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ON PERIODIC SHADOWING, TRANSITIVITY, CHAIN MIXING AND EXPANSIVITY IN UNIFORM DYNAMICAL SYSTEMS

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Abstract. In this paper we extend some results on the notions such as Expansiveness, Pseudo Orbit Tracing Property (P.O.T.P.), Chain Transitivity, Periodic Shadowing property to uniform dynamical system (X, f), where X is a compact uniform space. We prove that if an expansive dynamical system (X, f) on compact uniform space has P.O.T.P., then it has periodic shadowing.

1. Introduction and preliminaries

A topological dynamical system usually consists of a compact metric space X with metric d and a continuous function f from X to itself. The most generalised case would be when X is a topological space, and f a continuous function on X. However, when X is a compact metric space and f a continuous self map, there are many mathematical results regarding various notions. In this paper, we want to extend some interesting results to the case when X is a compact uniform space and f, a continuous self map on X. D. Alcaraz and M. Sanchiz first started to study dynamical systems on uniform spaces [4]. They extended the notions such as transitivity, minimality and chaos in uniform spaces. In [14], the notions such as expansivity, topological shadowing, topological chain recurrence were introduced in uniform spaces. Shah et al.[31] gave a sufficient condition for a map to have specification property on uniform spaces. Ahmadi [2] introduced the notions of ergodic shadowing, chain transitivity, and topological ergodicity on non compact and non metrizable spaces. In [28], Pramod Das and Tarun Das extended the notions of topological pseudo orbit specification, topological weak specification, topological ergodic shadowing, topological $\underline{d}$-shadowing for continuous maps on uniform spaces. They show that these notions are equivalent for uniformly continuous maps with topological shadowing on totally bounded uniform spaces. In [37], the notions such as rigidity and sensitivity were extended and studied in uniform spaces. The notion of positively expansive maps were introduced in uniform spaces [33, 16]. There are more results of dynamical systems

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on uniform spaces in [3, 36].

In order to make the paper self contained, we give definitions of some important notions of topological dynamics. A sequence $\langle x_i \rangle_{i\geq 0}$ is said to be an orbit in (X, f) if $x_{i+1} = f(x_i)$ for all $i \in \mathbb{Z}_+$, where $\mathbb{Z}_+$ denotes the set of non negative integers. When f is a homeomorphism, we say that a bisequence $\langle x_i \rangle_{i\in\mathbb{Z}}$ is an orbit in (X, f) if $x_{i+1} = f(x_i)$ for all $i \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers. A sequence $\langle x_i \rangle_{i\geq 0}$ is said to be a δ-pseudo orbit if $d(f(x_i), x_{i+1}) < \delta$ for all $i \geq 0$. A finite sequence $\langle x_i \rangle_{i=0}^{n}$ is said to be a δ-chain of length n if $d(f(x_i), x_{i+1}) < \delta$ for all $0 \leq i \leq n-1$. The idea of shadowing was first introduced by Anosov in [10] (See also [1, 8, 12, 13, 15, 18, 19, 21, 27, 24, 29, 25, 30, 34, 35]). Shadowing property is an important topic in dynamical system and it states that: a dynamical system (X, f) is said to have shadowing property on a set $Y \subset X$ if for each $\epsilon > 0$, there exists $\delta > 0$ such that any δ-pseudo orbit in Y is ϵ-shadowed by some point x i.e., $d(f^i(x), x_i) < \epsilon$ for all $i \geq 0$. When $Y = X$, we say that the dynamical system (X, f) has shadowing property or pseudo orbit tracing property (P.O.T.P.). Various results regarding various notions of shadowing have been studied by many Mathematician. The notion of periodic shadowing is one of the various notions of shadowing. It was introduced by Koscielniak [20]. A dynamical system (X, f), where f is a homeomorphism, is said to have periodic shadowing property if for each $\epsilon > 0$ there exists $\delta > 0$ such that every periodic δ-pseudo orbit is ϵ-shadowed by a periodic point p i.e., $d(f^i(p), x_i) < \epsilon$ for all $i \in \mathbb{Z}$. A dynamical system (X, f) is said to be topologically transitive if for any pair of non-empty open sets U, V in X, there exists a positive integer $n \geq 1$ such that $f^n(U) \bigcap V \neq \phi$. The concept of topological transitivity goes back to G.D.Birkhoff in 1920 [17] (See also [11, 22, 7, 23, 32]). A dynamical system (X, f) is said to be topologically mixing if for any pair of non-empty open sets U, V in X, there exists a positive integer N such that $f^n(U) \bigcap V \neq \phi$, for all $n \geq N$. A dynamical system (X, f) is said to be chain transitive if for any $\delta > 0$ and any pair of points $x, y \in X$ there exists a δ-chain from x to y. We say that (X, f) is chain mixing if for each pair of points $x, y \in X$ and for $\delta > 0$ there exists a positive integer N such that $\forall n \geq \mathbb{N}$, there exists a δ-chain from x to y of length n.

Uniform space is a generalization of the notion of metric space. The concept of uniformity was introduced by Andrie Weil [9]. Let X be a non-empty set and U, V subsets of $X \times X$. Define $U \circ V$ by $U \circ V = \{(x, y) \in X \times X : \exists z \in X$, such that $(x, z) \in U$, and $(z, y) \in V\}$ and U^{-1} by $U^{-1} = \{(x, y) \in X \times X : (y, x) \in U\}$. U^{-1} is called the inverse of U. If $U = U^{-1}$, we say that U is symmetric. It is obvious that $U \cap U^{-1}$ is symmetric. A non-empty collection $\mathcal{U}$ of subsets $U \subset X \times X$ is said to have uniform structure (or a uniformity) if the following are satisfied:
a) Each member U of $\mathcal{U}$ contains the diagonal $\triangle$,where $\triangle = \{(x, x) : x \in X\}$,
b) $U \in \mathcal{U} \Rightarrow U^{-1} \in \mathcal{U}$,
c) $U \in \mathcal{U} \Rightarrow V \circ V \subset U$ for some $V \in \mathcal{U}$,
d) $U \in \mathcal{U}$ and $V \in \mathcal{U} \Rightarrow U \bigcap V \in \mathcal{U}$,
e) $U \in \mathcal{U}$ and $U \subset V \subset X \times X \Rightarrow V \in \mathcal{U}$.

$\mathcal{U}$ is called a uniformity for X and the pair $(X,\mathcal{U})$ is called a uniform space. Members of $\mathcal{U}$ are known as entourages. Let $x \in X$ and $U \in \mathcal{U}$. The set $U[x] = \{y \in X : (x,y) \in U\}$ is called the U-neighbourhood of x. The collection of all such $U[x]$, where x runs over all points in X and U runs over all entourages $U \in \mathcal{U}$, forms a base for a topology on X. The topology generated by this base is called the uniform topology induced by the uniformity $\mathcal{U}$. By a uniform space $(X,\mathcal{U})$, we mean the underlying topology is the uniform topology. It is known that a uniform space is Hausdorff if and only if $\bigcap \mathcal{U} = \triangle$. Let U be an entourage, then it can be verified that $\overline{U} \subset U \circ U \circ U$. For a given entourage $U \in \mathcal{U}$, there exists a pseudo metric d and $\epsilon > 0$ such that $U_{d,\epsilon} = \{(x,y) \in X \times X : d(x,y) < \epsilon\} \subset U$ and $U_{d,\epsilon} \in \mathcal{U}$. In a uniform space $(X,\mathcal{U})$ for every entourage U there exists a symmetric entourage V such that $VoV \subset U$. Proceeding in this way there exists a symmetric entourage V such that $\overbrace{VoVo.....oV}^{2^n-times} \subset U$. We know that for each positive integer n, $V \subset VoV \subset ..... \subset \overbrace{VoVo....oV}^{(n-1)-times} \subset \overbrace{VoVo....oV}^{n-times} \subset .... \subset \overbrace{VoVo.....oV}^{2^n-times} \subset U$. Therefore, for each positive integer n and for each entourage U, there exists a symmetric entourage V such that $\overbrace{VoVo....oV}^{n-times} \subset U$. We use the notation nV to denote $\overbrace{VoVo....oV}^{n-times}$.

1.1. **Definitions.**

In this section, we define some of the notions of a dynamical system to a uniform dynamical system (X,f), where X is a compact uniform space with uniformity $\mathcal{U}$. A dynamical system (X,f) on a compact uniform space is said to be expansive if there exists an entourage D such that for any pair of distinct points $x,y \in X$ there is $n \geq 0$ such that $(f^n(x), f^n(y)) \notin D$. Such an entourage D is called a topological positive expansivity entourage for f. From the definition it follows that if $(f^n(x), f^n(y)) \in D$ for every $D \in \mathcal{U}$, then $x = y$. A D-chain is a finite sequence $\{x_0, x_1, ..., x_m\}$ such that $(f(x_i), x_{i+1}) \in D$, for $0 \leq i \leq (m-1)$. $\{x_0, x_1, ..., x_m\}$ is called a D-chain of length m from x to y. Let $\xi = \langle x_i \rangle_{i \in \mathbb{Z}_+}$ be a sequence in X. ξ is said to be D-pseudo orbit if $(f(x_i), x_{i+1}) \in D$ for all $i \geq 0$. A pseudo orbit $\xi = \langle x_i \rangle_{i \in \mathbb{Z}_+}$ is said to be E-shadowed by a point $y \in X$ if $(f^i(y), x_i) \in E$ for all $i \geq 0$. A uniform dynamical system (X,f) is said to have pseudo orbit tracing property (P.O.T.P.) if for every entourage E there is an entourage D such that every D-pseudo orbit is E-shadowed by some point in X. A dynamical system (X,f) is said to be topologically transitive if for any pair U,V of entourages in $\mathcal{U}$ and for any pair x,y of points in X there exists a positive integer $n \geq 1$ such that $f^n(U[x]) \bigcap V[y] \neq \phi$.

Definition 1.1. A D-pseudo orbit is said to be periodic D-pseudo orbit if there exists a positive integer N such that $x_{n+N} = x_n$ for all $n \geq 1$. (X,f) has periodic shadowing property if for each entourage E there exists an entourage D

such that every periodic D-pseudo orbit is E-shadowed by a periodic point p i.e.,$(f^i(p), x_i) \in E$ for all $i \in \mathbb{Z}_+$.

Definition 1.2. A dynamical system (X, f) on a compact uniform space $(X,\mathcal{U})$ is said to be chain transitive if for each pair of points $x, y \in X$ and for each entourage $D \in \mathcal{U}$, there exists a D-chain $\{x_0 = x, x_2, ..., x_n = y\}$ from x to y.

Definition 1.3. A dynamical system (X, f) on a compact uniform space $(X,\mathcal{U})$ is said to be chain mixing if for each pair of points $x, y \in X$ and for each entourage $D \in \mathcal{U}$, there exists a positive integer N such that $\forall n \geq \mathbb{N}$, there exists a D-chain of length n from x to y.

Definition 1.4. Let $(X,\mathcal{U})$ be a uniform space. A function $f : X \longrightarrow X$ is said to be Lipschitz continuous if there exists a positive integer n such that for each entourage E, the following condition holds

$$(f(x), f(y)) \in nE \quad whenever \quad (x, y) \in E.$$

In this paper we extend some familiar results on the notions such as Expansiveness, Pseudo Orbit Tracing Property (P.O.T.P.), Chain Transitivity and Periodic Shadowing property in uniform dynamical system (X, f), where X is a compact uniform space. In theorem 2.1, we prove that if an expansive dynamical system (X, f) on a compact uniform space has the P.O.T.P. then it has periodic shadowing property. It has been proved in lemma 2.2 that if a continuous self map f on a compact uniform space has finite shadowing, then f has P.O.T.P. We prove in theorem 2.3 that a chain transitive dynamical system (X, f) on compact uniform space has shadowing property if (X, f) has periodic shadowing. In theorem 2.4, we prove that if f is chain mixing on a compact uniform space $(X,\mathcal{U})$, then f^n is chain transitive for each $n \geq 1$. In theorem 2.5 we prove that if f has the periodic shadowing then f^n has periodic shadowing for all $n > 1$. In theorem 2.6 we prove that periodic shadowing is invariant of topological conjugacy provided that the conjugacy and its inverse are Lipschitz.

2. Results

In [6], Darabi and Fourouzanfar show that if f is an expansive map on a compact metric space and f has the shadowing property, then it has the periodic shadowing. We extend the result in uniform dynamical systems as follows:

Theorem 2.1. *Let f be an expansive homeomorphism on a compact uniform space $(X,\mathcal{U})$. If f has the pseudo-orbit tracing property, then it has the periodic shadowing.*

Proof. Let $C \in \mathcal{U}$ be an expansive entourage of f. Without loss of generality, it is sufficient to prove for a symmetric entourage E such that $EoE \subset C$. By pseudo orbit tracing property, there exists $D \in \mathcal{U}$ such that if $\langle x_i \rangle_{i\in\mathbb{Z}}$ is a periodic D-pseudo orbit of f, then there is $z \in X$ such that

$$(f^i(z), x_i) \in E \quad \forall i \in \mathbb{Z}. \tag{2.1}$$

Suppose μ is the period of $\langle x_i\rangle$ i.e., $x_{i+\mu} = x_i$ for all $i \in \mathbb{Z}$. It follows that $(f^{i+\mu}(z), x_{i+\mu}) = (f^{i+\mu}(z), x_i) \in E$ for all $i \in \mathbb{Z}$. Let $q = f^{\mu}(z)$. Then,

$$(f^i(q), x_i) \in E \quad \forall i \in \mathbb{Z}. \tag{2.2}$$

From equations 2.1 and 2.2 we have
$(f^i(z), f^i(q)) \in EoE \subset C$ for all $i \in \mathbb{Z}$ i.e.,$(f^i(z), f^i(q)) \in C$ for all $i \in \mathbb{Z}$. By expansivity of f, we have, $z = q$ i.e.,$z = f^{\mu}(z)$. Hence f has periodic shadowing. □

In [26][Lemma 1.1.1], It is proved that if X is a compact metric space and f has finite shadowing property on $Y \subset X$, then f has the P.O.T.P on Y. The result is proved when f is a homeomorphism. We now give the equivalent lemma in uniform dynamical systems when f is a continuous function.

Lemma 2.2. *Let $(X,\mathcal{U})$ be a compact uniform space and $f : X \longrightarrow X$, a continuous self map. Suppose f has finite shadowing, then f has pseudo orbit tracing property.*

Proof. Let E be an entourage. Let $E^{'}$ be a symmetric entourage such that $E^{'}oE^{'}oE^{'} \subset E$. Let D be the entourage given by the finite shadowing property corresponding to $E^{'}$. Let $\langle x_k\rangle$ be a D-pseudo orbit for f. For $n > 0$, by hypothesis there is $y_n \in X$ such that $(f^k(y_n), x_k) \in E^{'}$, $0 \leq k \leq n$. Let w be a limit point of the sequence $\langle y_n\rangle$. Passing to the limit as $n \to \infty$, we get $(f^k(w), x_k) \in \overline{E'} \subset E^{'}oE^{'}oE' \subset E$ for all $k \geq 0$ i.e., $(f^k(w), x_k) \subset E$ for all $k \geq 0$. Hence (X, f) has shadowing property. □

It is shown in [6] that, if f is a chain transitive map on a compact metric space X and f has the periodic shadowing, then f has the shadowing property and so transitivity. We extend the result in uniform dynamical systems as follows:

Theorem 2.3. *Let f be a chain transitive dynamical system on a compact uniform space. If f has the periodic shadowing , then f has the shadowing property and so transitivity.*

Proof. It is sufficient to prove that f has the finite shadowing property. Let E be an entourage. By periodic shadowing property, there exists an entourage D such that if $\langle x_i\rangle_{i=0}^{m}$ is a finite D-pseudo orbit of f, then it is E-shadowed by some periodic point in X. By chain transitivity of f, there is a D-chain from x_m to x_0 of length n i.e., there are points $x_m, x_{m+1}, ..., x_{m+n-1}, x_{m+n} = x_\circ$ so that $(f(x_i), x_{i+1}) \in D$, $0 \leq i \leq n-1$. Now,$\{x_\circ, x_1, ..., x_{m+n-1}, x_{m+n}\}$ is a D-pseudo orbit of length $m+n$ from x_0 to x_0. We extend the $(m+n)D$-pseudo orbit to a periodic D-pseudo orbit $\xi = \langle y_i\rangle_{i\in\mathbb{N}}$. By hypothesis, there is a periodic point $z \in X$ such that $(f^i(z), y_i) \in E$, $i \in \mathbb{N}$. It follows that $(f^i(z), x_i) \in E$, $0 \leq i \leq n$. □

In [6] it is shown that if f is chain mixing on a compact metric space, then f^n is chain transitive for each $n \geq 1$. The following theorem is the extension of the result in uniform dynamical systems.

Theorem 2.4. *Suppose f is chain mixing on a compact uniform space $(X,\mathcal{U})$, then f^n is chain transitive for each $n \geq 1$.*

Proof. Fix $n \geq 1$. Take an arbitrary entourage E. By uniform continuity we can find an entourage D such that if $(u, v) \in D$, then $(f^i(u), f^i(v)) \in V$, $i = 1, 2, ...n$, where V is a symmetric entourage such that $nV \subset E$. Without loss of generality, we assume that $D \subset V$. Let x, y be two points in X. Since f is chain mixing, we can find a D-chain $x = t_o, t_1, ..., t_N = y$ from x to y of length $N = kn$, where k is some positive integer. Now, $(f(t_i), t_{i+1}) \in D$, $0 \leq i \leq N-1$. It follows that

$$\begin{aligned}(f^n(t_o), f^{n-1}(t_1)) &\in V,\\ (f^{n-1}(t_1), f^{n-2}(t_2)) &\in V,\\ &\cdot\\ &\cdot\\ &\cdot\\ (f(t_{n-1}), t_n) &\in D \subset V.\end{aligned}$$

Therefore, $(f^n(t_o), t_n) \in nV \subset E$. In general, we can show that $(f^n(t_{in}), t_{(i+1)n}) \in E, \quad 0 \leq i \leq k-1$. It follows that $x = t_0, t_n, t_{2n}, ..., t_{kn} = t_N = y$ is an E-chain from x to y with respect to f^n. Hence, f^n is chain transitive for each $n \geq 1$. □

In [6] Darabi and Fourounzanfer prove that, if f has periodic shadowing on compact metric space then so does f^n for all $n > 1$. In the following, we extend the result in uniform dynamical systems.

Theorem 2.5. *If f has the periodic shadowing then f^n has periodic shadowing for all $n > 1$.*

Proof. Fix $n \geq 1$. Let E be an arbitrary entourage. By the periodic shadowing of f, there exists an entourage D such that every D-pseudo orbit for f is E-shadowed by some periodic point. Let $\langle x_i \rangle_{i \geq 0}$ be a periodic D-pseudo orbit for f^n. Define the sequence $\eta = \langle y_i \rangle_{i \geq 0}$ by

$$y_i = \begin{cases} x_q & \text{for} \quad i = nq, \\ f^{i-nq}(x_q) & \text{for} \quad nq < i < n(q+1) \end{cases}$$

i.e.,$x_0 = y_0, y_1 = f(x_0), f^2(x_0) = y_2, ...y_{n-1} = f^{n-1}(x_0), y_n = x_1, y_{n+1} = f(x_1), ..., y_{2n-1} = f^{n-1}(x_1), ..., y_{2n} = x_2, ...$ Indeed $\eta = \langle y_i \rangle_{i \geq 0}$ is also a periodic sequence of period mn, where m is the period of $\langle x_i \rangle_{i \geq 0}$. Now,

$$(f(y_i), y_{i+1}) = \begin{cases} (f(x_q), f(x_q)) & \text{if} \quad i = nq \\ (f^{i+1-nq}(x_q), f^{i+1-nq}(x_q)) & \text{if} \quad nq < i < n(q+1). \end{cases}$$

Therefore $\langle y_i \rangle_{i \geq 0}$ is a D-pseudo orbit with respect to f. By the periodic shadowing of f, we can find a periodic point $z \in X$ such that $(f^i(z), y_i) \in E$. Putting $i = nq$, we get $y_i = x_q$ therefore, $((f^n)^q(z), x_q) \in E$. Hence, f^n has the periodic shadowing. □

In [5], Darabi proves that periodic shadowing is invariant of topological conjugacy provided the conjugacy and its inverse are Lipschitz. We want to extend the result in uniform dynamical systems.

Theorem 2.6. *The periodic shadowing is invariant of topological conjugacy provided that the conjugacy and its inverse are Lipschitz.*

Proof. Let $(X,\mathcal{U})$ be a compact uniform space. f and g are two continuous self maps on X. Let h be a conjugacy of f and g i.e., h is a homeomorphism such that $hof = goh$. Suppose h and h^{-1} are Lipschitz continuous with Lipschitz constants $n^{'}$ and n respectively. We assume that f has periodic shadowing. Suppose E is a given entourage. Let E' be an entourage such that $n'E' \subset E$ and D' the corresponding entourage such that every D'-pseudo periodic orbit in (X, f) is E'-shadowed by a periodic point. Let D be an entourage such that $nD \subset D'$. Let $\langle x_i\rangle_{i\geq 0}$ be a D-pseudo periodic orbit in (X, g). Then,$\langle h^{-1}(x_i)\rangle_{i\geq 0}$ is a D'-pseudo periodic orbit in (X, f) because $(f(h^{-1}(x_i)), h^{-1}(x_{i+1})) = (h^{-1}g(x_i), h^{-1}(x_{i+1})) \in nD \subset D'$. Therefore there exists a periodic point z in (X, f)
such that $(f^i(z), h^{-1}(x_i)) \in E'$ for all $i \geq 0$. It follows that $(hf^i(z), hh^{-1}(x_i)) \in n'E'$. Therefore $(g^i(h(z)), x_i) \in n^{'}E^{'} \subset E$, where $h(z)$ is a periodic point in (X, g). □

2.1. **Conclusion.** It has been found that if an expansive dynamical system (X, f) on a compact uniform space has the P.O.T.P. then it has periodic shadowing property. It has been proved that if a continuous self map f on a compact uniform space has finite shadowing, then f has P.O.T.P. As in usual dynamical system on compact metric space, a chain transitive dynamical system (X, f) on compact uniform space has shadowing property if (X, f) has periodic shadowing. It has been found that if f is chain mixing on a compact uniform space $(X,\mathcal{U})$, then f^n is chain transitive for each $n \geq 1$. It has been found that if a dynamical system (X, f) on compact uniform space has periodic shadowing then (X, f^n) has periodic shadowing for all $n > 1$. Lastly, it has been proved that periodic shadowing is invariant of topological conjugacy provided that the conjugacy and its inverse are Lipschitz.

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