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THE TWO-DIMENSIONAL DIVISOR FUNCTION OVER PIATETSKI-SHAPIRO SEQUENCES

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Abstract. We study the values of the two-dimensional divisor function and its related functions on the elements of a Piatetski-Shapiro sequence.

1. Introduction and results

The Piatetski-Shapiro sequence of parameter c is defined by

$$\mathbb{N}^c = \{\lfloor n^c \rfloor\}_{n\in\mathbb{N}} \qquad (c > 1, c \notin \mathbb{N}),$$

where $\lfloor z \rfloor$ is the integer part of $z \in \mathbb{R}$. The Piatetski-Shapiro sequence was introduced by Piatetski-Shapiro [10] to study prime numbers in a sequence of the form $\lfloor f(n) \rfloor$, where $f(n)$ is a polynomial. The study of the distribution of arithmetical functions on Piatetski-Shapiro is studied by many authors; see, for example, [1, 4, 5, 6, 8, 11, 14, 15, 17] and the references contained therein. In particular, the author studied the distribution of square-full and (k, r)- integers in Piatetski-Shapiro sequence in [12, 13], respectively. This is a special problem related to the two-dimensional divisor problem. Thus, it is interesting to extend this to the two-dimensional divisor functions and its related functions on the elements of a Piatetski-Shapiro sequence. Let $1 \leq a \leq b$ be fixed integers, and we denote $d(a, b; n)$ the number of representations of k as $k = n_1^a n_2^b$, where n_1, n_2 are natural numbers. We prove the following theorem.

Theorem 1.1. *Let a and b be fixed integers such that $1 \leq a < b < 3a$. Let exponent pairs (κ_1, λ_1) and (κ_2, λ_2) satisfying $\frac{\lambda_1}{1+\kappa_1} < \frac{a}{b} < \frac{\lambda_2}{1+\kappa_2}$ and $\kappa_1 > \kappa_2$. For $1 < c < \frac{a(\lambda_2-\lambda_1)-(\kappa_2-\kappa_1)}{\lambda_2(1+\kappa_1)-\lambda_1(1+\kappa_2)}$, we have*

$$\sum_{n\leq x} d(a, b; \lfloor n^c \rfloor) = \frac{ac}{a + c - ac}\zeta(b/a)x^{1-c+c/a} + O\Big(x^{\frac{c}{b}+\frac{\kappa_2}{1+\kappa_2}+(\frac{a}{b}-\frac{\lambda_2}{1+\kappa_2})\frac{\kappa_1-\kappa_2}{\lambda_1(1+\kappa_2)-\lambda_2(1+\kappa_1)}} \log x\Big).$$

For example, for $(\kappa_1, \lambda_1) = (2/7, 4/7)$ and $(\kappa_2, \lambda_2) = (1/14, 2/14)$, we have

$$\sum_{n\leq x} d(2, 3; \lfloor n^c \rfloor) = \frac{2c}{2 - c}\zeta(3/2)x^{1-c/2} + O(x^{c/3+4/39}),\ 1 < c < 14/13.$$

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However, we can not apply Theorem 1.1 with $a = 1$, $b \geq 3$. To deal this, we use the several good estimations for the number of integer n up to x such that functions $\lfloor n^c \rfloor$ belongs to an arithmetic progression. We obtain the following theorem.

Theorem 1.2. *Let $b \geq 3$ be fixed integer. For $1 < c < 2$, we have*

$$\sum_{n \leq x} d(1, b; \lfloor n^c \rfloor) = \zeta(b)x + O\big(x^{\frac{c+1}{3}} \log x\big).$$

For an integer $n \geq 1$, let $\tau(n)$ be the divisor function, which is the number of positive divisors of n. The distribution of the divisor function over Piatetski-Shapiro sequences was studied by several authors. In [3] Arkhipov, Soliba and Chubarikov showed that, for $1 < c < \frac{8}{7}$,

$$\sum_{n \leq x} \tau(\lfloor n^c \rfloor) = xQ(\log x) + O\Big(\frac{x}{\log x}\Big),$$

where $Q(x)$ is a polynomial of degree 1. In 2003, Lü and Zhai [9] used the exponent pair method to improve the range of c to $1 < c < \frac{495}{433}$. Very recently, Wang and Zhang [16] improved the range of c to $1 < c < \frac{6}{5}$ by using some results of Jutila. Here, we obtain the following theorem.

Theorem 1.3. *Let $1 < c < 2$ and (κ, λ) be an exponent pair, such that $c(1 + \kappa + \lambda) < 2$, we have*

$$\sum_{n \leq x} \tau(\lfloor n^c \rfloor) = cx \log x + (2\gamma - c)x + O(x^{(c+ck+c\lambda+2\kappa)/(2+2\kappa)} \log x).$$

For example, for $(\kappa, \lambda) = (1/6, 2/3)$, we have

$$\sum_{n \leq x} \tau(\lfloor n^c \rfloor) = cx \log x + (2\gamma - c)x + O(x^{(11c+2)/14} \log x),\ \ 1 < c < 12/11.$$

For $(\kappa, \lambda) = (2/7, 4/7)$, we have

$$\sum_{n \leq x} \tau(\lfloor n^c \rfloor) = cx \log x + (2\gamma - c)x + O(x^{(13c+4)/18} \log x),\ \ 1 < c < 14/13.$$

Furthermore, we study the average order of the divisor function $\sigma_1(\lfloor n^c \rfloor)$, where $\sigma_1(n)$ is the sum of divisors of n. We obtain the following theorem.

Theorem 1.4. *Let $1 < c < 2$ and (κ, λ) be an exponent pair, such that $c\lambda < 1$, we have*

$$\sum_{n \leq x} \sigma_1(\lfloor n^c \rfloor) = \frac{\zeta(2)}{c+1} x^{c+1} + O(x^{c+(c\lambda+\kappa)/(1+\kappa)}).$$

2. Lemmas and notations

Throughout this paper, implied constants in symbols O and $\ll$ may depend on the parameters, $\alpha, \beta, c, \epsilon$, but are absolute otherwise. For given functions F and G, the notations $F \ll G$ and $F = O(G)$ are all equivalent to the statement that the inequality $|F| \leq C|G|$ holds with some constant $C > 0$.

Lemma 2.1. *[see Theorem 3.2 in [2]] Let $x \geq 1$. If $s > 1$, then*

$$\sum_{n \leq x} \frac{1}{n^s} = \zeta(s) + O(x^{1-s}).$$

Lemma 2.2. *[see Theorem 3.3 in [2]] Let $x \geq 1$, then*

$$\sum_{n \leq x} \tau(n) = x \log x + (2\gamma - 1)x + O(x^{1/2}),$$

where γ is Euler's constant.

Lemma 2.3. *[see Eq. 14.44 in [7]] Let $x \geq 1$, then*

$$\sum_{n \leq x} d(a, b; n) = \zeta(b/a)x^{1/a} + \zeta(a/b)x^{1/b} + O(x^{1/(a+b)}).$$

To prove Theorem 1.3 and 1.1, we use the method of Cao and Zhai in [5] that is used to study square-full and (k, r)- integers in Piatetski-Shapiro sequence in [12, 13].

Lemma 2.4. *[see Lemma 3 in [5]] Let $\psi(t) = t - \lfloor t \rfloor - 1/2$ for $t \in \mathbb{R}$. Let $y > 0, X > 1, 0 \leq \sigma < 1, g(n) = (n + \sigma)^\gamma$. Then, for any exponent pair (κ, λ),*

$$\sum_{n \sim X} \psi(yg(n)) \ll y^{\frac{\kappa}{1+\kappa}} X^{\frac{\lambda+\gamma\kappa}{1+\kappa}} + y^{-1}X^{1-\gamma}.$$

The proof of Theorem 1.2 makes use of the following estimate, due originally to Rieger [11], for the number of integers n up to x such that $\lfloor n^c \rfloor$ belongs to an arithmetic progression. This lemma is proved in [6].

Lemma 2.5. *([6]) For $1 < c < 2$. Let $x \in \mathbb{R}$ and $a, q \in \mathbb{Z}$ such that $0 \leq a < q \leq x^c$. Then*

$$\sum_{\substack{n \leq x \\ \lfloor n^c \rfloor \equiv a (\bmod q)}} 1 = \frac{x}{q} + O\Big(\min\Big(\frac{x^c}{q}, \frac{x^{(c+1)/3}}{q^{1/3}}\Big)\Big).$$

3. Proof of Theorems

Proof of Theorem 1.1. For $1 < c < 2$, let $\nu = 1/c$. Let a and b be fixed integers such that $1 \leq a < b < 3a$. For any two exponent pairs (κ_1, λ_1) and (κ_2, λ_2) satisfying $\frac{\lambda_1}{1+\kappa_1} < \frac{a}{b} < \frac{\lambda_2}{1+\kappa_2}$ and $\kappa_1 > \kappa_2$ and $1 < c < \frac{a(\lambda_2-\lambda_1)-(\kappa_2-\kappa_1)}{\lambda_2(1+\kappa_1)-\lambda_1(1+\kappa_2)}$. We note that $\lfloor n^c \rfloor$ is a integer m, if and only if $m^\nu \leq n < (m+1)^\nu$. Therefore,

$$\begin{aligned}\sum_{n \leq x} d(a, b; \lfloor n^c \rfloor) &= \sum_{m \leq x^c} d(a, b; m)\Big(\lfloor -m^\nu \rfloor - \lfloor -(m+1)^\nu \rfloor\Big) \\ &= \sum_{m \leq x^c} d(a, b; m)\Big((m+1)^\nu - m^\nu\Big) + \sum_{m \leq x^c} d(a, b; m)\Big(\psi(-(m+1)^\nu) - \psi(-m^\nu)\Big) \\ &:= B_1 + B_2.\end{aligned}$$

We apply Abel's identity and Lemma 2.3, we have

$$B_1 = \frac{ac}{a+c-ac}\zeta(b/a)x^{1-c+c/a} + \frac{bc}{b+c-bc}\zeta(a/b)x^{1-c+c/b} + O(x^{1-c+c/(a+b)}).$$

Now we bound the second sum B_2. Let $M = x^{\frac{c}{b}+\frac{a(\kappa_1-\kappa_2)}{b(\lambda_1(1+\kappa_2)-\lambda_2(1+\kappa_1))}}$. We write

$$\begin{aligned}
&\sum_{m\le x^c} d(a,b;m)\Big(\psi(-(m+1)^\nu)-\psi(-m^\nu)\Big)\\
&=\sum_{m^a n^b\le x^c}\Big(\psi(-(m^a n^b+1)^\nu)-\psi(-(m^a n^b)^\nu)\Big)\\
&=\sum_{m\le x^{c/b}}\ \sum_{n\le x^{c/b}m^{-a/b}}\Big(\psi(-(m^a n^b+1)^\nu)-\psi(-(m^a n^b)^\nu)\Big)\\
&=: C_1+C_2,
\end{aligned}$$

where

$$C_1 = \sum_{m\le M}\ \sum_{n\le x^{c/b}m^{-a/b}}\Big(\psi(-(m^a n^b+1)^\nu)-\psi(-(m^a n^b)^\nu)\Big),$$

and

$$C_2 = \sum_{M<m\le x^{c/b}}\ \sum_{n\le x^{c/b}m^{-a/b}}\Big(\psi(-(m^a n^b+1)^\nu)-\psi(-(m^a n^b)^\nu)\Big).$$

In view of Lemma 2.4 with the exponent pair $(\kappa,\lambda)=(\kappa_1,\lambda_1)$, we have

$$\begin{aligned}
C_1 &\ll \log x\sum_{m\le M}\left(m^{b\nu\kappa_1/(1+\kappa_1)}\left(\frac{x^{c/a}}{m^{b/a}}\right)^{(\lambda_1+a\nu\kappa_1)/(1+\kappa_1)} + m^{-b\nu}\left(\frac{x^{c/a}}{m^{b/a}}\right)^{1-a\nu}\right)\\
&\ll \log x\sum_{m\le M}\Big(x^{(c\lambda_1/a+\kappa_1)/(1+\kappa_1)}m^{-b\lambda_1/a(1+\kappa_1)} + x^{c-1}m^{-b/a}\Big).
\end{aligned}$$

In view of $\frac{\lambda_1}{1+\kappa_1}<\frac{a}{b}$, we have

$$\begin{aligned}
C_1 &\ll x^{(c\lambda_1/a+\kappa_1)/(1+\kappa_1)}M^{1-b\lambda_1/a(1+\kappa_1)}\log x + x^{c-1}\log x\\
&\ll x^{\frac{c}{b}+\frac{\kappa_2}{1+\kappa_2}+(\frac{a}{b}-\frac{\lambda_2}{1+\kappa_2})\frac{\kappa_1-\kappa_2}{\lambda_1(1+\kappa_2)-\lambda_2(1+\kappa_1)}}\log x.
\end{aligned}$$

Analogue, from Lemma 2.4 with the exponent pair $(\kappa,\lambda)=(\kappa_2,\lambda_2)$, we have

$$\begin{aligned}
C_2 &\ll \log x\sum_{M<m\le x^{c/b}}\left(m^{b\nu\kappa_2/(1+\kappa_2)}\left(\frac{x^{c/a}}{m^{b/a}}\right)^{(\lambda_2+a\nu\kappa_2)/(1+\kappa_2)} + m^{-b\nu}\left(\frac{x^{c/a}}{m^{b/a}}\right)^{1-a\nu}\right)\\
&\ll \log x\sum_{M<m\le x^{c/b}}\Big(x^{(c\lambda_2/a+\kappa_2)/(1+\kappa_2)}m^{-b\lambda_2/a(1+\kappa_2)} + x^{c-1}m^{-b/a}\Big).
\end{aligned}$$

In view of $\frac{\lambda_2}{1+\kappa_2}>\frac{a}{b}$, we have

$$\begin{aligned}
C_2 &\ll x^{(c\lambda_2/a+\kappa_2)/(1+\kappa_2)}M^{1-b\lambda_2/a(1+\kappa_2)}\log x + x^{c-1}\log x\\
&\ll x^{\frac{c}{b}+\frac{\kappa_2}{1+\kappa_2}+(\frac{a}{b}-\frac{\lambda_2}{1+\kappa_2})\frac{\kappa_1-\kappa_2}{\lambda_1(1+\kappa_2)-\lambda_2(1+\kappa_1)}}\log x.
\end{aligned}$$

□

Proof of Theorem 1.2. Let $x > 1$, and $b \geq 3$. In view of Lemma 2.5, we have

$$\begin{aligned}
\sum_{n\leq x} d(1,b;\lfloor n^c\rfloor) &= \sum_{n\leq x}\sum_{m^b|\lfloor n^c\rfloor} 1 \\
&= \sum_{m\leq x^{c/b}} \sum_{\substack{n\leq x\\ \lfloor n^c\rfloor\equiv 0(\bmod m^b)}} 1 \\
&= \sum_{m\leq x^{c/b}} \Big(\frac{x}{m^b} + O\Big(\min\Big\{\frac{x^c}{m^b}, \frac{x^{(c+1)/3}}{m^{b/3}}\Big\}\Big)\Big) \\
&= \sum_{m\leq x^{c/b-1/2b}} \Big(\frac{x}{m^b} + O\Big(\frac{x^{(c+1)/3}}{m^{b/3}}\Big)\Big) + \sum_{x^{c/b-1/2b}<m<x^{c/b}} \Big(\frac{x}{m^b} + O\Big(\frac{x^c}{m^b}\Big)\Big) \\
&= x\sum_{m\leq x^{c/b}} \frac{1}{m^b} + O(x^{(c+1)/3}\log x) + O(x^{c/b+1/2-1/2b}).
\end{aligned}$$

In view of Lemma 2.1, and $c < 2$, we have

$$\sum_{n\leq x} d(1,b;\lfloor n^c\rfloor) = x\zeta(b) + O(x^{(c+1)/3}\log x).$$

□

Proof of Theorem 1.3. For $1 < c < 2$, let $\nu = 1/c$. We note that $\lfloor n^c\rfloor$ is a integer m, if and only if $m^\nu \leq n < (m+1)^\nu$. Therefore

$$\begin{aligned}
\sum_{n\leq x} \tau(\lfloor n^c\rfloor) &= \sum_{m\leq x^c} \tau(m)\Big(\lfloor -m^\nu\rfloor - \lfloor -(m+1)^\nu\rfloor\Big) \\
&= \sum_{m\leq x^c} \tau(m)\Big((m+1)^\nu - m^\nu\Big) + \sum_{m\leq x^c} \tau(m)\Big(\psi(-(m+1)^\nu) - \psi(-m^\nu)\Big) \\
&:= A_1 + A_2.
\end{aligned}$$

From $(m+1)^\nu - m^\nu = \nu m^{\nu-1} + O(m^{\nu-2})$, we have

$$A_1 = \nu\sum_{m\leq x^c} \tau(m)m^{\nu-1} + O(\sum_{m\leq x^c} m^{\nu-2+\varepsilon}).$$

The last sum follows from the bound $\tau(m) \ll m^\varepsilon$. From Lemma 2.2 and Abel's identity, we have

$$A_1 = cx\log x + (2\gamma - c)x + O(x^{1-c/2}).$$

In view of Lemma 2.4 with the exponent pair (κ, λ), we have

$$
\begin{aligned}
A_2 &= \sum_{ab\le x^c} \Big(\psi(-(ab+1)^\nu) - \psi(-(ab)^\nu)\Big) \\
&\le \sum_{ab\le x^c} \Big|\psi(-(ab+1)^\nu) - \psi(-(ab)^\nu)\Big| \\
&\le 2 \sum_{a\le x^{c/2}} \sum_{b\le x^c a^{-1}} \Big|\psi(-(ab+1)^\nu) - \psi(-(ab)^\nu)\Big| \\
&\ll \log x \sum_{a\le x^{c/2}} \Big((a^\nu)^{\frac{\kappa}{1+\kappa}}\Big(x^c a^{-1}\Big)^{\frac{\lambda+\nu\kappa}{1+\kappa}} + (a^\nu)^{-1}\Big(x^c a^{-1}\Big)^{1-\nu}\Big) \\
&\ll x^{\frac{c\lambda+\kappa}{1+\kappa}} \log x \sum_{a\le x^{c/2}} a^{\frac{-\lambda}{1+\kappa}} + x^{c-1}\log x \\
&\ll x^{\frac{c\kappa+c+c\lambda+2\kappa}{2+2\kappa}} \log x + x^{c-1}\log x.
\end{aligned}
$$

Since $\kappa+\lambda \le 1$, we have $c \le \frac{2}{1+\kappa+\lambda}$. Thus,

$$\frac{c\kappa+c+c\lambda+2\kappa}{2+2\kappa} \ge c-1.$$

This proves Theorem 1.3. $\square$

Proof of Theorem 1.4. For $1<c<2$, let $\nu = 1/c$.

$$
\begin{aligned}
\sum_{n\le x} \sigma_1(\lfloor n^c\rfloor) &= \sum_{m\le x^c} \sigma_1\Big(\lfloor -m^\nu\rfloor - \lfloor -(m+1)^\nu\rfloor\Big) + O(1) \\
&= \sum_{m\le x^c} \sigma_1(m)\Big((m+1)^\nu - m^\nu\Big) + \sum_{m\le x^c} \sigma_1(m)\Big(\psi(-(m+1)^\nu) - \psi(-m^\nu)\Big) + O(1) \\
&:= A_1 + A_2 + O(1).
\end{aligned}
$$

From $(m+1)^\nu - m^\nu = \nu m^{\nu-1} + O(m^{\nu-2})$, we have

$$A_1 = \nu \sum_{m\le x^c} \nu_1(m) m^{\nu-1} + O(\sum_{m\le x^c} \sigma_1(m) m^{\nu-2}).$$

From the well known formula

$$\sum_{n\le x} \sigma_1(n) = \frac{1}{2}\zeta(2)x^2 + O(x\log x)$$

and using partial summation formula, we have

$$\nu \sum_{m\le x^c} \sigma_1(m) m^{\nu-1} = \frac{\zeta(2)}{c+1} x^{c+1} + O(x\log x)$$

and

$$\sum_{m\le x^c} \sigma_1(m) m^{\nu-2} \ll x.$$

Thus,

$$A_1 = \frac{\zeta(2)}{c+1}x^{c+1} + O(x\log x).$$

Next, we bound A_2.

$$\begin{aligned} A_2 &= \sum_{m\le x^c}\sum_{a|m} a\Big(\psi(-(m+1)^\nu) - \psi(-m^\nu)\Big) \\ &= \sum_{ab\le x^c} a\Big(\psi(-(ab+1)^\nu) - \psi(-ab^\nu)\Big) \\ &= \sum_{b\le x^c}\sum_{a\le x^c b^{-1}} a\Big(\psi(-(ab+1)^\nu) - \psi(-ab^\nu)\Big). \end{aligned}$$

In view of Lemma 2.4 with the exponent pair (κ,λ), we have

$$\sum_{a\le x^c b^{-1}}\Big(\psi(-(ab+1)^\nu) - \psi(-ab^\nu)\Big) \ll b^{\frac{-\lambda}{1+\kappa}} x^{\frac{c\lambda+\kappa}{1+\kappa}} + x^{c-1}b^{-1}.$$

Thus, by partial summation formula, we have

$$A_2 \ll x^{c+\frac{c\lambda+\kappa}{1+\kappa}}\sum_{b\le x^c} b^{-1-\frac{\lambda}{1+\kappa}} + x^{2c-1}\sum_{b\le x^c} b^{-2} \ll x^{c+\frac{c\lambda+\kappa}{1+\kappa}} + x^{2c-1}.$$

This proves Theorem 1.4. □

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