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EQUITABLE ISOLATE DOMINATION IN GRAPHS

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Abstract. A subset $S \subseteq V(G)$ is said to an equitable isolate dominating set of a graph G if it is both equitable dominating set of G and isolate dominating set of G. The minimum cardinality of an equitable isolate dominating set is called equitable isolate domination number of G and is denoted by $\gamma_{e0}(G)$. An equitable isolate dominating set S of G is called γ_{e0}-set of G. In this paper, the authors give characterizations of equitable isolate dominating set of some graphs and graphs obtained from the join and corona of two graphs. Furthermore, the equitable isolate domination numbers of these graphs is determined, and the graphs with no equitable isolate dominating sets are investigated.

1. Introduction and preliminaries

1.1. **Background of the study.** Recently, there has been a growing interest in the applications of one of the most extensively studied topics in graph theory: the study of domination in graphs. This concept was introduced by Claude Berge in 1958 when he defined the coefficient of external stability, which is now known as domination [4]. Due to the richness of research and its diverse applications to graphs, numerous variants of domination have emerged, as highlighted in [9], [14], [20], and [21]. Among these variants are isolate domination and equitable domination.

In 2013, the concept of isolate domination in graphs was studied by Hamid and Balamurugan [15] as a generalization of the work introduced by Ariola [2]. This study opened numerous opportunities for exploring various research topics, prompting many mathematicians to investigate different variants of this concept. Armada and Hamja, in [3], explored perfect isolate domination in graphs, where they defined domination that is both perfect and isolate simultaneously. Several authors have also conducted extensive studies on these different variants, as documented in [8].

Another prominent variant of domination is equitable domination. It is believed to have been introduced by A. Anitha et al. in [1] and was further discussed in the work of G. Deepak et al. in [11]. This concept has inspired numerous interesting studies, including the notion of perfect equitable domination, introduced by Caay

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and Arugay in 2017 [5]. Their work was motivated by the concept of equitable domination and the study of perfect domination introduced by M. Livingston and Stout in [18], which stemmed from supplementary research conducted by the same authors in [19]. This field of study has since motivated extensive research. For instance, the work of Caay and Durog in [6] developed from the interplay between independent sets and equitable domination. Numerous other studies and research topics have also been inspired by equitable domination, such as those discussed in [22]. Additionally, various authors have contributed significantly to this area, as reflected in [7], [12], and [23].

In this paper, we study the equitable isolate domination in graphs. A subset $S \subseteq V(G)$ is said to be an equitable isolate dominating set if it is both an isolate dominating set and an equitable dominating set. To give clarity, the flow of our paper is as follows: In this section, we introduce the necessary notations and basic concepts that are used in this study. We also introduce the isolate domination and equitable dominations, and some of their results from the references that are used in the discussion of the study. We also established the case when these two dominations implies each other and so we come up with our formal working definition. In Section 2, we show our results of our study.

1.2. **Preliminaries and the Working Definitions.** Throughout this paper, the graph we consider here is a connected simple graph. That means, there are no loops and multiple edges. A pair $G = (V(G), E(G))$ is called a graph (on V). The elements of $V(G)$ are called the vertices of G and the elements of $E(G)$ are called the edges of G. If no confusion arises, we can use V and E to denote the set of vertices and set of edges of G, respectively. Suppose $v \in V$, the neighborhood of v is the set $N_G(v) = \{u \in V : uv \in E.\}$. Given $D \subseteq V$, the set $N_G(D) = N(D) = \bigcup_{v \in D} N_G(v)$ and the set $N_G[D] = N[D] = D \bigcup N(D)$ are the *open neighborhood* and the *closed neighborhood* of D respectively.

We denote $\Delta(G)$ and $\delta(G)$ to be the maximum and minimum degree of G, respectively. We denote P_n, C_n, K_n, T_n for the path graph, cycle graph, complete graph and trees of order n, respectively.

Theorem 1.1. [10] *A graph G is a cycle graph if and only if every vertex of G is adjacent to two other vertices.*

Definition 1.2. [10] A **spanning subgraph** of a graph G is a subgraph obtained by deleting some edges of G with the same vertex set.

Example 1.3. A cycle C_n is a spanning subgraph of a complete graph K_n.

The following are the definitions of the binary operations in graphs used in this study: join, corona and cartesian product.

Definition 1.4. [16] The **join** $G + H$ of the two graphs G and H is the graph with vertex set

$$V(G + H) = V(G) + V(H)$$

and the edge set

$$E(G + H) = E(G) \cup E(H) \cup \{uv \ : \ u \in V(G), \ v \in V(H)\}.$$

Definition 1.5. [13] The **corona** $G \circ H$ of two graphs G and H is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the ith vertex of G to every vertex in the ith copy of H.

We will now introduce the following important notions for this paper.

Definition 1.6. [4] A subset D of $V(G)$ is a **dominating set** of G if for every $v \in V(G) \backslash D$, there exists $u \in D$ such that $uv \in E(G)$. That is, $N[D] = V(G)$. The minimum cardinality of the dominating set D of G is called a **domination number** of G and is denoted by $\gamma(G)$. In this case, D is called **γ-set** of G.

In [1] and [11], a dominating set S of G is said to be an *equitable dominating set* of G if for every $u \in S$, there exists $v \in V(G) \setminus S$ with $uv \in E(G)$ such that $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of an equitable dominating set S of G is called an *equitable domination number* of G and is denoted by $\gamma_e(G)$. In this case, we say S a γ_e-set of G.

Theorem 1.7. [24] *A γ_e-set S of G is minimal if and only if for every $u \in S$, one of the following holds:*

(i) *Either $N(u) \cap S = \varnothing$ or $|\deg(v) - \deg(u)| \geq 2$ for all $N(u) \cap S$.*
(ii) *There exists a vertex $v \in V(G) \setminus S$ such that $N(v) \cap S = \{u\}$ and*

$$|\deg(v) - \deg(u)| \leq 1.$$

The following result has a similar result of [6], and thus works the same with equitable domination.

Proposition 1.8. *Given a complete graph K_n, $n \geq 3$, $\gamma_e(K_n) = 1$.*

Remark 1.9. If a singleton set $\{v\}$ is a γ-set of G, then $\{v\}$ is a γ_0-set of G.

The following result has a similar result of [3], and thus works the same with isolate domination.

Theorem 1.10. [2] *Let G and H be any nontrivial graphs. The $S \subseteq V(G+H)$ is a γ_0-set of $G+H$ if and only if either $S \subseteq V(G)$ is a γ_0-set of G or $S \subseteq V(H)$ is a γ_0-set of H.*

Proposition 1.11. *Given a complete graph K_n, $n \geq 3$, $\gamma_0(K_n) = 1$.*

There are graphs where the equitable domination number and the isolate domination number coincide. By Proposition 1.8 and Proposition 1.11, we have the following example to this claim.

Proposition 1.12. *Let $n \geq 2$. Then $\gamma_0(K_n) = \gamma_e(K_n) = 1$.*

By virtue of Proposition 1.12, we now introduce the formal defintion of our paper.

Definition 1.13. A dominating set $S \subseteq V(G)$ of G is said to be an **equitable isolate dominating set** of G (or **γ_{e0}-set** of G) if there exists $u \in S$ such that $uv \in E(G)$ for some $v \in S$, and for every $w \in V(G) \setminus S$, there exists $u \in S$ such that $uw \in E(G)$ and $|\deg(u) - \deg(w)| \leq 1$. The minimum cardinality of an equitable isolate dominating set of G is called **equitable isolate domination number** of G and is denoted by $\gamma_{e0}(G)$.

2. Main results

In this section, we present the results for Equitable Isolate dominations in graphs. The minimality of an equitable isolate dominating set S follows from Theorem 1.7 with the additional property that it acquires at least one vertex in S that is not adjacent to the other vertices in S.

To provide the results of the paper, we present the following definition.

Definition 2.1. A dominating set $S \subseteq V(G)$ of G is said to be a **strong equitable isolate dominating set** of G (or **strong $\boldsymbol{\gamma_{e0}}$-set** of G) if there exists $u \in S$ such that $uv \in E(G)$ for some $v \in S$, and for every $w \in V(G) \setminus S$, there exists $u \in S$ such that $uw \in E(G)$ and $|\deg(u) - \deg(w)| = 0$.

Proposition 2.2. *Given a path P_n and cycle C_n, $n \geq 3$, $\gamma_{e0}(P_n) = \gamma_{e0}(C_n) = \left\lceil \frac{n}{3} \right\rceil$.*

Theorem 2.3. *Let G be any graph of order $n \geq 3$. If G is $(n-1)$-regular, then $\gamma_{e0}(G) = 1$. That is, $\gamma_{e0}(K_n) = 1$.*

Proof. The proof follows from Proposition 1.12 □

Theorem 2.4. *Let G be a k-regular graph of order n. Then $\gamma_{e0}(G) = j$, where j is the least integer satisfying $jk \geq n$.*

Proof. Let $v_i \in V(G)$. Since G is k-regular, v_i dominates k vertices of G. Hence, there are $n - (k+1)$ vertices that are not grouped when choosing v_i. Choosing another vertex $v_i + 1$ and dominates k vertices from the remaining $n - (k+1)$ ungrouped vertices, there are remaining $n - 2(k+1)$. Repeating the process, we have $\dfrac{n}{k+1}$ steps to dominate all the vertices of G. Hence, there must be $\dfrac{n}{k+1}$ vertices that dominates the other vertices.

Observe that C_n is an induced subgraph of G. By Proposition 2.2, we have

$$\gamma_{e0}\left(C_n\right) = \left\lceil \frac{n}{3} \right\rceil.$$

Let $\gamma_{e0}\left(C_n\right) = \left\lceil \frac{n}{3} \right\rceil = j$. That is, j is the least integer satisfying $j \geq \dfrac{n}{3}$. Observe that $k \geq 3$ and so $\dfrac{n}{k+1} \leq \dfrac{n}{k} \leq \dfrac{n}{3} \leq j$. Thus, we have $j \geq \dfrac{n}{k}$. Therefore, we have $kj \geq n$. □

Proposition 2.5. *There does not exist γ_e-set and γ_{e0}-set of a star graph of order S_n, $n \geq 4$. However, $\gamma_{e0}(S_n) = 1$.*

Theorem 2.6. *Let G be a graph of order n. Then G is k-regular, with nk an even integer, if and only if G has a strong γ_{e0}-set.*

Proof. Assume G is a k-regular graph of order n with $nk = 2m$ for some positive integer m. By Theorem 2.4, G admits a γ_{e0}-set. Since G os k-regular, every vertices of G have degrees k. Thus, for every $u, v \in V(G)$, we have $|\deg(u) - \deg(v)| = 0$. Hence, S is a strong γ_{e0}-set. Conversely, suppose G

has a strong γ_{e0}-set S. Then for every $u \in S$ and $v \in V(G) \setminus S$, we have $|\deg(u) - \deg(v)| = 0$. Hence, G is a regular graph. □

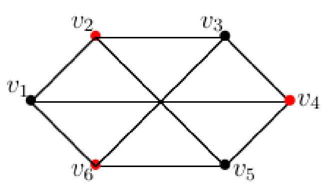
Proposition 2.7. *Let S_1 and S_2 be the minimal nontrivial γ_{e0}-sets of G and H, respectively. That is, $|S_1| \neq 1$ and $|S_2| \neq 1$. Then $S_1 \cup S_2$ is a not a γ_{e0}-set of $G + H$ but a γ_e-set of $G + H$.*

Proof. Let S_1 and S_2 be the minimal nontrivial γ_{e0}-sets of G and H, respectively. Then for every $u \in V(G) \setminus S_1$, there exists $v \in S_1$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. Similarly, for every $x \in V(H) \setminus S_2$, there exists $y \in S_2$ such that $xy \in E(G)$ and $|\deg(x) - \deg(y)| \leq 1$. Then $S_1 \cup S_2 := \{v_i, y_j,\ v_i \in S_1,\ y_j \in S_2,\ \text{for some } i, j\} \subseteq V(G + H)$. Thus, for all $w \in V(G + H) \setminus (S_1 \cup S_2)$, there exists $z \in S_1 \cup S_2$ such that $wz \in E(G + H)$ and $|\deg(w) - \deg(z)| \leq 1$. Hence, $S_1 \cup S_2$ is a γ_e-set of $G + H$.

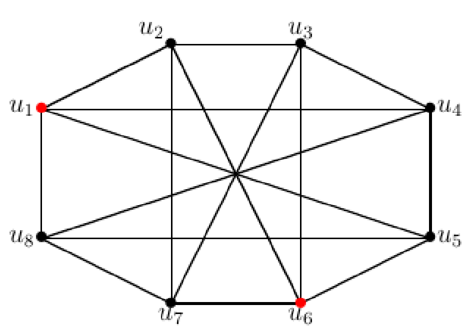
Now if $v_i \in S_1$ is an isolated vertex of S_1, then $v_i u_j \in E(G+H)$ for all $u_j \in S_2$, and $v_k u_j \in E(G + H)$, for all $v_k \in S_1$ with $v_i \neq v_k$. This means that v_i is no longer isolated. Since v_i is arbitrary, this holds for all isolated dominating vertices. Hence, $S_1 \cup S_2$ is not γ_{e0}-set of $G + H$. □

Remark 2.8. $S_1 \cup S_2$ is a γ_e-set of $G + H$ of Proposition 2.7 is not necessarily minimal.

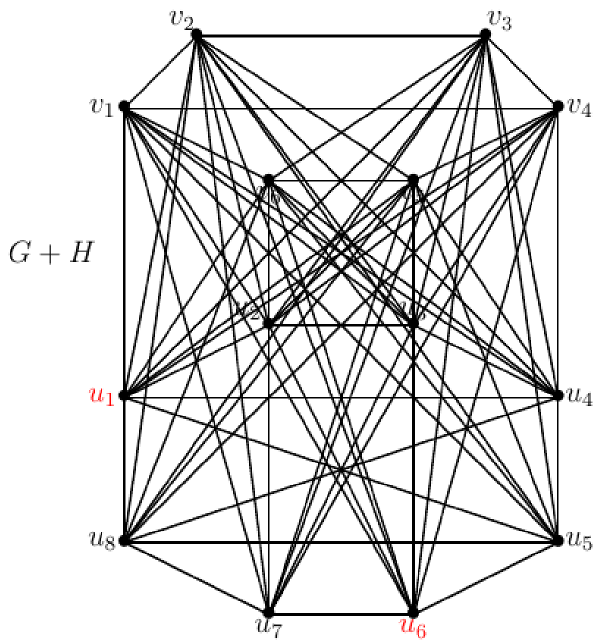
To see this, consider the following graphs G and H below.



G



H



Note that $S_1 = \{v_2, v_6, v_4\}$ and $S_2 = \{u_1, u_6\}$ are the minimal γ_{e0}-set of G and H, respectively. However, $S_1 \cup S_2 = \{v_2, v_6 v_4, u_1, u_6\}$ is not a γ_{e0}-set, but is a γ_e-set. Moreover, $S_1 \cup S_2$ is not a minimal γ_e-set since S_2 is a minimal set of $G + H$. □

Theorem 2.9. *Let G and H be any graph. Then $G+H$ has a γ_{e0}-set if and only if either of the following holds*

(i) $|(\deg(u) - \deg(v)) + (|V(G)| - |V(H)|)| \leq 1$ *for every* $u \in V(H)$ *and for some* $v \in S_1 \subseteq V(G)$ *where* S_1 *is a* γ_{e0}*-set of* G*; or*

(ii) $|(\deg(w) - \deg(z)) + (|V(G)| - |V(H)|)| \leq 1$ *for every* $w \in V(G)$ *and for some* $z \in S_2 \subseteq V(H)$ *where* S_2 *is a* γ_{e0}*-set of* H.

Proof. Let $\overline{S}$ be a γ_{e0}-set of $G+H$. By Theorem 1.10, $\overline{S} \subseteq V(G)$ is γ_{e0}-set of G or $\overline{S} \subseteq V(H)$ is γ_{e0}-set of H. Without loss of generality, assume $\overline{S} \subseteq V(G)$ is γ_{e0}-set of G. Then for every $u \in V(G+H) \setminus \overline{S}$, there exists $v \in \overline{S}$ such that

$$|\deg(u_{G+H}) - \deg(v_{G+H})| \leq 1. \tag{2.1}$$

Since $\overline{S} \subseteq V(G)$ is γ_{e0}-set of G, we have

$$\deg(v_{G+H}) = \deg(v_G) + |V(H)|$$

Since $u \in \{u_i, u_j \ : \ u_i \in V(G) \setminus \overline{S}, \ u_j \in V(H)\}$, we have

$$\deg(u_{G+H}) = \deg(u_G) + |V(H)| \quad \text{or}$$
$$\deg(u_{G+H}) = \deg(u_H) + |V(G)|$$

Thus,

$$\begin{aligned}
&|(\deg(u_H) - \deg(v_G)) + (|V(G)| - |V(H)|)| \\
&= |[(\deg(u_{G+H}) - |V(G)|) - (\deg(v_{G+H}) - |V(H)|)] + (|V(G)| - |V(H)|)| \\
&= |\deg(u_{G+H}) - \deg(v_{G+H})| \\
&\leq 1.
\end{aligned}$$

Now since $\overline{S} \subseteq V(G)$ is γ_{e0}-set of G, it follows that for all $w \in V(G+H)$, there exists $z \in \overline{S}$ such that $|\deg(w) - \deg(z)| \leq 1$. Thus, $|\deg(w)| - |\deg(z)| \leq 1$. Note that $w \in \{w_i, w_j \ : \ w_i \in V(G),\ w_j \in V(H)\}$. Thus, we have

$$\begin{aligned}
|\deg(w_i)| - |\deg(z)| \leq 1 \leq 1 \quad \text{and} \\
|\deg(w_j)| - |\deg(z)| \leq 1 \leq 1.
\end{aligned}$$

Let $x \in V(G) \setminus \overline{S}$. Then $x \in V(G+H) \setminus \overline{S}$, and so we have

$$|\deg(x_{G+H}) - \deg(z_{G+H})| \leq 1,$$

for some $z \in \overline{S}$. But we know that

$$\begin{aligned}
\deg(x_{G+H}) = \deg(x_G) + |V(H)| \\
\deg(z_{G+H}) = \deg(z_G) + |V(H)|
\end{aligned}$$

Thus,

$$\deg(x_G) - \deg(z_G) = \deg(x_{G+H}) - \deg(z_{G+H}) \leq 1.$$

This means that $\overline{S}$ is a γ_{e0}-set of G. The same proof follows for the other case. This proves the claim. □

Corollary 2.10. *For any $n \geq 3$ and $m \geq 3$, $\gamma_{e0}(K_n + K_m) = 1$.*

Theorem 2.11. *Let G and H be any graph such that G and H have γ_{e0}-set. Then $G \circ H$ has γ_{e0}-set.*

Proof. Suppose G and H have γ_{e0}-set. Let S_G be a γ_{e0}-set of G and S_H be a γ_{e0}-set of H. □

Corollary 2.12. *Let G and H be any graph such that G of order n has a minimal γ_{e0}-set. If there exists $u \in V(H)$ such that $|\deg(u) - \deg(v_i)| \leq 1$ for all $v_i \in V(H)$ with $v_i \neq u$, $S \cup \{u\}^n$ γ_{e0}-set of $G \circ H$, where $\{u\}^n$ denotes the n copies of the same vertex u in H. Consequently,*

$$\gamma_{e0}(G \circ H) = |S| + n.$$

Proof. Let S be a γ_{e0}-set of a graph G of order n. If there exists $u \in V(H)$ such that $|\deg(u) - \deg(v_i)| \leq 1$ for all $v_i \in V(H)$ with $v_i \neq u$, then u must dominate all vertices of H. Since S is a γ_{e0}-set of G, for all $y_i \in V(G) \setminus S$, there exists $x \in S$ such that the $|\deg(x) - \deg(y_i)| \leq 1$. Hence, $S \cup \{u\}^n$ dominates all vertices of $V(G \circ H) \setminus (S \cup \{u\}^n)$, where n is the number of copies of u since there are $|V(G)| = n$ copies of H in $G \circ H$. Since S is γ_{e0}-set of G, it follows that for all $y_i \in V(G) \setminus S$, there exists $x \in S$ with $|\deg(x) - \deg(y_i)| \leq 1$, and $y_i u \in E(G \circ H)$ for some u in the ith copy of H. Hence, there exists $u_j \in S \cup \{u\}^n$ such that $u_j u_t \notin E(G)$ for some $u_t \in S \cup \{u\}^n$. This further implies that $S \cup \{u\}^n$

is not connected subgraph of $G \circ H$. Therefore, $S \cup \{u\}^k$ is γ_{e0}-set of $G \circ H$. Consequently, $\gamma_{e0}(G \circ H) = |S| + n$. □

Corollary 2.13. *Let G and H be any graph such that G has a minimal γ_{e0}-set S. If for every $u \in V(H)$, $|\deg(u) - \deg(v)| \leq 1$ for some $v \in S$, then $\gamma_{e0}(G \circ H) = |V(G)|$.*

Proof. By Corollary 2.12, it follows that $|S| \cup \{u\}^n$ is a γ_{e0}-set of $G \circ H$. If for all $u \in V(H)$, $|\deg(u) - \deg(v)| \leq 1$ for some $v \in S$, S already dominates all vertices of $V(H)$ which are adjacent to $v_i \in S$ for some ith copy $V(H)$. But there are some copies of $V(H)$ which are not dominated by some vertices of S. Since for all $w \in V(H)$, $|\deg(w) - \deg(v)| \leq 1$, there exists $v \in S$ it follows that $|\deg(u) - \deg(u_j)| \leq 1$, for all $u_j \in V(H)$, $u_j \neq u$. Hence, u dominates all other vertices of H and so $\{u\}$ is a γ_{e0}-set. By Remark 1.9, $\{u\}$ is a γ_{e0}-set. Thus, $\{u\}$ is γ_{e0}-set. Hence,

$$S \cup \bigcup_{i=1}^{|V(G)|-|S|} \{u\}_i$$

forms a minimal γ_{e0}-set of $G \circ H$. Moreover, since $\{u\}$ and S are disjoints

$$S \cup \bigcup_{i=1}^{|V(G)|-|S|} \{u\}_i = |S| + \sum_{i=1}^{|V(G)|-|S|} i = |S| + (n - |S|) = n.$$

This proves the claim. □

Corollary 2.14. *For any $n \geq 3$ and $m \geq 3$, $\gamma_{e0}(K_n \circ K_m) = \min\{n, m\}$.*

3. Conclusion

This paper has introduced the concept of equitable isolate domination in graphs and thus, has open a lot of opportunity to widen the research in the area of Graph Theory. This gives characterization to graphs and discusses the existence since there are also graphs that don't admit this domination. With this, the authors recommend to further explore this domination in many graphs and binary operation that are not discussed in this paper, and to give characterization and properties that can show relationship of this type of domination to the other well-known dominations.

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