Gulf Journal of Mathematics
Vol 19, Issue 2 (2025) 277-292

ISSN: 2309-4966
https://doi.org/10.56947/gjom.v19i2.2633

THRESHOLD DYNAMICS AND IMPULSIVE TREATMENT IN STOCHASTIC NEET POPULATION MODEL

ALI EL MYR[1*]

ABSTRACT. This paper introduces the dynamics of the NEET (Not in Employment, Education, or Training) population through a stochastic SNE model. Using a compartmental epidemic framework, we evaluate the impact of impulsive treatment strategies designed to facilitate the rapid reintegration of NEET individuals into education and employment. The theoretical analysis, supported by numerical simulations, demonstrates that when the threshold parameter $\mathcal{R}$ is below 1, the NEET population tends to become extinct. Conversely, when $\mathcal{R}$ exceeds 1, the NEET status persists, thereby highlighting the need for periodic impulsive interventions. These findings offer valuable insights for policymakers to design targeted and effective strategies to address the NEET phenomenon.

1. INTRODUCTION

The status of young people known as NEET poses a significant global challenge. It threatens their ability to take part in meaningful pursuits, such as work, and restricts their access to crucial areas such as education and training that make them ready for the job market. This question is particularly obvious in many countries, where NEET youth often face serious socio-economic hardships. As stated by the ILO's Global Employment Trends for Youth 2024 report [12], about 20.4% of young people aged 15 to 24 worldwide are NEET. Rates are high in regions like the Middle East and North Africa, where they can reach 30%, exceeding the global average (ILO).

Several studies have identified many important factors influencing NEET status. These range from socio-economic conditions [2, 11, 14] to psychological factors [17], along with the role of public policies. Moreover, Poverty and a lack of family support networks are particularly essential, forcing many youths toward long-term marginalization. Economic crises have aggravated the situation by driving up youth unemployment and limiting job opportunities. Furthermore, NEET youth often struggle with psychological factors such as depression and anxiety, making it even tougher for them to reintegrate into society [13]. While the former studies have looked at general trends, they often overlook the transition dynamics young people experience between education, work, and unemployment [14].

Date: Received: Mar 5, 2025; Accepted: Mar 24, 2025.

* Corresponding author.

2020 *Mathematics Subject Classification.* Primary 60H10; Secondary 34D05, 92D25.

Key words and phrases. Mathematical Modeling, Stochastic Perturbations, Threshold Dynamics, impulsive Treatment, NEETs.

In Spain, for example, many NEET individuals move rapidly between precarious employment, unemployment, and training, without being able to stabilize their situation [15]. These frequent changes often reflect structural issues like poverty and economic crises.

Recent studies have emphasized the significant impact of socio-economic background on the length of NEET status. Individuals from more privileged backgrounds tend to transition into employment at a faster rate compared to those from underprivileged backgrounds, leading to extended periods of exclusion and worsening intergenerational disparities [4].

To understand these dynamics and the influence of the various factors on being NEET, we conceptualize the transmission of NEET status among young people as a social epidemic, characterized by its viral and contagious nature. Recent studies, such as those by Franzen and Mader, support this analogy and highlight the role of social interactions in spreading behaviors [5]. Building on this perspective, we introduce a dynamic model inspired by a stochastic epidemic framework. Unlike traditional static analyses, this multi-compartment mathematical model enables a detailed examination of NEET experiences and the impact of public policies and initiatives on their reintegration. Furthermore, the model incorporates thresholds or conditions to analyze the extinction or persistence of the NEET phenomenon, similar to the thresholds used in epidemic models [1, 3, 16].

2. Formulation of the SEN model

In the following, we introduce a stochastic SNE model (Susceptible, NEET, Employed). This compartmental model consists of three groups: susceptible individuals (S), representing youth involved in precarious employment, education, or training and vulnerable to transitioning into NEET status; NEETs (N), referring to those disconnected from employment, education, and training; and employed individuals (E), who have successfully transitioned out of NEET status into stable employment. The model aims to capture the dynamics of transitions between these compartments, reflecting the movement of individuals between employment vulnerability, NEET status, and stable integration into the workforce. The population considered has a constant size $\mathcal{P}$, and the variables are normalized to $\mathcal{P} = 1$.

Based on the compartmental diagram and the associated flows, the following stochastic differential system is presented. A more detailed explanation of the

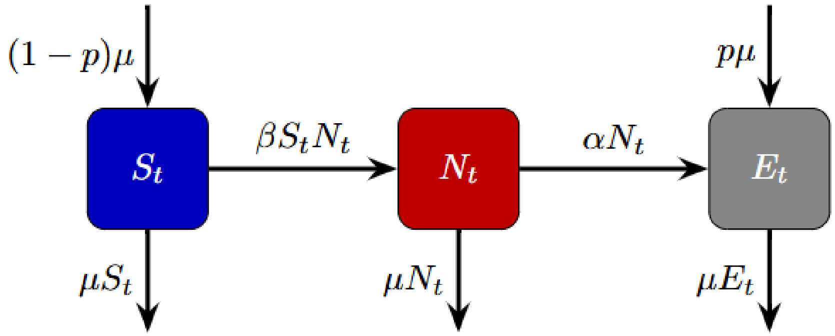

FIGURE 1. SNE Model Schematic Diagram

system will follow in the subsequent discussion.

$$\begin{cases} \left.\begin{aligned} dS_t &= [(1-p)\mu - \beta S_t N_t - \mu S_t]\,dt - \sigma S_t N_t\,dB_t, \\ dN_t &= [\beta S_t N_t - \alpha N_t - \mu N_t]\,dt + \sigma S_t N_t\,dB_t, \\ dE_t &= [p\mu - \mu E_t + \alpha N_t]\,dt, \end{aligned}\right\}, \quad t \neq t_k \\ \\ \left.\begin{aligned} N_{t_k^+} &= N_{t_k^-} - \theta_k N_{t_k^-}, \\ E_{t_k^+} &= E_{t_k^-} + \theta_k N_{t_k^-}, \\ S_{t_k^+} &= S_{t_k^-}. \end{aligned}\right\}, \quad t = t_k,\ k \in \mathbf{N} \end{cases} \tag{2.1}$$

The model is based on the following assumptions and constraints:

(1) **Compartments and parameters of the Model**:
- S_t: the population of young people involved in precarious employment, education, or training and vulnerable to transitioning into NEET status at time t.
- E_t: the young population who have successfully transitioned out of NEET status into stable employment, reflecting the resolution of their socioeconomic vulnerability at time t.
- N_t: the population of young individuals disconnected from employment, education, or training (NEET). These individuals are temporarily outside the productive field and the education system.

The parameter β measures the intensity with which young individuals leave the education system under the influence of NEETs.

The parameter μ represents a constant recruitment rate, ensuring a constant population size. This recruitment is distributed into the system as follows:

Table 1. Description of model parameters

Parameter	Description
$\beta \in \mathbb{R}^+$	$Transmission\ rate\ between\ susceptible\ and\ NEET\ individuals$
$\mu \in \mathbb{R}^+$	$Natural\ exit\ rate\ from\ the\ system$
$p \in (0,1)$	$Proportion\ of\ young\ individuals\ directed\ towards\ education$
$\alpha \in \mathbb{R}^+$	$Transition\ rate\ from\ NEET\ to\ employment$
$\sigma \in \mathbb{R}^+$	$Magnitude\ of\ fluctuations\,(noise\ intensity)$

- A fraction $(1-p)\mu$ enters the Susceptible (S) compartment, representing youth engaged in precarious employment, education, or training.
- A fraction $p\mu$ enters directly into the Employed (E) compartment, representing individuals who secure stable employment upon entry.

μ also represents the natural exit rate (due to aging or other external causes) from the S, E, and N compartments, maintaining a constant total population.

These parameters will be estimated at the end of Section (3) using available data and appropriate statistical method.

(2) **Stochastic Transitions**:

The transition between compartments in the model depends on several stochastic factors. The latter model the random aspect and uncertainty associated with unforeseen events (economic crises or policy reforms). Accordingly, stochastic models are central in the solution of such problems. In this paper, we consider a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$, where $\{\mathcal{F}_t\}_{t\geq 0}$ is a filtration satisfying the usual conditions. It is assumed that stochastic perturbations mainly influence the coefficient β, leading to fluctuations modeled as:

$$\beta \to \beta + \sigma \dot{B}_t,$$

where B_t is a standard Brownian motion and σ quantifies the intensity of these fluctuations.

(3) **Impulse Effects**:

Impulses occur at specific times t_k, representing external interventions aimed at addressing the NEET population through targeted treatment measures [7]. These impulses induce instantaneous modifications in the compartmental populations:

$\theta_k N_{t_k^-}$ a discrete transition from (N) to (E), representing the impact of impulsive treatments designed to facilitate the reintegration of NEET individuals into employment. These interventions may include policy-driven measures, or direct job placement schemes.

We assume that:

- $\theta_k \in (0,1)$,

- $0 = t_0 < t_1 < ... < t_k < t_{k+1} < ...,$
- $\lim\limits_{k\to+\infty} t_k = +\infty.$

3. Model Without Impulses

We begin by analyzing the mathematical properties of the following SEN model without impulses:

$$\begin{cases} dS_t = [(1-p)\mu - \beta S_t N_t - \mu S_t]\,dt - \sigma S_t N_t\,dB_t, \\ dN_t = [\beta S_t N_t - \alpha N_t - \mu N_t]\,dt + \sigma S_t N_t\,dB_t, \\ dE_t = [p\mu - \mu E_t + \alpha N_t]\,dt. \end{cases} \tag{3.1}$$

Theorem 3.1. *Given any initial condition $(S_0, N_0, E_0) \in \mathbb{R}^3_+$, a unique solution (S_t, N_t, E_t) to the system (3.1) exists for all $t \geq 0$. Moreover, with probability 1, the solution will remain in the non-negative region $\mathbb{R}^3_+$ for all $t \geq 0$.*

Proof. The system (3.1) has coefficients that are continuous and locally Lipschitz. By [10], there exists a unique solution on $[0, \tau_e)$, where τ_e represents the explosion time.

By using similar techniques as in [8, 16], we can establish that $\tau_e = \infty$: Indeed, it is sufficient to consider the Lyapunov function defined by

$$V(S, N, E) = -\ln(S) - \ln(N) - \ln(E)$$

as well as the sequence of stopping times

$$\tau_n = \inf\left\{t \geq 0 \mid \max(S_t, N_t, E_t) \geq n \text{ or } \min(S_t, N_t, E_t) \leq \frac{1}{n}\right\},$$

which is increasing and whose limit τ_∞ satisfies $\tau_\infty \leq \tau_e$.

We prove, by contradiction, that $\tau_\infty = \infty$, which implies that $\tau_e = \infty$, ensuring that the (S_t, N_t, E_t) exists for all $t \geq 0$ and remains almost a.s. for any initial condition $(S_0, N_0, E_0) \in \mathbb{R}^3_+$.

□

Remark 3.2. $\Gamma_0 = \{(s, n, e) \in \mathbb{R}^3_+ / s+e+n = 1\}$ and $\Gamma_1 = \{(s, n, e) \in \mathbb{R}^3_+ / s+n \leq (1-p)\}$. Γ_0 and Γ_1 are a.s. positively invariants under the system (3.1), then from this point forward, we will always assume the initial value $(S_0, N_0, E_0) \in \Gamma \triangleq \Gamma_0 \cap \Gamma_1$.

The threshold parameter $\mathcal{R}$ is a key factor in shaping the long-term dynamics of (3.1):

$$\mathcal{R} = \frac{\beta(1-p)}{\mu + \alpha + \frac{1}{2}\sigma^2(1-p)^2}$$

The two following theorems formalize this by establishing the asymptotic behavior of the compartments under specific conditions.

Theorem 3.3. *For any initial values $(S_0, N_0, E_0) \in \Gamma$, if $\mathcal{R} < 1$ and $\beta(1-p) > \sigma^2$ then we have:*

$$(i)\ \lim_{t\to+\infty} N_t = 0 \ a.s., \tag{3.2}$$

$$(ii)\ \lim_{t\to+\infty} E_t = p \ a.s., \tag{3.3}$$

$$(iii)\ \lim_{t\to+\infty} S_t = (1-p) \ a.s. \tag{3.4}$$

Proof. (i) Define $\langle S_t\rangle = \frac{1}{t}\int_0^t S_s ds$. We apply Itô's formula to obtain the following:

$$d\log N_t = \left[\beta S_t - \alpha - \mu - \frac{1}{2}\sigma^2 S_t^2\right] dt + \sigma S_t dB_t \tag{3.5}$$

Next, integrating (3.5) we get

$$\frac{\log N_t}{t} - \frac{\log N_0}{t} = \beta\langle S_t\rangle - \frac{1}{2}\sigma^2\langle S_t^2\rangle - (\alpha+\mu) + \sigma\frac{1}{t}\int_0^t S_s dB_s \tag{3.6}$$

using the Cauchy-Schwarz inequality one obtains

$$\frac{\log N_t}{t} - \frac{\log N_0}{t} \le \beta\langle S_t\rangle - \frac{1}{2}\sigma^2\langle S_t\rangle^2 - (\alpha+\mu) + \sigma\frac{1}{t}\int_0^t S_s dB_s \tag{3.7}$$

By the condition $\beta > \frac{\sigma^2}{1-p}$ and the fact that $\langle S_t\rangle \in (0,(1-p)]$, the function $\beta x - \frac{1}{2}\sigma^2 x^2 - (\alpha+\mu)$ reaches its maximum on $(0,(1-p)]$ at $1-p$ Which gives

$$\frac{\log N_t}{t} \le \beta(1-p) - \frac{1}{2}\sigma^2(1-p)^2 - (\alpha+\mu) + \frac{\log N_0}{t} + \sigma\frac{1}{t}\int_0^t S_s dB_s \text{ a.s.} \tag{3.8}$$

thus

$$\frac{\log N_t}{t} \le \left(\frac{1}{2}\sigma^2(1-p)^2 + \alpha + \mu\right)(\mathcal{R}-1) + \frac{\log N_0}{t} + \sigma\frac{1}{t}\int_0^t S_s dB_s \text{ a.s.} \tag{3.9}$$

using Remark 3.2 and using the large number theorem for martingales [10] one obtains:

$$\limsup_{t\to+\infty} \frac{\log N_t}{t} \le \left(\frac{1}{2}\sigma^2(1-p)^2 + (\mu+\lambda)\right)(\mathcal{R}-1) \triangleq \xi \quad \text{a.s.} \tag{3.10}$$

By the condition $\mathcal{R} < 1$, we have $\xi < 0$. Therefore, for any arbitrarily small constant $0 < \kappa < \frac{\xi}{2}$, there exists a positive constant $T = T(\omega)$ and a set Ω_κ such that:

$$\mathbb{P}(\Omega_\kappa) \ge 1 - \kappa,$$

and for all $t \ge T$ and $\omega \in \Omega_\kappa$, we have:

$$\log N_t \le -\frac{\xi}{2}t.$$

As a result, this implies that:

$$N_t \le e^{-\frac{\xi}{2}t}.$$

Consequently, we obtain:

$$\limsup_{t\to\infty} N_t \le 0 \quad \text{a.s.}\ .$$

This inequality, together with the positivity of the solution, leads to the conclusion that:

$$\lim_{t\to\infty} N_t = 0 \quad \text{a.s.} \tag{3.11}$$

(ii) (ii) Referring to the second equation in system (3.1), we derive for all $t \geq 0$:

$$E_t = p(1-e^{-\mu t}) + e^{-\mu t}E_0 + e^{-\mu t}\int_0^t e^{s\mu}N_s ds \tag{3.12}$$

According to (3.11), for sufficiently large t, we have:

$$e^{-\mu t}\int_0^t e^{s\mu}N_s ds \leq \frac{1}{\mu}\epsilon e^{-\mu t}(e^{\mu t}-1) \text{ a.s.} \tag{3.13}$$

for any $\epsilon > 0$. The result follows from (3.12) and (3.13).

(iii) Follows immediately from $S_t + N_t + E_t = 1$. □

Theorem 3.4. *Let $\langle N_t\rangle = \frac{1}{t}\int_0^t N_s ds$. For any initial values $(S_0, N_0, E_0) \in \Gamma$, if $\mathcal{R} > 1$ then we have*

$$\liminf_{t\to+\infty} \langle N_t\rangle > 0 \text{ } a.s. \tag{3.14}$$

Proof. By (3.5) we have :

$$d\log N_t = \left[\beta S_t - \alpha - \mu - \frac{1}{2}\sigma^2 S_t^2\right]dt + \sigma S_t dB_t \tag{3.15}$$

on the other hand

$$\frac{\beta}{\mu}dS_t = \left[(1-p)\beta - \beta S_t - \beta^2 S_t N_t\right]dt - \beta\sigma S_t N_t dB_t, \tag{3.16}$$

since $S_t \in (0,1)$, we get:

$$\frac{\beta}{\mu}dS_t \geq \left[(1-p)\beta - \beta S_t - \beta^2 N_t\right]dt - \frac{\beta}{\mu}\sigma S_t N_t dB_t \tag{3.17}$$

By combining equations (3.14) and (3.15), we obtain

$$d(\frac{\beta}{\mu}S_t + \log N_t) \geq (1-p)\beta - \mu - \alpha - \frac{1}{2}\sigma^2 S_t^2 - \beta^2 N_t + \sigma S_t dB_t - \frac{\beta}{\mu}\sigma S_t N_t dB_t \tag{3.18}$$

using Remark 3.2

$$d(\frac{\beta}{\mu}S_t + \log N_t) \geq (1-p)\beta - \mu - \alpha - \frac{1}{2}\sigma^2(1-p)^2 - \beta^2 N_t + \sigma S_t dB_t - \frac{\beta}{\mu}\sigma S_t N_t dB_t \tag{3.19}$$

Integrating (3.19) from 0 to $t > 0$, we obtain

$$\log N_t \geq \left((1-p)\beta - \mu - \alpha - \frac{1}{2}\sigma^2(1-p)^2\right)t - \beta^2\int_0^t N_s ds + L(t). \tag{3.20}$$

where $L(t) = \log N_0 + \frac{\beta}{\mu}(S_0 - S_t) + \sigma\int_0^t S_s dB_s - \frac{\beta}{\mu}\sigma\int_0^t S_s N_s dB_s$.
Introducing $\mathcal{R}$, we obtain

$$\log N_t \geq \left(\frac{1}{2}\sigma^2(1-p)^2 + \alpha + \mu\right)(\mathcal{R}-1)\,t - \beta^2\int_0^t N_s ds + L(t). \tag{3.21}$$

Noting that

$$\lim_{t\to\infty} \frac{L(t)}{t} = 0 \quad \text{a.s.} \tag{3.22}$$

Thus, by a direct application of Lemma 5.1 in [6] we get

$$\liminf_{t\to+\infty} \frac{1}{t}\int_0^t N_s ds > \frac{\xi}{\beta^2} \text{ a.s.} \tag{3.23}$$

where

$$\xi = \left(\frac{1}{2}\sigma^2(1-p)^2 + \alpha + \mu\right)(\mathcal{R} - 1),$$

which is positive under condition $\mathcal{R} > 1$. This concludes the proof. □

Remark 3.5. If we take $p = 0$ in Theorem 3.3 we recover the work done in [9].

To understand the dynamics of the SEN model without impulsive treatment (3.1), it is crucial to accurately estimate the model parameters from the available data. To achieve this, we applied the nonlinear least squares method to a stochastic model, taking into account the stochasticity of the Susceptible (S) and NEET (N) compartments. This method consists of minimizing the discrepancy between observed and predicted values.

The data used comes from official and reliable sources, including the "Haut Commissariat au Plan" (HCP) and the World Bank. These data cover a 8-year period, with annual observations of the S, E, and N compartments. We applied an interpolation technique to increase the temporal resolution of the dataset.

The parameter estimation of the model yielded the following results:

$$(\beta, \alpha, \mu, \sigma, p) = (0.379, 0.059, 0.168, 0.08, 0.1) \tag{3.24}$$

The calculation of the critical threshold $\mathcal{R}$ with these parameters yielded:

$$\mathcal{R} \approx 1.485.$$

Theorem 3.4 states that if the threshold is greater than 1, the natural dynamics of the labor market and current policies are not enough to reverse the trend, leading to a persistent NEET phenomenon (see Fig 6) that requires external intervention.

This observation motivates the introduction of an impulsive treatment, aimed at inducing sudden but controlled changes in the compartments E and N. This approach allows bringing the critical threshold $\mathcal{R}$ below 1, thereby ensuring a gradual extinction of the NEET phenomenon.

4. Analysis of the Model with Impulsive Treatment

We start by proving that system (2.1) possesses a globally unique solution that stays non-negative. This guarantees that the proposed model (2.1) is mathematically well-defined.

Theorem 4.1. *The system (2.1) admits a unique solution (S_t, N_t, E_t) for $t \geq 0$ given any initial condition $(S_0, N_0, E_0) \in \mathbb{R}^3_+$. Moreover, with probability 1, the solution will remain in the non-negative region $\mathbb{R}^3_+$ for all $t \geq 0$.*

Proof. For $t \in [t_0, t_1]$ and any initial condition $(S_0, N_0, E_0) \in \mathbb{R}^3_+$, by Theorem 3.1, system (3.1) has a unique solution

$$(S^0_t, N^0_t, E^0_t) \triangleq Z^0_t \in \mathbb{R}^3_+$$

on the interval $t \in [t_0, t_1]$.

At $t = t_1$, an impulse occurs, shifting the solution from $Z^0_{t_1^-}$ to a new state given by

$$Z_{t_1^+} = (S_{t_1^+}, N_{t_1^+}, E_{t_1^+}) = (S_{t_1^-}, (1-\theta_k)N_{t_0^-}, E_{t_0^-} + \theta_k N_{t_0^-})$$

Applying the same reasoning, given that $Z_{t_1^+} \in \mathbb{R}^3_+$, a unique positive solution Z^1_t is defined on the interval $t \in (t_1, t_2]$.

Repeating this procedure leads to the conclusion that system (2.1) possesses a unique global positive over the interval $[t_0, \infty)$ for any positive initial condition , which is given by:

$$Z_t = \begin{cases} Z^0_t, t_0 \le t \le t_1; \\ Z^1_t, t_1 < t \le t_2; \\ \qquad \vdots \\ Z^k_t, t_k < t \le t_{k+1}; \\ \qquad \vdots \end{cases}$$

□

To study the dynamics of system (2.1), we examine the following system without impulses:

$$\begin{cases} dS_t = \left[(1-p)\mu - \prod_{0<t_k<t}(1-\theta_k)X_t\left(\beta S_t + \mu\right)\right]dt - \sigma \prod_{0<t_k<t}(1-\theta_k)X_tS_tdB_t, \\ dX_t = \left[\beta X_tS_t - \mu X_t - \alpha X_t\right]dt + \sigma X_tS_tdB_t \\ dY_t = \left[p\mu - \mu Y_t + \beta \prod_{0<t_k<t}(1-\theta_k)X_tS_t\right]dt + \sigma \prod_{0<t_k<t}(1-\theta_k)X_tdB_t. \end{cases} \tag{4.1}$$

The following lemma provides an equivalent reformulation of system (2.1).

Lemma 4.2. *Let $N_t = X_t\prod_{0<t_k<t}(1-\theta_k)$ and $E_t = Y_t - X_t\prod_{0<t_k<t}(1-\theta)$. Then:*

(i) *If (S_t, X_t, Y_t) is a solution of (4.1) then, (S_t, N_t, E_t) is a solution of (2.1).*
(ii) *If (S_t, N_t, E_t) is a solution of (2.1) then, (S_t, X_t, Y_t) is a solution of (4.1).*

Proof. (i) From the classical theory of SDE [10, 8], system (2.1) admits a unique global positive solution. Let

$$N_t = X_t \prod_{0<t_k<t}(1-\theta_k) \text{ and } E_t = Y_t - X_t \prod_{0<t_k<t}(1-\theta_k).$$

In fact, since N_t is continuous on each interval (t_k, t_{k+1}) and for $t \neq t_k$, $k \in \mathbb{N}$,

$$\begin{aligned} dN_t &= \prod_{0<t_k<t} (1-\theta_k) dX_t \\ &= [\beta S_t N_t - \alpha N_t - \mu N_t]\, dt + \sigma S_t N_t\, dB_t \end{aligned}$$

In a similar manner, we can check that:

$$\begin{aligned} dE_t &= dY_t - \prod_{0<t_k<t} (1-\theta_k) dX_t \\ &= [p\mu + \alpha N_t - \mu E_t]\, dt. \end{aligned}$$

However, for each $t_k \in \mathbb{R}^+$, we have:

$$\begin{aligned} N_{t^-} &= \prod_{0<t_i<t_k} (1-\theta_k) X_{t^-} \\ &= \prod_{0<t_i<t_k} (1-\theta_k) X_t, \\ &= N_t. \end{aligned}$$

and,

$$\begin{aligned} N_{t^+} &= \prod_{0<t_i\leq t_k} (1-\theta_k) dY_{t^+} \\ &= (1-\theta_k) \prod_{0<t_i<t_k} (1-\theta) dY_t, \\ &= (1-\theta_k) N_{t^-}. \end{aligned}$$

Furthermore:

$$\begin{aligned} E_{t_k^-} &= Y_{t_k} - \prod_{0<t_i<t_k} (1-\theta_i) X_{t_k} \\ &= E_{t_k}, \end{aligned}$$

and,

$$\begin{aligned} E_{t_k^+} &= Y_{t_k} - \prod_{0<t_i\leq t_k} (1-\theta_i) X_{t_k} \\ &= Y_{t_k} - (1-\theta_k) \prod_{0<t_i<t_k} (1-\theta_i) X_{t_k} \\ &= E_{t_k} + \theta_k N_{t_k}. \end{aligned}$$

Using the same method, we can prove (ii), so we omit the details. This completes the proof. □

Remark 4.3. We have $\mathcal{P}_t = Y_t + S_t = \mu - \mu\mathcal{P}_t$ and $\mathcal{P}_{t^+} = \mathcal{P}_{t^+}$. This implies that the set Γ_0 is a.s. positively invariant under the system (2.1).

Theorem 4.4. *For anay initial values $(S_0, N_0, E_0) \in \Gamma_0$, if $(1-p)\beta > \sigma^2$ then we have*

$$\limsup_{t\to+\infty} \frac{\log N_t}{t} \leq \limsup_{t\to+\infty} \frac{1}{t} \sum_{0<t_k<t} \log(1-\theta_k) + \xi \qquad a.s. \tag{4.2}$$

where $\xi = \left(\frac{1}{2}\sigma^2(1-p)^2 + \alpha + \mu\right)(\mathcal{R} - 1)$

Proof. According to Itô's formula, we have

$$d\log X_t = \beta S_t - (\alpha+\mu) - \frac{1}{2}\sigma^2 S_t^2 + \sigma S_t dB_t \tag{4.3}$$

And by a similar argument in the proof of the Theorem 3.3, we have

$$\log X_t \leq \log X_0 + \xi t + \sigma \int_0^t S_s dB_s \quad \text{a.s.} \tag{4.4}$$

Moreover, for all $t > 0$

$$\log N_t \leq \sum_{0<t_k<t} \log(1-\theta_k) + \log X_0 + \xi t + \sigma \int_0^t S_s dB_s \quad \text{a.s.} \tag{4.5}$$

Which gives, by taking the limit superior

$$\limsup_{t\to+\infty} \frac{\log N_t}{t} \leq \limsup_{t\to+\infty} \frac{1}{t} \sum_{0<t_k<t} \log(1-\theta_k) + \xi \quad \text{a.s.} \tag{4.6}$$

Hence the result. □

Corollary 4.5. *If* $\limsup_{t\to+\infty} \frac{1}{t}\sum_{0<t_k<t} \log(1-\theta_k) + \xi < 0$ *then we have*

$$\lim_{t\to+\infty} N_t = 0 \text{ a.s.} \tag{4.7}$$

5. Discussion

In this work, we examined a compartmental SNE model, introducing this innovative framework to capture the transmission dynamics among three distinct populations: susceptible individuals, employees and NEET groups. This study is particularly valuable for analyzing the potential impact of impulsive treatments within these populations. In our analysis, the persistence in mean and extinction of NEET status are assessed through the threshold $\mathcal{R}$. This parameter provides a measure of the likelihood of leaving or remaining in NEET status. Specifically:

- $\mathcal{R} < 1$ indicates that the share of NEETs is gradually decreasing as shown in Fig 3.
- $\mathcal{R} > 1$ suggests that their share remains almost stationary as shown in Fig 4, Fig 5 and Fig 6.

This threshold is of major importance, as they could serve as critical indicator for public policies aimed at this vulnerable segment of the population.

By introducing impulsive treatment strategies it is possible to achieve periodic impacts that influence the system dynamics. The theorem demonstrates that if the condition of Corollary 4.5 is met, then the compartment N decreases and this behavior is clearly confirmed by Fig 7.

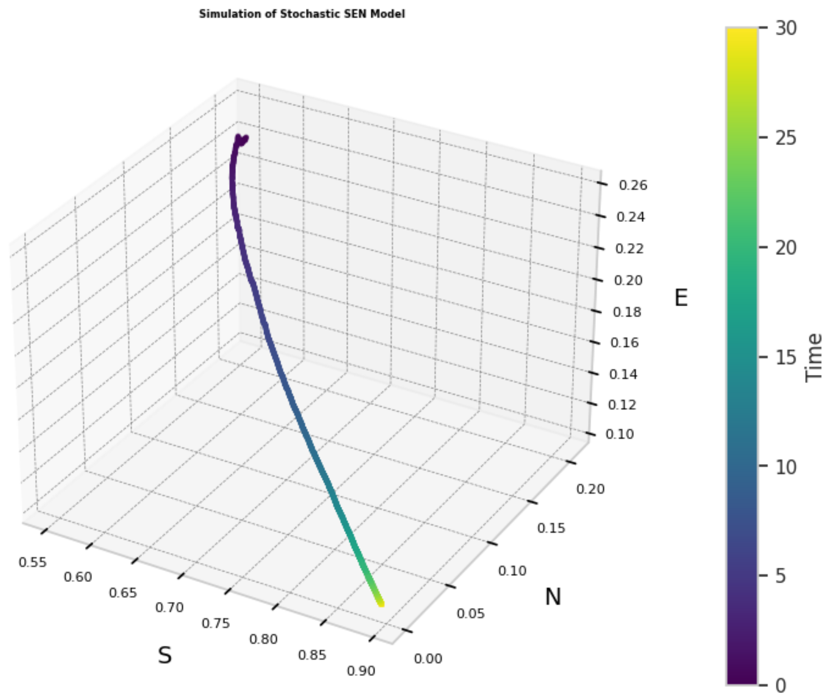

FIGURE 2. Simulation of one path of the solution (S_t, N_t, E_t) for the system (3.1) with $\mathcal{R} < 1$, meeting the conditions of Theorem 3.3.

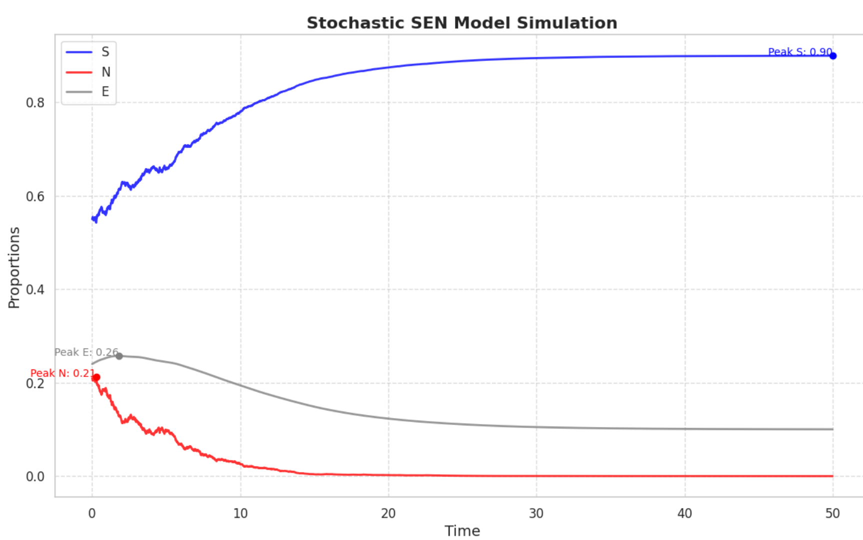

FIGURE 3. Temporal Evolution of the solution (S_t, N_t, E_t) of the system (3.1), with $\mathcal{R} < 1$, meeting the conditions of Theorem 3.3.

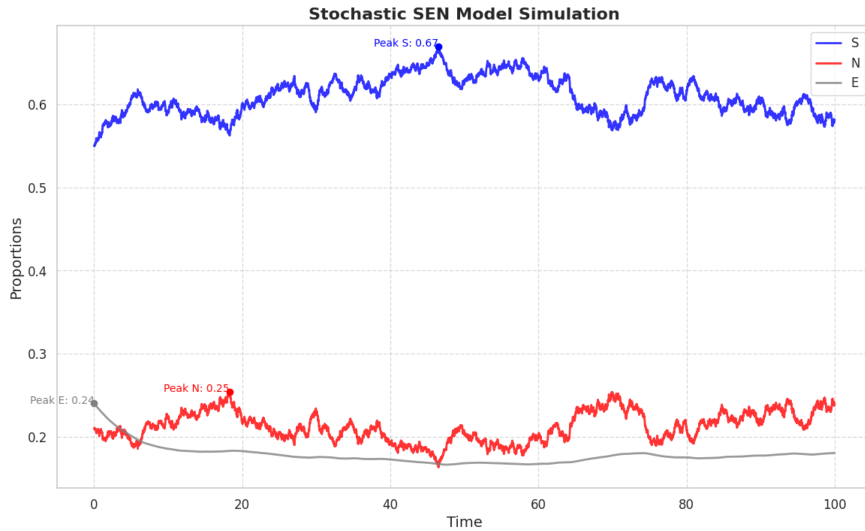

FIGURE 4. Simulation of one path of the solution (S_t, N_t, E_t) for the system (3.1) with the estimated parameters (3.24), where $\mathcal{R} > 1$.

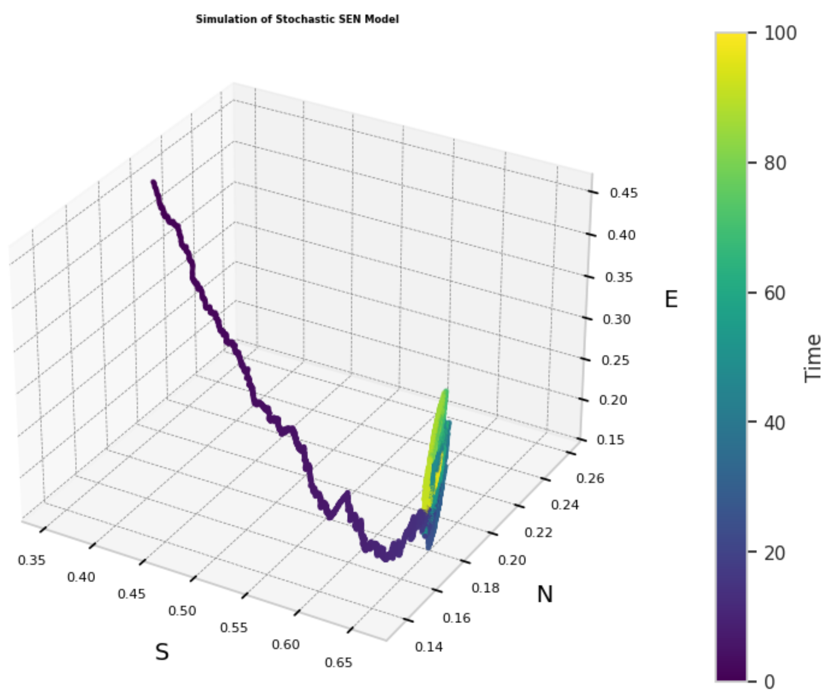

FIGURE 5. Temporal Evolution of the solution (S_t, N_t, E_t) of the system (3.1), under Estimated Parameters (3.24) with $\mathcal{R} > 1$.

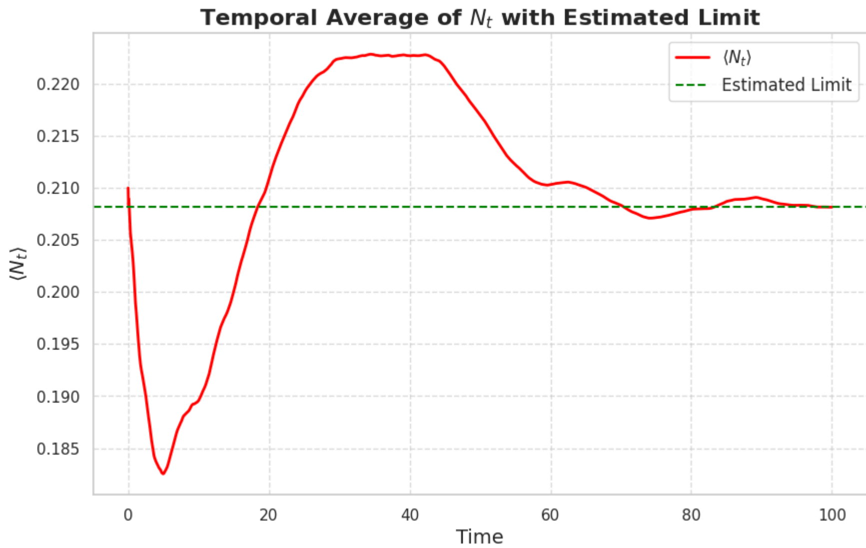

FIGURE 6. Simulation of the temporal average of the NEET status N_t using the estimated parameters (3.24), under $\mathcal{R} > 1$, where the conditions of Theorem 3.4 are satisfied.

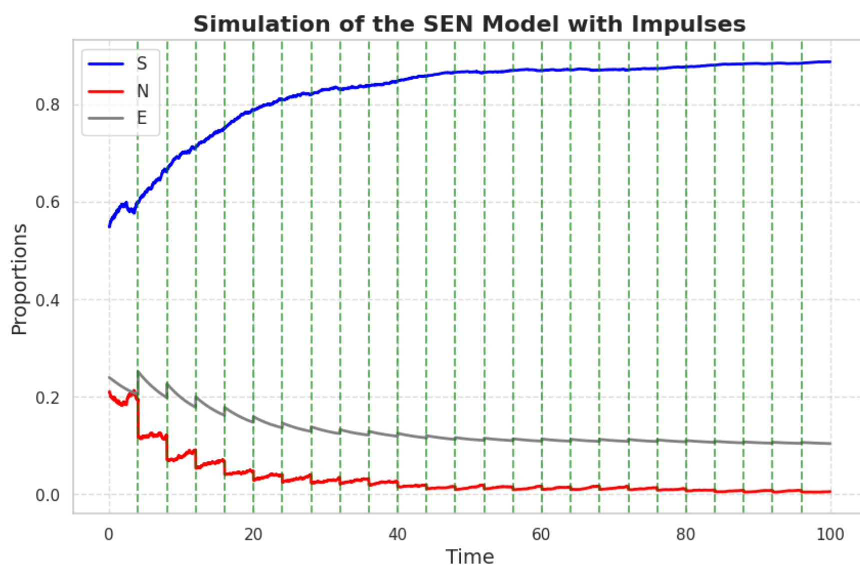

FIGURE 7. Simulation of the trajectory (S_t, N_t, E_t) for the system (2.1) using the estimated parameters (3.24) and θ_k such that the condition of Corollary 4.5 is satisfied.

Acknowledgement. We thank the anonymous reviewers for their helpful comments that improved the manuscript.

References

1. Babrhou, Y., Cherkaoui, F., El Boukari, B., Hilal, K., & Kajouni, A. (2025). Analysis and optimal control of a fractional-order SEAIR epidemic model with two-strains. *Gulf Journal of Mathematics*, 19(1), 251-284. https://doi.org/10.56947/gjom.v19i1.2561.
2. Berigel, M., Boztaş, G. D., Rocca, A., & Neagu, G. (2024). Using machine learning for NEETs and sustainability studies: Determining best machine learning algorithms. *Socio-Economic Planning Sciences*, 94, 101921. https://doi.org/10.1016/j.seps.2024.101921.
3. El Myr, A., Assadouq, A., Omari, L., Settati, A., & Lahrouz, A. (2018). A stochastic SIR epidemic system with a nonlinear relapse. *Discrete Dynamics in Nature and Society*, 2018, 5493270. https://doi.org/10.1155/2018/5493270.
4. Fabrizi, E., & Rocca, A. (2024). NEET status duration and socio-economic background. *Socio-Economic Planning Sciences*, 95, 101986. https://doi.org/10.1016/j.seps.2024.101986.
5. Franzen, A., & Mader, S. (2023). The power of social influence: A replication and extension of the Asch experiment. *PLoS ONE*, 18(11), e0294325. https://doi.org/10.1371/journal.pone.0294325.
6. Ji, C., & Jiang, D. (2014). Threshold behaviour of a stochastic SIR model. *Applied Mathematical Modelling*, 38(21), 5067–5079. https://doi.org/10.1016/j.apm.2014.03.037.
7. Jin, Z., Haque, M., & Liu, Q. (2008). Pulse vaccination in the periodic infection rate SIR epidemic model. *International Journal of Biomathematics*, 1(4), 409–432. https://doi.org/10.1142/S1793524508000370.
8. Jiang, D., Yu, J., Ji, C., & Shi, N. (2011). Asymptotic behavior of global positive solution to a stochastic SIR model. *Mathematical and Computer Modelling*, 54(1), 221–232. https://doi.org/10.1016/j.mcm.2011.02.004.
9. Lahrouz, A., & Settati, A. (2014). Necessary and sufficient condition for extinction and persistence of SIRS system with random perturbation. *Applied Mathematics and Computation*, 233, 10–19. https://doi.org/10.1016/j.amc.2014.01.158.
10. Mao, X. (1997). *Stochastic differential equations and applications*. Horwood Publishing.
11. Manhica, H., Yacaman-Mendez, D., Sjöqvist, H., Lundin, A., Agardh, E., & Danielsson, A.-K. (2022). Trajectories of NEET (Not in Education, Employment, and Training) in emerging adulthood, and later drug use disorder - a national cohort study. *Drug and Alcohol Dependence*, 233, 109350. https://doi.org/10.1016/j.drugalcdep.2022.109350.
12. Organisation Internationale du Travail: Global Employment Trends for Youth 2024: Decent Work, Brighter Futures. International Labour Organization, Genève (2024). https://www.ilo.org/publications/major-publications/global-employment-trends-youth-2024.
13. Parola, A., Marcionetti, J., & Savickas, M. (2023). Psychometric properties of the Career Adapt-Abilities Scale in Italian young people not in education, employment, or training (NEET). *International Journal for Educational and Vocational Guidance*, 23. https://doi.org/10.1007/s10775-023-09604-z.
14. Pitkänen, J., Junna, L., & Martikainen, P. (2024). Adolescent psychiatric inpatient episodes and subsequent labor market trajectories. *Journal of Adolescent Health*, 74, 1175-1183. https://doi.org/10.1016/j.jadohealth.2024.02.001.
15. Strecker, T., López, J., & Cabasés, M. À. (2021). Examining NEET situations in Spain: Labour Market, Discourses and Policies. *Journal of Applied Youth Studies*, 4, 119–134. https://doi.org/10.1007/s43151-021-00048-2.

16. Ullah, R., Al Mdallal, Q., Khan, T., Ullah, R., Al Alwan, B., Faiz, F., & Zhu, Q. (2023). The dynamics of novel coronavirus disease via stochastic epidemiological model with vaccination. *Scientific Reports*, 13, 3805. https://doi.org/10.1038/s41598-023-30647-3.
17. Veldman, K., Reijneveld, S. A., Andersen, J. H., Winding, T. N., Labriola, M., Lund, T., & Bültmann, U. (2022). The timing and duration of depressive symptoms from adolescence to young adulthood and young adults' NEET status: The role of educational attainment. *Social Psychiatry and Psychiatric Epidemiology*, 57, 83–93. https://doi.org/10.1007/s00127-021-02142-5.

[1] MASI Laboratory, École Normale Supérieure, Sidi Mohamed Ben Abdellah University, Fez, Morocco.
Email address: elmyrali@gmail.com