

NON-LIPSCHITZ STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS ON RIEMANNIAN MANIFOLDS

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ABSTRACT. In this paper, we investigate the existence and uniqueness of solution for a class of stochastic functional differential equations with delay on compact manifolds. The results are established under non-Lipschitz conditions on the coefficients. As an application, a geometrical example is provided to illustrate the obtained results.

1. INTRODUCTION

The last decades, several researchers have paid much attention to the existence and uniqueness of solution of stochastic differential equations under some kind of weak sufficient conditions [14, 2]. Yong and Shigeng [19] discussed neutral stochastic functional differential equations on \mathbb{R}^d with infinite delay in abstract space and they proved the existence and uniqueness of solution under Lipschitz conditions. Non-Lipschitz stochastic differential equations on Hilbert spaces have also been investigated e.g [20, 18, 17, 3, 12]. A theory of differential equations in a space of semimartingales on manifolds was developed by many authors [5, 8, 9, 13, 11]. Moreover, Bernardin et al. [1] proved the existence and uniqueness of solution of second-order multivariate stochastic differential equations (SDE) without delay on Riemannian manifolds under Lipschitz conditions. Recently, Ouahra et al. [10] discussed the existence and uniqueness of solution of Stochastic Functional Differential Equations (SFDE) with solutions constrained to lie in a smooth compact Riemannian manifold under Lipschitz conditions.

In this paper, we study the existence and uniqueness of solution of SFDE on compact Riemannian manifolds under non-Lipschitz conditions. Our main result is an extension of Ouahra's results [10] to non-Lipschitz case.

The rest of the paper is organized as follows. In Section 2, we summarize important working tools on Riemannian manifolds and Wiener processes. Section 3 is devoted to the existence and uniqueness of solution of the class of equations considered using Picard approximation. In Section 4, we give a geometrical example to illustrate the effectiveness of the obtained results.

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2. PRELIMINARIES

In this section, we define a large class of SFDE on compact Riemannian manifolds. The general theory of Riemannian manifolds can be found in textbooks on Riemannian geometry like [4, 15, 16]. Let $d \in \mathbb{N} \setminus \{0\}$ and I an index set.

Definition 2.1. (*Manifold*) A manifold \mathcal{M} of dimension d is a Hausdorff space for which every point has a neighborhood U that is homeomorphic to an open subset V of \mathbb{R}^d . Such a homeomorphism $\varphi : U \rightarrow V$ is called a coordinate chart and (U, φ) is called a local chart.

Definition 2.2. (*Atlas*) Let \mathcal{M} be a topological manifold and $\mathcal{A} = (U_i, \varphi_i)_{i \in I}$ a family of local charts of \mathcal{M} .

- \mathcal{A} is called an atlas of \mathcal{M} if $\mathcal{M} = \cup_{i \in I} U_i$.
- \mathcal{A} is a \mathcal{C}^k -atlas, $k \in \{1, 2, \dots\} \cup \{\infty\}$, if all transition maps

$$\varphi_j \circ \varphi_i^{-1} : \varphi_i(U_i \cap U_j) \rightarrow \varphi_j(U_i \cap U_j)$$

are differentiable of class \mathcal{C}^k (in case $U_i \cap U_j \neq \emptyset$).

Definition 2.3. (*Smooth manifold*) A smooth manifold (or \mathcal{C}^∞ -differentiable manifold) \mathcal{M} is a manifold equipped with a \mathcal{C}^∞ -atlas.

For $p \in \mathcal{M}$, $\mathcal{C}^\infty(p)$ is the set of functions whose restriction to some open neighborhood U of p is in $\mathcal{C}^\infty(U)$.

Definition 2.4. (*Tangent map*) The tangent space $\mathbb{T}_p\mathcal{M}$ to \mathcal{M} at p is the set of all maps $X_p : \mathcal{C}^\infty(p) \rightarrow \mathbb{R}$ such that for all $f, g \in \mathcal{C}^\infty(p)$

- $X_p(\alpha f + \beta g) = \alpha X_p(f) + \beta X_p(g)$, $\alpha, \beta \in \mathbb{R}$
- $X_p(fg) = X_p(f)g(p) + f(p)X_p(g)$.

X_p is called tangent vector and $\mathbb{T}_p\mathcal{M}$ naturally carries the structure of a vector space.

Definition 2.5. (*Vector field*) A vector field on \mathcal{M} is a map $X : \mathcal{C}^\infty(\mathcal{M}) \rightarrow \mathcal{C}^\infty(\mathcal{M})$ such that for all $f, g \in \mathcal{C}^\infty(\mathcal{M})$

- $X(\alpha f + \beta g) = \alpha X(f) + \beta X(g)$, $\forall \alpha, \beta \in \mathbb{R}$
- $X(fg) = X(f)g + fX(g)$,

and for all $p \in \mathcal{M}$, we denote $X(f)(p)$ by $X_p(f)$.

Let \mathcal{M} be a smooth manifold of dimension d , $p \in \mathcal{M}$ and U a neighborhood of p with local coordinates x^1, \dots, x^d , then any smooth vector field on U may be uniquely represented in the form $X = \sum_{k=1}^d a^k \frac{\partial}{\partial x^k}$, where $\frac{\partial}{\partial x^k}$, ($k = 1, \dots, d$) is a vector field on U such that $(\frac{\partial}{\partial x^k})_p(f) = \frac{\partial f}{\partial x^k}(p)$ and a^k , ($k = 1, \dots, d$) are smooth functions on U .

Notation 2.6. Let $p \in \mathcal{M}$ and (x^1, \dots, x^d) be a local system of coordinates of p . $(\partial_1, \dots, \partial_d)$ is a positively oriented basis of $\mathbb{T}_p\mathcal{M}$ with $\partial_i := \frac{\partial}{\partial x^i}$ for all $i \in \{1, \dots, d\}$, and dx^i is the linear form on $\mathbb{T}_p\mathcal{M}$ defined by

$$dx^i(\partial_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if not} \end{cases}.$$

Definition 2.7. (Riemannian manifold)

- A Riemannian metric g on a smooth manifold \mathcal{M} is a smoothly chosen inner product $g_p : \mathbb{T}_p\mathcal{M} \times \mathbb{T}_p\mathcal{M} \rightarrow \mathbb{R}$, ($p \in \mathcal{M}$) on each of the tangent spaces $\mathbb{T}_p\mathcal{M}$ of \mathcal{M} . We note $\langle u, v \rangle_p := g_p(u, v)$ for all $u, v \in \mathbb{T}_p\mathcal{M}$.
- A Riemannian manifold (\mathcal{M}, g) is a smooth manifold \mathcal{M} endowed with a Riemannian metric g .

Now, we introduce the notations used throughout this paper. Let \mathcal{M} be a smooth compact d -dimensional Riemannian manifold, $\zeta > 0$ and $T > 0$. Suppose that $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq -\zeta}, \mathbb{P})$ is a filtered probability space such that:

- $(\Omega, \mathcal{F}, \mathbb{P})$ is complete and $\mathcal{F}_{-\zeta}$ contains all the \mathbb{P} -null sets,
- For all $t \in [-\zeta, T]$, $\mathcal{F}_t = \bigcap_{u > t} \mathcal{F}_u$.

Let $(W(t))_{t \geq -\zeta}$ be a m -dimensional Brownian motion on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq -\zeta}, \mathbb{P})$ adapted to the filtration $(\mathcal{F}_t)_{t \geq -\zeta}$ satisfying $W(-\zeta) = 0$. For any smooth finite-dimensional Riemannian manifold N , we will denote by $L^0(\Omega, N)$ the space of all N -valued \mathcal{F} -measurable random variables $X : \Omega \rightarrow N$ equipped with the topology of convergence in probability. For all $x \in N$, denote by $\mathcal{S}(N, x)$ the space of all N -valued $(\mathcal{F}_t)_{t \geq -\zeta}$ -adapted continuous semimartingales $\tilde{p} : [-\zeta, T] \times \Omega \rightarrow N$ with $\tilde{p}(-\zeta, \cdot) = x$.

Let $x \in \mathcal{M}$. Define the Itô map by the association

$$\mathcal{S}(\mathcal{M}, x) \ni \tilde{p} \mapsto p \in \mathcal{S}(\mathbb{T}_x\mathcal{M}, 0)$$

where

$$\begin{cases} dp(t) = \tau_t^{-\zeta}(\tilde{p}) \circ d\tilde{p}(t), & -\zeta \leq t \leq T \\ p(-\zeta) = 0. \end{cases} \quad (2.1)$$

The differential in the above equation is in the Stratonovich sense and $\tau_t^{-\zeta}(\tilde{p})$ denotes the stochastic parallel transport from $T_{\tilde{p}(t)}\mathcal{M}$ to $T_x\mathcal{M}$ along the semimartingale p (see, [6, 7]). Observe that the Itô map is a bijection (see [10]).

Let \mathcal{S}_2^T be the Hilbert space of all semimartingales $p \in \mathcal{S}(\mathbb{T}_x\mathcal{M}, 0)$ such that $p(t) = \int_{-\zeta}^t A(s)ds + \int_{-\zeta}^t B(s)dW(s)$, $-\zeta \leq t \leq T$ and

$$\|p\|^2 := \mathbb{E} \left[\int_{-\zeta}^T \|A(s)\|^2 ds \right] + \mathbb{E} \left[\int_{-\zeta}^T \|B(s)\|^2 ds \right] < \infty, \quad (2.2)$$

where $A : [-\zeta, T] \times \Omega \rightarrow \mathbb{T}_x\mathcal{M}$ and $B : [-\zeta, T] \times \Omega \rightarrow L(\mathbb{R}^m, \mathbb{T}_x\mathcal{M})$ are adapted, previsible processes such that $\|B(s)\|^2 = \text{trace}(B(s)^T B(s))$. In the sequel, we shall refer to the pair (A, B) as the characteristic of p or \tilde{p} . Note that the Hilbert norm $\|\cdot\|$ induces on \mathcal{S}_2^T a topology slightly different from the traditional semimartingale topologies that are often used in stochastic analysis. Let $\tilde{\mathcal{S}}_2^T$ be the image of \mathcal{S}_2^T under the Itô map with the induced topology. For any $t \in [-\zeta, T]$ and $\tilde{p} \in \tilde{\mathcal{S}}_2^T$, the map

$$\begin{aligned} \tilde{p}^t : [-\zeta, T] &\rightarrow \mathcal{M} \\ s &\mapsto \tilde{p}^t(s) := \tilde{p}(t \wedge s) \end{aligned}$$

also belongs to $\tilde{\mathcal{S}}_2^T$.

3. MAIN RESULTS

In this section, we prove the existence and uniqueness of solution of the following SFDE with delay on compact Riemannian d -dimensional manifold \mathcal{M} :

$$\begin{cases} d\tilde{x}(t) = \tilde{f}(t, \tilde{x}^t)dt + \tilde{g}(t, \tilde{x}^t)dW(t), t \in [0, T] \\ \tilde{x}^0(t) = \tilde{p}^0(t), t \in [-\zeta, 0] \end{cases} \quad (3.1)$$

where $\tilde{f} : [0, T] \times \tilde{\mathcal{S}}_2^T \rightarrow L^0(\Omega, T\mathcal{M})$, $\tilde{g} : [0, T] \times \tilde{\mathcal{S}}_2^T \rightarrow L^0(\Omega, L(\mathbb{R}^m, T\mathcal{M}))$ are non-anticipating i.e. $\tilde{f}(t, \tilde{p}) = \tilde{f}(t, \tilde{p}^t)$ and $\tilde{g}(t, \tilde{p}) = \tilde{g}(t, \tilde{p}^t)$ for all $(t, \tilde{p}) \in [0, T] \times \tilde{\mathcal{S}}_2^T$. Moreover the flat versions of \tilde{f} and \tilde{g} defined by:

$$\begin{aligned} f(t, p) &= \tau_t^{-\zeta}(\tilde{p})\tilde{f}(t, \tilde{p}) \\ g(t, p) &= \tau_t^{-\zeta}(\tilde{p})\tilde{g}(t, \tilde{p}) \end{aligned}$$

for all $(t, \tilde{p}) \in [0, T] \times \tilde{\mathcal{S}}_2^T$ satisfy:

for each $p \in \mathcal{S}_2^T$, the processes $[0, T] \ni t \mapsto f(t, p) \in T_x\mathcal{M}$ and $[0, T] \ni t \mapsto g(t, p) \in L(\mathbb{R}^k, T_x\mathcal{M})$ are $(\mathcal{F}_t)_{0 \leq t \leq T}$ -semimartingales.

In order to establish the existence of a unique solution of (3.1), we consider non-Lipschitz conditions on \tilde{f} and \tilde{g} . We shall make the following assumptions:

- (H1)** (a) There exists a function $H : [0, +\infty) \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $(t, y) \mapsto H(t, y)$ is locally integrable in $t \geq 0$ and is continuous monotone nondecreasing with respect to y for any fixed $t \in [0, T]$. Moreover, for all $t \in [0, T]$, $p \in \mathcal{S}_2^T$, the following inequality is satisfied:

$$\mathbb{E}\|f(t, p^t)\|^2 \vee \mathbb{E}\|g(t, p^t)\|^2 \leq H(t, \|p^t\|^2).$$

- (b) For any constant $K > 0$, the differential equation

$$\frac{dx}{dt} = KH(t, x), \quad t \in [0, T]$$

has a global solution for any initial value x_0 .

- (H2)** (a) There exists a function $L : [0, +\infty) \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $(t, y) \mapsto L(t, y)$ is locally integrable in $t \geq 0$ and is continuous monotone nondecreasing with respect to y for any fixed $t \in [0, T]$. $L(t, 0) = 0 \forall t \in [0, T]$. Moreover for all $t \in [0, T]$, $p, q \in \mathcal{S}_2^T$, the following inequality is satisfied:

$$\mathbb{E}\|f(t, p^t) - f(t, q^t)\|^2 \vee \mathbb{E}\|g(t, p^t) - g(t, q^t)\|^2 \leq L(t, \|p^t - q^t\|^2).$$

- (b) For any constant \bar{K} , if a nonnegative function $z : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfies

$$z(t) \leq \bar{K} \int_0^t L(s, z(s))ds, \quad \forall t \in [0, T]$$

then $z(t) = 0; \quad \forall t \in [0, T]$.

Under **(H1)**-**(H2)**, we obtain the existence and uniqueness of solution of (3.1) as follows:

Theorem 3.1. *Assume that the geometrical stochastic differential equation (3.1) satisfies **(H1)**-**(H2)**. Suppose that $\tilde{p}^0 \in \tilde{\mathcal{S}}_2^T$ has characteristic $(A^0(t), B^0(t))$, $t \in [-\zeta, 0]$ which is adapted and satisfies*

$$\mathbb{E} \left[\int_{-\zeta}^0 \|A^0(s)\|^2 ds \right] + \mathbb{E} \left[\int_{-\zeta}^0 \|B^0(s)\|^2 ds \right] < \infty.$$

Then there exists a unique global solution of (3.1).

Proof. Using the Itô map, we pullback the geometrical stochastic differential equation (3.1) to an Euclidean stochastic differential equation on the flat space $\mathbb{T}_x \mathcal{M}$ as follows

$$\begin{cases} dx(t) = f(t, x^t)dt + g(t, x^t)dW(t), t \in [0, T] \\ x^0(t) = p^0(t), t \in [-\zeta, 0] \end{cases} \quad (3.2)$$

It is then sufficient to prove the existence and uniqueness of the solution of (3.2). To do this, we use successive approximations. Define the sequence $(x_n)_{n \in \mathbb{N}^*} \subset \mathcal{S}_2^T$

inductively by setting $x_1(t) = \begin{cases} p^0(t), t \in [-\zeta, 0] \\ p^0(0), t \in [0, T] \end{cases}$ and

$$\begin{cases} dx_{n+1}(t) = f(t, x_n^t)dt + g(t, x_n^t)dW(t), & t \in [0, T], \\ x_{n+1}^0(t) = p^0(t), t \in [-\zeta, 0], \end{cases} \quad (3.3)$$

for all $n \geq 1$.

Step 1. We show that the sequence $\{x_n(\cdot), n \in \mathbb{N}^*\} \subset \mathcal{S}_2^T$. From (3.3) and **(H1)**, for $0 \leq t \leq T$,

$$\begin{aligned} \mathbb{E} \int_{-\zeta}^t \|f(s, x_n^s)\|^2 ds &\leq \int_{-\zeta}^t H(s, \|x_n^s\|^2) ds \\ &\leq \int_{-\zeta}^0 H(s, \|x_n^s\|^2) ds + \int_0^t H(s, \|x_n^s\|^2) ds \\ &\leq \int_{-\zeta}^0 H(s, \|p^0(s \wedge \cdot)\|^2) ds \\ &\quad + \int_0^t H(s, \|x_n^s\|^2) ds. \end{aligned} \quad (3.4)$$

By (3.4), we obtain

$$\mathbb{E} \int_{-\zeta}^t \|f(s, x_n^s)\|^2 ds \leq \int_{-\zeta}^0 H(s, \|p^0(s \wedge \cdot)\|^2) ds + \int_0^t H(s, \|x_n^s\|^2) ds.$$

We similarly have,

$$\mathbb{E} \int_{-\zeta}^t \|g(s, x_n^s)\|^2 ds \leq \int_{-\zeta}^0 H(s, \|p^0(s \wedge \cdot)\|^2) ds + \int_0^t H(s, \|x_n^s\|^2) ds.$$

Since $\|p^0\| < \infty$, then $\|p^0(s \wedge \cdot)\| < \infty$, for all $s \in [-\zeta, 0]$. Moreover $\|x_n^s\|^2 \leq \|x_n\|^2$ for all $s \geq 0$. Hence, by (2.2)

$$\|x_{n+1}^t\|^2 \leq C_1 + 2 \int_0^t H(s, \|x_n^s\|^2) ds, \quad (3.5)$$

where $C_1 = 2 \int_{-\zeta}^0 H(s, \|p^0(s \wedge \cdot)\|^2) ds$. Assumption ((H1) – (b)) indicates that there is a solution u that satisfies

$$u(t) = C_1 + 2 \int_0^t H(r, \|u(r)\|^2) dr, t \in [0, T]. \quad (3.6)$$

We have $\|x_{n+1}^t\|^2 \leq u(t) \leq u(T) < \infty$, and for $t = T$, $\|x_{n+1}\|^2 \leq u(T) < \infty$. Hence $\{x_n(\cdot), n \in \mathbb{N}^*\} \subset \mathcal{S}_2^T$.

Step 2. We show that $\{x_n(\cdot), n \in \mathbb{N}^*\}$ is a Cauchy sequence. For all $n, m \geq 0$ and $t \in [0, T]$, from (3.3), Step 1 and (H2) we have

$$\mathbb{E} \int_{-\zeta}^t \|f(s, x_n^s) - f(s, x_m^s)\|^2 ds \leq \int_0^t L(s, \|x_n^s - x_m^s\|^2) ds,$$

and

$$\mathbb{E} \int_{-\zeta}^t \|f(s, x_n^s) - f(s, x_m^s)\|^2 ds \leq \int_0^t L(s, \|x_n^s - x_m^s\|^2) ds.$$

Then

$$\|x_{n+1}^t - x_{m+1}^t\|^2 \leq 2 \int_0^t L(s, \|x_n^s - x_m^s\|^2) ds. \quad (3.7)$$

Let

$$b(t) = \lim_{n, m \rightarrow +\infty} \|x_n^t - x_m^t\|^2.$$

By Assumption ((H2)-(b)) and Fatou lemma, we have

$$b(t) \leq 2 \int_0^t L(s, b(s)) ds.$$

By Assumption ((H2)-(b)), we have $b(t) = 0, \forall t \in [0, T]$. This proves that $\{x_n(\cdot), n \in \mathbb{N}^*\}$ is a Cauchy sequence.

Step 3. We show the existence and uniqueness of the solution of (3.2). The Borel-Cantelli lemma shows that, as $n \rightarrow +\infty$, $x_n \rightarrow x$ uniformly. Moreover, by Assumption (H2) the functions $f(t, \cdot)$ and $g(t, \cdot)$ are continuous for all $t \in [0, T]$. Hence, taking limits on both sides of (3.3), we get that x is a solution of (3.2). This shows the existence. The uniqueness of the solution is obtained by a procedure similar to Step 2. This completes the proof. \square

Remark 3.2. If $L(t, x) = K_1 x$ for some constant K_1 , then Condition (H2) implies a global Lipschitz condition.

4. APPLICATION

We consider the geometrical stochastic differential equation on a Hadamard manifold \mathcal{M} :

$$\begin{cases} d\tilde{q}(t) = \tau_{t-\zeta}^t \lambda(t) F_1(\tilde{q}(t-\zeta)) ds + \tau_{t-\zeta}^t(\tilde{q}) \gamma q(t-\zeta) dW(s), & 0 \leq t \leq T \\ \tilde{q}^0(t) = \theta(t), & -\zeta \leq t \leq 0 \end{cases} \quad (4.1)$$

where $\gamma \in \mathbb{R}$, W denotes a \mathbb{R} -valued Brownian motion, $\lambda(\cdot)^2 > 0$ is locally integrable function, $\tau_s^t(\tilde{q}) : \mathbb{T}_{\tilde{q}(s)}\mathcal{M} \rightarrow \mathbb{T}_{\tilde{q}(t)}\mathcal{M}$ is the stochastic parallel transport, F_1 is a bounded smooth section of $\mathbb{T}\mathcal{M}$, and $\theta : [-\zeta, 0] \rightarrow \mathcal{M}$ is continuous semimartingale with characteristic $(A^0(t), B^0(t))$, $t \in [-\zeta, 0]$ that satisfies

$$\mathbb{E} \left[\int_{-\zeta}^0 \|A^0(s)\|^2 ds \right] + \mathbb{E} \left[\int_{-\zeta}^0 \|B^0(s)\|^2 ds \right] < \infty$$

and q is the image of \tilde{q} by Itô map defined by (2.1). Let ϕ be as follows:

$$\phi(x) = \begin{cases} 0 & \text{if } x = 0, \\ cx \left(\log \frac{1}{x} \right) & \text{if } 0 < x \leq \delta, \\ c\delta \left(\log \frac{1}{x} \right) & \text{if } x > \delta \end{cases}$$

or

$$\phi(x) = \begin{cases} 0 & \text{if } x = 0, \\ cx \left(\log \frac{1}{x} \right)^{\frac{1}{x}} \log \log \frac{1}{x} & \text{if } 0 < x \leq \delta, \\ c\delta \left(\log \frac{1}{x} \right)^{\frac{1}{x}} \log \log \frac{1}{x} & \text{if } x > \delta \end{cases}$$

with $c > 0$ and $0 < \delta \leq 1$ is sufficiently small. Assume that the following inequality is satisfied:

$$\|F_1(\tilde{p}_1(t)) - F_1(\tilde{p}_2(t))\|^2 \leq \phi(\|p_1 - p_2\|^2)$$

for all $\tilde{p}_1, \tilde{p}_2 \in \tilde{\mathcal{S}}_2^T$, $t \in [-\zeta, T]$. For $t \in [0, T]$, $\tilde{p} \in \tilde{\mathcal{S}}_2^T$, define the functions

$$\begin{aligned} F(t, \tilde{p}^t) &= \tau_{t-\zeta}^t(\tilde{p}) \lambda(t) F_1(\tilde{p}(t-\zeta)) \\ G(t, \tilde{p}^t) &= \tau_{t-\zeta}^t(\tilde{p}) \gamma p(t-\zeta). \end{aligned}$$

Then the equation (4.1) takes the form

$$\begin{cases} d\tilde{q}(t) = F(t, \tilde{q}^t) dt + G(t, \tilde{q}^t) dW(t), & t \in [0, T] \\ \tilde{q}^0(t) = \theta(t), & t \in [-\zeta, 0]. \end{cases}$$

For any nonnegative real number r , we set

$$\Phi(r) = \gamma^2 r.$$

We define ρ by

$$\rho(t) = \begin{cases} 1 & \text{if } \lambda(t) \leq 1, \\ \lambda^2(t) & \text{if } \lambda(t) > 1. \end{cases}$$

Then ρ is a locally integrable function and we have, for all $\tilde{p}_1, \tilde{p}_2 \in \tilde{\mathcal{S}}_2^T$,

$$\begin{aligned} & \mathbb{E} \|\tau_{t-\zeta}^t(\tilde{p}_1)\lambda(t)F_1(\tilde{p}_1(t-\zeta)) - \tau_{t-\zeta}^t(\tilde{p}_2)\lambda(t)F_1(\tilde{p}_2(t-\zeta))\|^2 \\ & + \mathbb{E} \|\tau_{t-\zeta}^t(\tilde{p}_1)\gamma p_1(t-\zeta) - \tau_{t-\zeta}^t(\tilde{p}_2)\gamma p_2(t-\zeta)\|^2 \\ & \leq \rho(t) [\phi(\|p_1 - p_2\|^2) + \Phi(\|p_1 - p_2\|^2)], \end{aligned}$$

since ϕ is a concave function. It follows that $\phi(r) > \phi(1)r$ for $0 \leq r < 1$. Thus it holds that

$$\begin{aligned} \int_{0+}^{\infty} \frac{1}{\phi(r) + \Phi(r)} dr &= \int_{0+}^{\infty} \frac{1}{\phi(r) + \gamma^2 r} dr \\ &\geq \frac{\phi(1)}{\phi(1) + \gamma^2} \int_{0+}^1 \frac{1}{\phi(r)} dr \\ &= \infty. \end{aligned}$$

Then by Lemma 3 in [17] (p.157), **(H2)** holds. Therefore by Theorem (3.1) there exists a unique global solution of (4.1).

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