submanifold of an indefinite cosymplectic manifold is called ξ -horizontal (resp. ξ -vertical) if $\xi_x \in D_x$ (resp. $\xi_x \in D_x^\perp$) for each $x \in M$ by [7]. The distribution D (resp. $D^\perp$) can be defined by a projector P (resp. Q), satisfying

$$P^2 = P, \quad Q^2 = Q, \quad PQ = QP = 0, \quad g \circ (P \times Q) = 0. \quad (1.7)$$

For a vector field X tangent to M , we write

$$\phi X = PX + QX, \quad (1.8)$$

where PX (resp. QX) is a tangential (resp. normal) component of ϕX .

Again we write

$$\phi N = BN + CN, \quad (1.9)$$

for a vector field N in the normal bundle, where BN (resp. CN) is a tangential (resp. normal) component of ϕN . The Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X, Y), \quad (1.10)$$

$$\tilde{\nabla}_X N = -A_N X + \nabla_X^\perp N, \quad (1.11)$$

where ∇ is the Riemannian connection on M and $\nabla^\perp$ is the connection on the normal bundle induced by ∇ and h is the second fundamental form of the immersion, satisfying the condition

$$g(A_N X, Y) = g(h(X, Y), N). \quad (1.12)$$

Definition 1.5. A submanifold M of a Riemannian manifold $\tilde{M}$ is said to be totally umbilical if

$$h(X, Y) = g(X, Y)H, \quad (1.13)$$

where H is the mean curvature vector.

Definition 1.6. An indefinite almost contact metric structure $(\phi, \xi, \eta, \tilde{g})$ is called an indefinite Sasakian structure if

$$(\tilde{\nabla}_Z \phi)W = \epsilon \eta(W)Z - g(Z, W)\xi, \quad (1.14)$$

Definition 1.7. An indefinite almost contact metric structure $(\phi, \xi, \eta, \tilde{g})$ is called an Kenmotsu structure if

$$(\tilde{\nabla}_Z \phi)W = g(\phi Z, W)\xi - \epsilon \eta(W)\phi Z, \quad (1.15)$$

Definition 1.8. An indefinite almost contact metric structure $(\phi, \xi, \eta, \tilde{g})$ is called an indefinite trans-Sasakian structure if

$$(\tilde{\nabla}_Z \phi)W = \alpha[g(Z, W)\xi - \epsilon \eta(W)Z] + \beta[g(\phi Z, W)\xi - \epsilon \eta(W)\phi Z] \quad (1.16)$$

for functions α and β on $\tilde{M}$ of type (α, β) ,

Definition 1.9. An indefinite almost contact metric structure $(\phi, \xi, \eta, \tilde{g})$ is called an ϵ -paracontact Sasakian structure if

$$(\tilde{\nabla}_Z \phi)W = -g(\phi Z, \phi W)\xi - \epsilon \eta(W)\phi^2 Z, \quad (1.17)$$

for all vector fields Z, W in respective $\tilde{M}$.

2. CONTACT CR-SUBMANIFOLD OF AN INDEFINITE SASAKIAN MANIFOLD

Here we consider a contact CR-submanifold M of an indefinite Sasakian manifold $\tilde{M}$. Then we prove the following theorem and corollary :

Theorem 2.1. *Let M be a totally umbilical contact CR-submanifold of an indefinite Sasakian manifold $\tilde{M}$. Then the anti invariant distribution $D^\perp$ is one dimensional i.e. $\dim D^\perp = 1$.*

Proof. For an indefinite Sasakian structure we have

$$(\tilde{\nabla}_Z \phi)W = \epsilon\eta(W)Z - g(Z, W)\xi. \quad (2.1)$$

Also we know

$$\tilde{\nabla}_Z \phi W = (\tilde{\nabla}_Z \phi)W + \phi \tilde{\nabla}_Z W. \quad (2.2)$$

From (2.1) and (2.2) we get

$$\tilde{\nabla}_Z \phi W = \phi \tilde{\nabla}_Z W + \epsilon\eta(W)Z - g(Z, W)\xi. \quad (2.3)$$

Since M is totally umbilical, by Weingarten and Gauss formula we have

$$\nabla_Z^\perp \phi W - g(H, \phi W)Z = \phi[\nabla_Z W + g(Z, W)H] + \epsilon\eta(W)Z - g(Z, W)\xi \quad (2.4)$$

for any $Z, W \in \Gamma(D^\perp)$.

Taking inner product with $Z \in \Gamma(D^\perp)$ in (2.4) we obtain

$$-g(H, \phi W)\|Z\|^2 = g(Z, W)g(\phi H, Z) + \epsilon\eta(W)\|Z\|^2 - g(Z, W)\epsilon\eta(Z). \quad (2.5)$$

Interchanging Z and W we have

$$-g(H, \phi Z)\|W\|^2 = g(W, Z)g(\phi H, W) + \epsilon\eta(Z)\|W\|^2 - g(W, Z)\epsilon\eta(W). \quad (2.6)$$

Substituting (2.5) in (2.6) and simplifying we have

$$g(H, \phi Z)\left[1 - \frac{g(Z, W)^2}{\|Z\|^2\|W\|^2}\right] + \epsilon\eta(Z)\left[1 - \frac{g(Z, W)^2}{\|Z\|^2\|W\|^2}\right] = 0. \quad (2.7)$$

The equation (2.7) has a solution if $Z \parallel W$ i.e. $\dim D^\perp = 1$. Hence the theorem is proved. $\square$

Corollary 2.2. *A contact CR-submanifold of an indefinite Sasakian manifold reduces to a cosymplectic manifold provided the vector field Z becomes the structure vector field ξ .*

Proof. In a contact CR-submanifold of an indefinite Sasakian manifold we have

$$(\tilde{\nabla}_Z \phi)W = \epsilon\eta(W)Z - g(Z, W)\xi. \quad (2.8)$$

Taking inner product with Z and using (1.3) in (2.8) we obtain

$$(\tilde{\nabla}_Z \phi)W = \epsilon\eta(W)g(Z, Z) - \epsilon\eta(Z)g(Z, W). \quad (2.9)$$

The equation vanishes provided $Z = \xi$ and therefore the contact CR-submanifold reduces to a cosymplectic manifold. $\square$

3. CONTACT CR-SUBMANIFOLD OF AN INDEFINITE KENMOTSU MANIFOLD

In this section we take a contact CR-submanifold M of an indefinite Kenmotsu manifold $\tilde{M}$. Then we obtain the following theorem and corollary :

Theorem 3.1. *Let M be a totally umbilical contact CR-submanifold of an indefinite Kenmotsu manifold $\tilde{M}$. Then the anti invariant distribution $D^\perp$ is one dimensional i.e. $\dim D^\perp = 1$.*

Proof. From (1.15) and (2.2) we get

$$\tilde{\nabla}_Z \phi W = \phi \tilde{\nabla}_Z W + g(\phi Z, W)\xi - \epsilon\eta(W)\phi Z. \quad (3.1)$$

Since M is totally umbilical, by Weingarten and Gauss formula we have

$$\nabla_Z^\perp \phi W - g(H, \phi W)Z = \phi[\nabla_Z W + g(Z, W)H] + g(\phi Z, W)\xi - \epsilon\eta(W)\phi Z \quad (3.2)$$

for any $Z, W \in \Gamma(D^\perp)$.

Taking inner product with $Z \in \Gamma(D^\perp)$ in (3.2) we have

$$g(H, \phi W)\|Z\|^2 = g(Z, W)g(H, \phi Z) + \epsilon\eta(W)g(\phi Z, Z) - g(\phi Z, W)\epsilon\eta(Z). \quad (3.3)$$

Interchanging Z and W we have

$$g(H, \phi Z)\|W\|^2 = g(W, Z)g(H, \phi W) + \epsilon\eta(Z)g(\phi W, W) - g(\phi W, Z)\epsilon\eta(W). \quad (3.4)$$

Substituting (3.3) in (3.4) and after a brief calculation we have

$$\begin{aligned} g(H, \phi W)\left[1 - \frac{g(Z, W)^2}{\|Z\|^2\|W\|^2}\right] + \frac{\epsilon}{\|Z\|^2}[g(\phi Z, W)\eta(Z) - g(\phi Z, Z)\eta(W)] + \\ \frac{\epsilon g(Z, W)}{\|Z\|^2\|W\|^2}[\eta(W)g(\phi W, Z) - \eta(Z)g(\phi W, W)] = 0. \end{aligned} \quad (3.5)$$

The Equation (3.5) has a solution if $Z \parallel W$ i.e. $\dim D^\perp = 1$. Hence the theorem. $\square$

Corollary 3.2. *A contact CR-submanifold of an indefinite Kenmotsu manifold reduces to a cosymplectic manifold if vector field Z becomes the structure vector field ξ .*

Proof. Taking inner product with Z in the equation (1.15) and using (1.3) we obtain

$$(\tilde{\nabla}_Z \phi)W = g(\phi Z, W)\epsilon\eta(Z) - \epsilon\eta(W)g(\phi Z, Z). \quad (3.6)$$

The equation vanishes provided $Z = \xi$ and hence the contact CR-submanifold reduces to a cosymplectic manifold. $\square$

4. CONTACT CR-SUBMANIFOLD OF AN INDEFINITE TRANS-SASAKIAN MANIFOLD

We consider a contact CR-submanifold M of an indefinite trans-Sasakian manifold $\tilde{M}$. Then we obtain the following theorem and corollary.

Theorem 4.1. *Let M be a totally umbilical contact CR-submanifold of an indefinite trans-Sasakian manifold $\tilde{M}$. Then the anti invariant distribution $D^\perp$ is one dimensional i.e. $\dim D^\perp = 1$.*

Proof. Using equations (1.16) and (2.2) we get

$$\tilde{\nabla}_Z \phi W = \alpha[g(Z, W)\xi - \epsilon\eta(W)Z] + \beta[g(\phi Z, W)\xi - \epsilon\eta(W)\phi Z] + \phi\tilde{\nabla}_Z W. \quad (4.1)$$

Since M is totally umbilical, by Weingarten and Gauss formula we have

$$\begin{aligned} \nabla_Z^\perp \phi W - g(H, \phi W)Z &= \alpha g(Z, W)\xi - \alpha\epsilon\eta(W)Z + \beta g(\phi Z, W)\xi \\ &\quad - \beta\epsilon\eta(W)\phi Z + \phi\nabla_Z W + g(Z, W)\phi H \end{aligned} \quad (4.2)$$

for any $Z, W \in \Gamma(D^\perp)$. Taking inner product with $Z \in \Gamma(D^\perp)$ in Equation (4.2) we have

$$\begin{aligned} -g(H, \phi W)g(Z, Z) &= \alpha g(Z, W)\epsilon\eta(Z) - \alpha\epsilon\eta(W)g(Z, Z) + \beta g(\phi Z, W)\epsilon\eta(Z) \\ &\quad - \beta\epsilon\eta(W)g(\phi Z, Z) + g(Z, W)g(\phi H, Z). \end{aligned} \quad (4.3)$$

Interchanging Z and W we have

$$\begin{aligned} -g(H, \phi Z)\|W\|^2 &= \alpha g(W, Z)\epsilon\eta(W) - \alpha\epsilon\eta(Z)\|W\|^2 + \beta g(\phi W, Z)\epsilon\eta(W) \\ &\quad - \beta\epsilon\eta(Z)g(\phi W, W) + g(W, Z)g(\phi H, W). \end{aligned} \quad (4.4)$$

$$\begin{aligned} g(H, \phi Z) &= -\frac{\alpha g(W, Z)\epsilon\eta(W)}{\|W\|^2} + \alpha\epsilon\eta(Z) - \frac{\beta g(\phi W, Z)\epsilon\eta(W)}{\|W\|^2} + \frac{\beta\epsilon\eta(Z)g(\phi W, W)}{\|W\|^2} + \\ &\quad \frac{g(W, Z)g(H, \phi W)}{\|W\|^2}. \end{aligned} \quad (4.5)$$

Substituting Equation (4.3) in Equation (4.5) we have after some steps of calculations

$$\begin{aligned} g(H, \phi Z) &= -\frac{\alpha g(W, Z)\epsilon\eta(W)}{\|W\|^2} + \alpha\epsilon\eta(Z) - \frac{\beta g(\phi W, Z)\epsilon\eta(W)}{\|W\|^2} + \frac{\beta\epsilon\eta(Z)g(\phi W, W)}{\|W\|^2} \\ &\quad + \frac{g(W, Z)}{\|W\|^2} \left[-\frac{\alpha g(Z, W)\epsilon\eta(Z)}{\|Z\|^2} + \alpha\epsilon\eta(W) - \frac{\beta g(\phi Z, W)\epsilon\eta(Z)}{\|Z\|^2} \right. \\ &\quad \left. + \frac{\beta\epsilon\eta(W)g(\phi Z, Z)}{\|Z\|^2} - \frac{g(Z, W)g(\phi H, Z)}{\|Z\|^2} \right], \end{aligned} \quad (4.6)$$

$$\begin{aligned}
& g(H, \phi Z) \left[1 - \frac{g(Z, W)^2}{\|Z\|^2 \|W\|^2} \right] - \alpha \epsilon \eta(Z) \left[1 - \frac{g(Z, W)^2}{\|Z\|^2 \|W\|^2} \right] - \frac{\beta g(\phi Z, W) \epsilon}{\|W\|^2} [\eta(W) \\
& - \frac{\eta(Z) g(Z, W)}{\|Z\|^2}] - \frac{\beta \epsilon}{\|W\|^2} [\eta(Z) g(\phi W, W) - \frac{\eta(W) g(\phi Z, Z) g(W, Z)}{\|Z\|^2}] = 0.
\end{aligned} \tag{4.7}$$

The equation Equation (4.7) has a solution if $Z \parallel W$ i.e. $\dim D^\perp = 1$. Hence the proof is complete. $\square$

Corollary 4.2. *A contact CR-submanifold of an indefinite trans-Sasakian manifold reduces to a cosymplectic manifold when the vector field Z becomes the structure vector field ξ .*

Proof. Taking inner product with Z in the equation (1.16) and using (1.3) we obtain

$$(\tilde{\nabla}_Z \phi)W = \epsilon \eta(Z) [\alpha g(Z, W) + \beta g(\phi Z, W)] - \epsilon \eta(W) [\alpha \|Z\|^2 + \beta g(\phi Z, Z)]. \tag{4.8}$$

The equation vanishes provided $Z = \xi$ and thus this contact CR-submanifold reduces to a cosymplectic manifold. $\square$

This is an example of a trans-Sasakian manifold, which proves the above corollary.

Example 4.3. Let R^3 be a 3-dimensional Euclidean space with rectangular coordinates (x, y, z) . In R^3 we define

$$\eta = dz - ydx, \xi = \frac{\partial}{\partial z}, \tag{4.9}$$

$$\phi\left(\frac{\partial}{\partial x}\right) = \frac{\partial}{\partial y}, \phi\left(\frac{\partial}{\partial y}\right) = -\frac{\partial}{\partial x} - y\frac{\partial}{\partial z}, \phi\left(\frac{\partial}{\partial z}\right) = 0. \tag{4.10}$$

The Riemannian metric g is defined by the matrix:

$$\begin{bmatrix}
\epsilon y^2 & 0 & -\epsilon y \\
0 & 0 & 0 \\
-\epsilon y & 0 & \epsilon
\end{bmatrix}$$

Then it can be easily seen that (ϕ, ξ, η, g) forms an indefinite trans-Sasakian structure in R^3 for some functions α and β and it reduces to a cosymplectic manifold when $Z = \xi$. The corollary (3.1) can be verified by assuming $\alpha = 0$ and $\beta = 1$.

5. CONTACT CR-SUBMANIFOLD OF AN EPSILON-PARACONTACT SASAKIAN MANIFOLD

In this section we consider a contact CR-submanifold M of an ϵ -paracontact Sasakian manifold $\tilde{M}$. Then we get the following theorem and corollary.

Theorem 5.1. *Let M be a totally umbilical contact CR-submanifold of an ϵ -paracontact Sasakian manifold $\tilde{M}$. Then the anti invariant distribution $D^\perp$ is one dimensional i.e. $\dim D^\perp = 1$.*

Proof. Substituting (1.17) in (2.2) we have

$$\tilde{\nabla}_Z \phi W = \phi \tilde{\nabla}_Z W - g(\phi Z, \phi W) \xi - \epsilon \eta(W) \phi^2 Z. \quad (5.1)$$

Since M is totally umbilical by Weingarten and Gauss formula we have

$$\nabla_Z^\perp \phi W - g(H, \phi W) Z = \phi [\nabla_Z W + g(Z, W) H] - g(\phi Z, \phi W) \xi - \epsilon \eta(W) \phi^2 Z \quad (5.2)$$

for any $Z, W \in \Gamma(D^\perp)$. Taking inner product with $Z \in \Gamma(D^\perp)$ in (5.2) we have

$$g(H, \phi W) \|Z\|^2 = g(Z, W) g(H, \phi Z) - \epsilon \eta(W) \|Z\|^2 + g(Z, W) \epsilon \eta(Z). \quad (5.3)$$

Interchanging Z and W we have

$$g(H, \phi Z) \|W\|^2 = g(W, Z) g(H, \phi W) - \epsilon \eta(Z) \|W\|^2 + g(Z, W) \epsilon \eta(W). \quad (5.4)$$

Substituting (5.3) in (5.4) and simplifying we have

$$g(H, \phi Z) \left[1 - \frac{g(Z, W)^2}{\|Z\|^2 \|W\|^2}\right] + \epsilon \eta(Z) \left[1 - \frac{g(Z, W)^2}{\|Z\|^2 \|W\|^2}\right] = 0. \quad (5.5)$$

The equation (5.5) has a solution if $Z \parallel W$ i.e. $\dim D^\perp = 1$. Hence the proof follows. $\square$

Corollary 5.2. *A contact CR-submanifold of an ϵ -paracontact Sasakian manifold reduces to a cosymplectic manifold if the vector field Z becomes the structure vector field ξ .*

Proof. Taking inner product with Z in equation (1.17) and using (1.3) we get

$$(\tilde{\nabla}_Z \phi) W = -g(\phi Z, \phi W) \epsilon \eta(Z) - \epsilon \eta(W) g(\phi^2 Z, Z). \quad (5.6)$$

The equation vanishes provided $Z = \xi$ and hence this contact CR-submanifold reduces to a cosymplectic manifold. $\square$

Acknowledgement. The first author has been sponsored by UGC-BSR fellowship, India. The authors are thankful to referee for valuable suggestions.

REFERENCES

1. A. Bejancu, *CR-submanifolds of a Kaehlerian manifold I*, Proc. of the Amer. Math. Society, **69** (1978), 135-142.
2. A. Bejancu, *CR-submanifolds of a Kaehlerian manifold II*, Transactions of the Amer. Math. Society, **250** (1979), 333-345.
3. D.E. Blair, *Contact manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, vol. **509**, Springer Verlag, Berlin, (1976).
4. D.E. Blair and D.K. Showers, *Almost contact manifolds with Killing structure tensors II*, Jour. of Diff. Geom., vol. **9**, (1974), 577-582.
5. B.Y. Chen, *Geometry of submanifolds*, M. Dekker, New York, (1973).
6. B.Y. Chen and K. Ogiue, *On totally real submanifolds*, Transaction of the Amer. Math. Society, **193** (1974), 257-266.
7. M. Kobayashi, *CR-submanifolds of a Sasakian manifold*, Tensor, N.S. vol. **35** (1981), 297-307.

8. M. Kon, *Invariant submanifolds in Sasakian manifolds*, Math. Ann. **219** (1976), 277-290.
9. G.D. Ludden, *Submanifolds of cosymplectic manifolds*, Jour.of Diff. Geom., vol. **4**, (1970), 237-244.
10. B.C. Montano, *Topology of 3-cosymplectic manifolds*, Quart.Jour. Math. **00** (2011), 1-24.
11. K. Ogiue, *Differential geometry of kaehler submanifolds*, Advances in Mathematics, **13** (1974), 73-114.
12. M.Tarafdar, Bandana Das, A.Bhattacharya, *Contact CR-submanifolds of an indefinite Sasakian manifold*, Tensor, N.S. vol. **72** (2010), 214-224.
13. K. Yano and M. Kon, *Anti-invariant submanifolds*, Marcel Dekker Inc., New York,(1976).

¹ DEPARTMENT OF MATHEMATICS, JADAVPUR UNIVERSITY, KOLKATA, INDIA.

E-mail address: barnali.laha87@gmail.com

² DEPARTMENT OF MATHEMATICS, JADAVPUR UNIVERSITY, KOLKATA, INDIA.

E-mail address: badan06@yahoo.co.in

³ DEPARTMENT OF MATHEMATICS, JADAVPUR UNIVERSITY, KOLKATA, INDIA.

E-mail address: bhattachar1968@yahoo.co.in