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METRIC AND STRONG METRIC DIMENSIONS OF GENERALIZED EDGE CORONA PRODUCT OF GRAPHS

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Abstract. This paper investigates the metric dimension and strong metric dimension problems for the generalized edge corona product of graphs. The bounds and exact values for the metric and strong metric dimensions of this graph product are derived.

Keywords. Metric basis, metric dimension, mutually maximally distant vertices, strong metric dimension, generalized edge corona product graphs
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1. Introduction and preliminaries

Throughout this paper, G = (V (G),E(G)) denotes a simple, connected, and undirected graph with n vertices. The terminology and notation used are consistent with those in [9].

For any two vertices $x_1, x_2 \in V(G)$, let $R\{x_1, x_2\} = \{z \in V(G) : d_G(z, x_1) \neq d_G(z, x_2)\}$. A set $W \subseteq V(G)$ is called a resolving set of G if $W \cap R\{x_1, x_2\} \neq \emptyset$ for any two distinct vertices $x_1, x_2 \in V(G)$. The minimum cardinality of all resolving sets of G is called the metric dimension of G and it is denoted by $dim(G)$.

The metric dimension was first studied by Slater [26] (independently by Harary and Melter [10]) and further studied in [4, 8, 11, 12, 14, 19, 21, 22, 27]. It was noted in [7] that determining the metric dimension of a graph is an NP-hard problem. The concept of metric dimension has several applications in chemistry for representing chemical components [4], the navigation of robots in networks [13], network optimization [2, 3] and mastermind game [5].

Another invariant, more restricted than the metric dimension was presented by Sebő and Tannier in [24], and studied further in several articles [6, 15, 16, 17, 20, 23, 25, 28, 29]. Following [20], for two vertices x and y in a connected graph G, the interval $I[x, y]$ between x and y is said to be the collection of all vertices that belong to some shortest $x - y$ path.

For $x_1, x_2 \in V(G)$, let $P\{x_1, x_2\} = \{s \in V(G) : x_1 \in I[x_2, s]$ or $x_2 \in I[x_1, s]\}$. A set $S \subseteq V(G)$ is called a strong resolving set of G if $S \cap P\{x_1, x_2\} \neq \emptyset$ for

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any two distinct vertices $x_1, x_2 \in V(G)$. The minimum cardinality of all strong resolving sets of G is called the strong metric dimension of G and it is denoted by $dim_s(G)$.

A vertex u of G is said to be maximally distant from v if for every $w \in N(u)$, $d_G(v, w) \leq d_G(v, u)$. If u is maximally distant from v and v is maximally distant from u, then the vertices u and v are said to be mutually maximally distant [17].

Let G be a simple graph of order n and size m. Let H_i, $1 \leq i \leq m$, be m simple graphs. The generalized edge corona product of graphs G and $H_1, H_2 \dots H_m$, denoted by $G[H_i]_1^m$ is obtained by taking one copy of graphs $G, H_1, H_2 \dots H_m$ and joining the two end vertices of the ith edge of G to every vertex of H_i, $1 \leq i \leq m$ [1, 18]. See Figures 1 and 2.

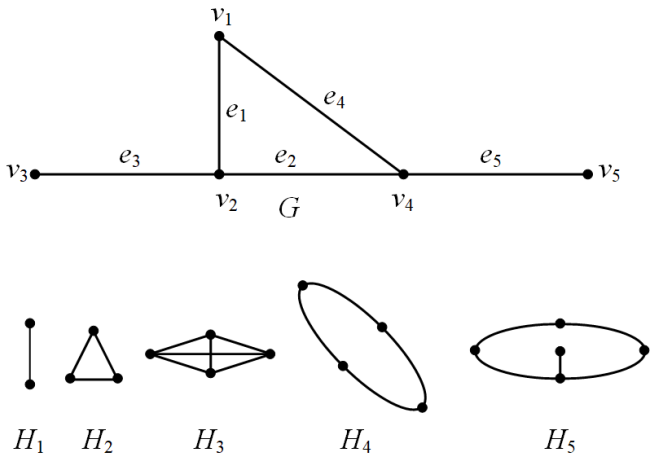


FIGURE 1. The graph G with labeled vertices $v_1, v_2 \dots v_5$ and edges $e_1, e_2 \dots e_5$, and a family of graphs $\{H_1, H_2, H_3, H_4, H_5\}$, where each H_i corresponds to the edge e_i of G.

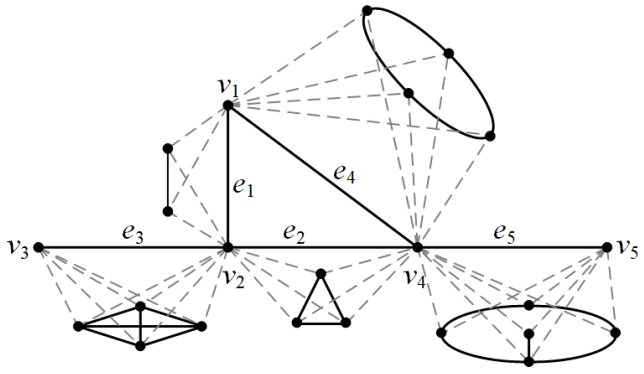


FIGURE 2. The generalized edge corona product $G[H_i]_1^5$, where the end vertices of the edge e_i are connected to each vertex of the corresponding H_i.

In this paper, the metric dimension and strong metric dimension problems for the generalized edge corona product of graphs are investigated.

2. Main results

Section 2.1 focuses on the metric dimension of the generalized edge corona product of graphs, presenting our main results and proofs. Section 2.2 discusses the strong metric dimension of the generalized edge corona product of graphs. We conclude the paper in Section 3 with a summary and directions for future research.

2.1. Metric dimension of $G[H_i]_1^m$.

Let G be a graph of order at least three and H a graph of order at least two. Let $e = uv$ be an edge of G. The graph formed by joining both u and v to every vertex of H is denoted by G_e.

Lemma 2.1. *For any two vertices of x and y of H and a vertex $z \in V(G) \setminus \{u, v\}$, $d_{G_e}(z, x) = d_{G_e}(z, y)$.*

Proof. By hypothesis, $d_{G_e}(u, w) = d_{G_e}(v, w) = 1$ for all $w \in V(H)$. Let $x, y \in V(H)$ and $z \in V(G) \setminus \{u, v\}$. Without loss of generality, assume that there exists a shortest path from z to x that passes through u. If $d_G(z, u) = l$, then $d_{G_e}(z, x) = d_G(z, u) + d_{G_e}(u, x) = l+1$ and $d_{G_e}(z, y) = d_G(z, u) + d_{G_e}(u, y) = l+1$. This proves the claim. □

Lemma 2.2. *Let $e = uv$ be any pendant edge of G such that $deg(v) = 1$. Then for any two vertices x, y of H and a vertex $z \in V(G) \setminus \{u, v\}$, $d_{G_e}(z, x) = d_{G_e}(z, y) = d_G(z, v)$.*

Proof. By hypothesis, $d_{G_e}(u, w) = d_{G_e}(v, w) = 1$ for all $w \in V(H)$. Let $x, y \in V(H)$ and let $z \in V(G) \setminus \{u, v\}$. Clearly a shortest path from z to x (or y) passes through u. If $d_G(z, u) = l$, then $d_{G_e}(z, x) = d_{G_e}(z, y) = d_G(z, v) = l + 1$, since $d_G(u, v) = 1$. □

Theorem 2.3. *Let G be a connected graph of order $n \geq 3$ and size m. Let $H_1, H_2 \ldots H_m$ be m simple graphs of order at least two. Let H_i be the graph corresponding to the edge e_i of G in $G[H_i]_1^m$. Then*

- **(i):** *If $x, y \in V(H_i)$, then $d_{G[H_i]_1^m}(z, x) = d_{G[H_i]_1^m}(z, y)$ for every $z \in V(G[H_i]_1^m) \setminus V(H_i)$.*
- **(ii):** *If W is a resolving set for $G[H_i]_1^m$, then $W \cap V(H_i) \neq \emptyset$ for every i, $1 \leq i \leq m$. Further $W \cap V(H_i)$ is a resolving set for H_i.*
- **(iii):** *If W is a metric basis for $G[H_i]_1^m$, then $W \cap V(G) = \emptyset$.*

Proof. (i) The proof is similar to that of Lemma 2.3.

(ii) Suppose that $W \cap V(H_i) = \emptyset$ for some i, $1 \leq i \leq m$. Let $x, y \in V(H_i)$. Then by (i), $d_{G[H_i]_1^m}(w, x) = d_{G[H_i]_1^m}(w, y)$ for every vertex $w \in V(G[H_i]_1^m) \setminus V(H_i)$; in particular for every $w \in W$, a contradiction.

Let $W_i = W \cap V(H_i)$. For $x \in W_i$ or $y \in W_i$, the result is straightforward. Suppose $x, y \in V(H_i) \setminus W_i$. Since W is a resolving set for $G[H_i]_1^m$,

$d_{G[H_i]_1^m}(w,x) \neq d_{G[H_i]_1^m}(w,y)$ for some $w \in W$. But by (i), $d_{G[H_i]_1^m}(z,x) = d_{G[H_i]_1^m}(z,y)$ for all $z \in V(G[H_i]_1^m) \setminus V(H_i)$. Hence the vertex $w \in W$ such that $d_{G[H_i]_1^m}(w,x) \neq d_{G[H_i]_1^m}(w,y)$, must belong to W_i.

(iii) Suppose that $W \cap V(G) \neq \emptyset$. We claim that $W' = W \setminus V(G)$ is a resolving set for $G[H_i]_1^m$. Since $|W'| < |W|$ and W is a metric basis for $G[H_i]_1^m$, this would result in a contradiction.

Let x, y be vertices of $G[H_i]_1^m$. Then the following cases arise.

Case 1: $x, y \in V(H_i)$. By (ii), there exists $w \in W_i = W \cap V(H_i) = W' \cap V(H_i)$ such that $d_{G[H_i]_1^m}(w,x) \neq d_{G[H_i]_1^m}(w,y)$.

Case 2: $x \in V(H_i)$ and $y \in V(H_j), i \neq j$. Let e_i and e_j be the edges corresponding to the graphs H_i and H_j in $G[H_i]_1^m$ respectively. Then we have the following two subcases.

Case 2.1: e_i and e_j are not adjacent. In this case each $w \in W' \cap V(H_i)$ satisfies $d_{G[H_i]_1^m}(w,x) \leq 2 < 3 \leq d_{G[H_i]_1^m}(w,y)$.

Case 2.2: e_i and e_j are adjacent. Let $x \in V(H_i)$ and $y \in V(H_j)$. For every $w \in V(H_i)$, it is clear that $d_{G[H_i]_1^m}(w,x) \leq 2$ and $d_{G[H_i]_1^m}(w,y) = 2$. If there exists a vertex $w \in W' \cap V(H_i)$ for which $d_{G[H_i]_1^m}(w,x) \neq d_{G[H_i]_1^m}(w,y)$, then w resolves the vertices x and y. Suppose that for every $w \in W' \cap V(H_i)$ we have $d_{G[H_i]_1^m}(w,x) = d_{G[H_i]_1^m}(w,y) = 2$. Then no vertex of W' resolves the vertices x and y, contradicting the fact that W' is a resolving set of $G[H_i]_1^m$. Therefore there exists a vertex $w \in W' \cap V(H_i)$ such that $d_{G[H_i]_1^m}(w,x) \neq d_{G[H_i]_1^m}(w,y)$. Hence the vertices x and y are resolved by a vertex of W'.

Case 3: Let $x, y \in V(G)$. Then we have the following subcases.

Case 3.1: $xy \notin E(G)$. Without loss of generality, let H_i be the graph corresponding to the edge $e_i = xz$ of G. Let $w \in V(H_i) \cap W'$. Then $d_{G[H_i]_1^m}(w,x) = 1 < d_{G[H_i]_1^m}(w,y)$.

Case 3.2: $xy \in E(G)$. Let H_i be the graph corresponding to the edge $e_i = xy$ of G. In this case, for every $w \in V(H_i) \cap W'$, $d_{G[H_i]_1^m}(w,x) = d_{G[H_i]_1^m}(w,y)$. Hence there exists some $w_j \in V(H_j) \cap W'$, $i \neq j$, $d_{G[H_i]_1^m}(w_j,x) \neq d_{G[H_i]_1^m}(w_j,y)$.

Case 4: Let $x \in V(H_i)$ and $y \in V(G)$. Then we have the following subcases.

Case 4.1: $xy \in E(G[H_i]_1^m)$.

Case 4.1.1: $deg(y) = 1$ with $e_i = yz$ and $deg(z) > 1$.

This is possible since order of G is at least three. In this case z cannot resolve x and y. Neither y nor z can resolve any two vertices of H_i. Hence to resolve x and y, x must be included in any resolving set.

Case 4.1.2: $deg(y) \neq 1$.

In this case, for some $j \neq i$, $w \in V(H_j) \cap W'$, $d_{G[H_i]_1^m}(w,x) = d_{G[H_i]_1^m}(w,y) + d_{G[H_i]_1^m}(y,x) = d_{G[H_i]_1^m}(w,y) + 1$.

Case 4.2: $xy \notin E(G[H_i]_1^m)$. Without loss of generality, let H_j be the graph corresponding to the edge $e_j = yz$ of G in $G[H_i]_1^m$. Clearly, for every $w \in V(H_j) \cap W'$, $d_{G[H_i]_1^m}(w,y) = 1$ and $d_{G[H_i]_1^m}(w,x) = d_{G[H_i]_1^m}(w,y) + d_{G[H_i]_1^m}(y,x) = 1 + d_{G[H_i]_1^m}(y,x)$. □

We obtain a lower bound for the metric dimension of $G[H_i]_1^m$.

Theorem 2.4. *Let G be a connected graph of size m. Let $H_1, H_2 \dots H_m$ be m simple graphs of orders at least two. Then*

(i): *If G has no pendent edges, $dim(G[H_i]_1^m) \geq \sum_{i=1}^{m} dim(H_i)$.*

(ii): *If G has a pendent edge say $e_t = u_t v_t$ such that $deg(v_t) = 1$ and H_t the graph corresponding to e_t, $dim(G[H_i]_1^m) \geq \sum_{i=1, i\neq t}^{m} dim(H_i) + dim(H_t + K_1)$.*

Proof. (i) Let W be a metric basis for $G[H_i]_1^m$. By Theorem 2.3(iii), $W \cap V(G) = \emptyset$ and by Theorem 2.3(ii), $W_i = W \cap V(H_i)$ is a nonempty resolving set for H_i, $1 \leq i \leq m$. Clearly $W = \bigcup_{i=1}^{m} W_i$. Therefore, $dim(G[H_i]_1^m) \geq \sum_{i=1}^{m} dim(H_i)$.

(ii) Let $x, y \in V(H_t)$. Then vertices of $\{x, y, v_t\}$ taken two at a time are equidistant from every $z \in V(G[H_i]_1^m) \setminus V(H_t)$, arguing as in Lemma 2.2. Hence it is necessary to consider a resolving set of $H_t + K_1$. This proves our claim. □

As a consequence of Theorem 2.4, we obtain the following result.

Theorem 2.5. *Let G be a connected graph of size m. Let e_i, $1 \leq i \leq m_1$ and e_j, $1 \leq j \leq m_2$, $m_1 + m_2 = m$, be the non-pendant and pendant edges of G respectively. Let $H_1, H_2 \dots H_m$ be m simple graphs of order at least two. Then $dim(G[H_i]_1^m) \geq \sum_{i=1}^{m_1} dim(H_i) + \sum_{j=1}^{m_2} dim(H_j + K_1)$.*

We now proceed to obtain an upper bound for the metric dimension of $G[H_i]_1^m$.

Theorem 2.6. *Let G be a connected graph of size m. Let e_i, $1 \leq i \leq m_1$ and e_j, $1 \leq j \leq m_2$, $m_1 + m_2 = m$, be the non-pendant and pendant edges of G respectively. Let $H_1, H_2 \dots H_m$ be m simple graphs of order at least two. Then $dim(G[H_i]_1^m) \leq \sum_{i=1}^{m_1} dim(H_i + K_2) + \sum_{j=1}^{m_2} dim(H_j + K_1)$.*

Proof. Let W be a resolving set for $G[H_i]_1^m$. By Theorem 2.3(ii), $W_k = V(H_k) \cap W$, $1 \leq k \leq m$ is a resolving set for H_k. By Theorem 2.3(iii), $V(G) \cap W = \emptyset$. Let $e_i = uv$ be a non-pendent edge of G and H_i be the graph corresponding to e_i. Then $dim(G[H_i]_1^{m_1}) \leq \sum_{i=1}^{m_1} dim(H_i + K_2)$.

Let $e_j = uv$ be a pendent edge of G such that $deg(v) = 1$. Let H_j be the graph corresponding to e_j. Then by Lemma 2.2, for any two vertices $x, y \in V(H_j)$ and a vertex $z \in V(G[H_i]_1^m) \setminus \{u, v\}$, $d_{G[H_i]_1^m}(z, x) = d_{G[H_i]_1^m}(z, y) = d_{G[H_i]_1^m}(z, v)$. Hence $dim(G[H_j]_1^{m_2}) \leq \sum_{j=1}^{m_2} dim(H_j + K_1)$. This completes the proof. □

2.2. **Strong metric dimension of $G[H_i]_1^m$.** The following Lemmas are useful in the sequel.

Lemma 2.7. [17] *For every pair of mutually maximally distant vertices x, y of a connected graph G and for every strong resolving set S of G, it follows that $x \in S$ or $y \in S$.*

Lemma 2.8. [17] *Let H be a connected graph of order $n \geq 2$, and let $\overline{\omega}(H)$ denote the twin-free clique number of H. Then $dim_s(H) \leq n - \overline{\omega}(H)$. Moreover, if H has diameter two, then $dim_s(H) = n - \overline{\omega}(H)$.*

Lemma 2.9. *Let G be a graph with m edges. Let $H_1, H_2 \dots H_m$ be m simple graphs of order at least two. For any two vertices $u, v \in V(G)$, if $d_G(u,w) < d_G(u,v)$ for all $w \in N(v)$ then v is maximally distant from u in $G[H_i]_1^m$.*

Proof. Let $d_G(u,v) = l$. Since $d_G(u,w) < d_G(u,v)$, $d_G(u,w) = l-1$ for all $w \in N(v)$ in G. Let H_j be the graph corresponding to the edge $e_j = vw_j$ in G. Let $v_{j_1}, v_{j_2} \dots v_{j_r}$ be the vertices of H_j. Since $w_jv \in E(G)$, $w_jv_{j_r} \in E(G[H_i]_1^m)$ for all r and $vv_{j_r} \in E(G[H_i]_1^m)$ for all r. This implies that $d_{G[H_i]_1^m}(u, v_{j_r}) = d_{G[H_i]_1^m}(u, w_j) + d_{G[H_i]_1^m}(w_j, v_{j_r}) = l - 1 + 1 = l = d_{G[H_i]_1^m}(u, v)$. Hence v is maximally distant from u.

□

Theorem 2.10. *Let G be any graph with m edges and H_i be the graphs of order at least two. Then a pair of vertices $x \in V(H_i)$ and $y \in V(H_j)$, $i \neq j$ are mutually maximally distant in $G[H_i]_1^m$.*

Proof. Let $e_i = u_iv_i$ and $e_j = u_jv_j$, $i \neq j$ be two edges of G. Let H_i and H_j be the graphs corresponding to e_i and e_j of G respectively. Let $x \in V(H_i)$ and $y \in V(H_j)$, $i \neq j$, $1 \leq i, j \leq m$. We show that y is maximally distant from x. Consider a shortest path from u_i to u_j. Let $d_{G[H_i]_1^m}(u_i, u_j) = l$. Without loss of generality, let the path be $u_i = p_0p_1p_2 \dots p_{l-1}p_l = u_j$. We have the following cases.

Case 1: $p_{l-1}v_j \notin E(G)$

In this case a shortest path from x to y passes through only $u_j = p_l$. Clearly $d_{G[H_i]_1^m}(x,y) = d_{G[H_i]_1^m}(x,u_i) + d_{G[H_i]_1^m}(u_i,u_j) + d_{G[H_i]_1^m}(u_j,y) = 1+l+1 = l+2$ and for all $w \in N(y)$, $d_{G[H_i]_1^m}(x,w) = d_{G[H_i]_1^m}(x,u_i) + d_{G[H_i]_1^m}(u_i,u_j) + d_{G[H_i]_1^m}(u_j,w) = 1+l+1 = l+2$. Hence y is maximally distant from x.

Case 2: $p_{l-1}v_j \in E(G)$

In this case there are shortest paths from x to y one passing through u_j and another passing through v_j. It is clear that $d_{G[H_i]_1^m}(x,y) = d_{G[H_i]_1^m}(x,u_i) + d_{G[H_i]_1^m}(u_i,u_j) + d_{G[H_i]_1^m}(u_j,y) = 1+l+1 = l+2$ and for all $w \in N(y) \setminus \{v_j\}$, $d_{G[H_i]_1^m}(x,w) = d_{G[H_i]_1^m}(x,u_i) + d_{G[H_i]_1^m}(u_i,u_j) + d_{G[H_i]_1^m}(u_j,w) = 1+l+1 = l+2$ and $d_{G[H_i]_1^m}(x,v_j) = d_{G[H_i]_1^m}(x,u_i) + d_{G[H_i]_1^m}(u_i,p_{l-1}) + d_{G[H_i]_1^m}(p_{l-1},v_j) = 1+l-1+1 = l+1$. Hence y is maximally distant from x. □

Theorem 2.11. *Let G be a simple graph with m edges. Let $H_1, H_2 \dots H_m$ be m simple graphs of order at least two. Then $dim_s(G[H_i]_1^m) \geq \sum_{i=1}^{m} |V(H_i)| + 2 - \overline{\omega}(H_k + K_2)$ where H_k is the graph for which $V(H_k)$ is not fully contained in any strong resolving set S.*

Proof. Let $x \in V(H_i)$ and $y \in V(H_j)$, $i \neq j$. Then by Theorem 2.10, x and y are mutually maximally distant in $G[H_i]_1^m$. By Lemma 2.7, S must fully contain either $V(H_i)$ or $V(H_j)$. Extending to m sets, S must fully contain at least $m-1$ of the sets $V(H_i)$. If $V(H_k)$ is the set that is not fully contained in any strong

resolving set S, then $|S| \geq \sum_{i \neq k} |V(H_i)| + dim_s(H_k + K_2)$. Since the diameter of $H_k + K_2$ is two, by Lemma 2.8, $dim_s(H_k + K_2) = |V(H_k + K_2)| + 2 - \overline{\omega}(H_k + K_2)$. Consequently, $|S| \geq \sum_{i=1}^{m} |V(H_i)| + 2 - \overline{\omega}(H_k + K_2)$. This completes the proof. □

Theorem 2.12. *Let G be a simple graph with m edges. For any fixed vertex $u \in V(G)$, let $S_v = \{v \in V(G) : d_G(u, w) < d_G(u, v), w \in N(v)\}$. Let $H_1, H_2 \ldots H_m$ be m simple graphs of order at least two. For any strong resolving set S, H_k is the graph such that $V(H_k)$ is not fully contained in S. Then $dim_s(G[H_i]_1^m) \geq \sum_{i=1}^{m} |V(H_i)| + 2 - \overline{\omega}(H_k + K_2) + |S_v|$.*

Proof. It follows directly from Lemma 2.9 and Theorem 2.11. □

3. Conclusion

In this paper, we have investigated the metric dimension and strong metric dimension problems for the generalized edge corona product of graphs. By expanding the study of these graph invariants to this graph operation, we have established bounds, providing new insights into their structural properties. The fault-tolerant versions of these parameters for the generalized edge corona product are currently under investigation.

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References

1. Anand, B. S., Dayap, J. A., Casinillo, L. F., Pepper, R., & Nair, R. S. (2025). On distance k-domination number of graphs under product operations. Gulf Journal of Mathematics, 19(1), 117-124. https://doi.org/10.56947/gjom.v19i1.2032
2. Beerliova, Z., Eberhard, F., Erlebach, T., Hall, A., Hoffmann, M., Mihalak, M., & Ram, L. (2006). Network discovery and verification. IEEE Journal on Selected Areas in Communications, 24, 2168–2181. https://doi.org/10.1109/JSAC.2006.884015
3. Caay, M., Maza, R., Duarte, R., Lana, S. R., Soriano, B., & Janer, A. (2025). Equitable isolate domination in graphs. Gulf Journal of Mathematics, 19(1), 337-346. https://doi.org/10.56947/gjom.v19i1.2603
4. Chartrand, G., Eroh, L., Johnson, M. A., & Oellermann, O. R. (2000). Resolvability in graphs and the metric dimension of a graph. Discrete Applied Mathematics, 105(1-3), 99-113. https://doi.org/10.1016/S0166-218X(00)00198-0
5. Chvátal, V. (1983). Mastermind. Combinatorica, 3(3), 325-329. https://doi.org/10.1007/BF02579188
6. Cui, C., Chen, J., & Lin, S. (2025). Metric and strong metric dimension in TI-power graphs of finite groups. AIMS Math., 10, 705-720. https://doi.org/10.3934/math.2025032.
7. Garey, M. R., & Johnson, D. S. (1979). Computers and intractability: A guide to the theory of NP-completeness. Freeman, New York.
8. Grigorious, C., Manuel, P., Miller, M., Rajan, B., & Stephen, S. (2014). On the metric dimension of circulant and Harary graphs. Applied Mathematics and Computation, 248, 47-54. https://doi.org/10.1016/j.amc.2014.09.045

9. Harary, F. (2018). Graph theory (on Demand Printing of 02787). CRC Press.
10. Harary, F., & Melter, R. A. (1976). On the metric dimension of a graph. Ars combin, 2(191-195), 1.
11. Hernando, C., Mora, M., Pelayo, I. M., Seara, C., Cáceres, J., & Puertas, M. L. (2005). On the metric dimension of some families of graphs. Electronic notes in discrete mathematics, 22, 129-133. https://doi.org/10.1016/j.endm.2005.06.023
12. Imran, M., Baig, A. Q., Shafiq, M. K., & Tomecu, L. (2014). On metric dimension of generalized Petersen graphs P (n, 3). Ars Combinatoria, 117, 113-130.
13. Khuller, S., Raghavachari, B., & Rosenfeld, A. (1996). Landmarks in graphs. Discrete applied mathematics, 70(3), 217-229. https://doi.org/10.1016/0166-218X(95)00106-2
14. Klein, D. J., & Yi, E. (2012). A comparison on metric dimension of graphs, line graphs, and line graphs of the subdivision graphs. European journal of pure and applied mathematics, 5(3), 302-316.
15. Kratica, J., Kovačević-Vujčić, V., Čangalović, M., & Stojanović, M. (2012). Minimal doubly resolving sets and the strong metric dimension of some convex polytopes. Applied Mathematics and Computation, 218(19), 9790-9801. https://doi.org/10.1016/j.amc.2012.03.047
16. Krishnan, S., Rajan, B., & Imran, M. (2017). On the strong metric Dimension of Certain Nanostructures. Journal of Computational and Theoretical Nanoscience, 14(1), 354-358.
17. Kuziak, D., Yero, I. G., & Rodríguez-Velázquez, J. A. (2013). On the strong metric dimension of corona product graphs and join graphs. Discrete Applied Mathematics, 161(7-8), 1022-1027. https://doi.org/10.1016/j.dam.2012.10.009
18. Luo, Y., & Yan, W. (2018). Spectra of the generalized edge corona of graphs. Discrete Mathematics, Algorithms and Applications, 10(01), 1850002. https://doi.org/10.1142/S1793830918500027
19. Manuel, P., Bharati, R., Rajasingh, I., & Chris Monica, M. (2008). On minimum metric dimension of honeycomb networks. Journal of Discrete algorithms, 6(1), 20-27. https://doi.org/10.1016/j.jda.2006.09.002
20. Oellermann, O. R., & Peters-Fransen, J. (2007). The strong metric dimension of graphs and digraphs. Discrete Applied Mathematics, 155(3), 356-364. https://doi.org/10.1016/j.dam.2006.06.009
21. Prabhu, S., Janany, T. J., & Klavžar, S. (2025). Metric dimensions of generalized Sierpiński graphs over squares. Applied Mathematics and Computation, 505, 129528. https://doi.org/10.1016/j.amc.2025.129528
22. Prabhu, S., Sagaya Rani Jeba, D., Arulperumjothi, M., & Klavžar, S. (2024). Metric dimension of irregular convex triangular networks. AKCE international journal of graphs and combinatorics, 21(3), 225-231. https://doi.org/10.1080/09728600.2023.2280799
23. Sathish, K., & Rajan, B. (2015). On strong metric dimension of certain triangular tessellations. International Journal of Pure and Applied Mathematics, 101(5), 637-645.
24. Sebő, A., & Tannier, E. (2004). On metric generators of graphs. Mathematics of Operations Research, 29(2), 383-393. https://doi.org/10.1287/moor.1030.0070
25. Shahriyari, R., Nikandish, R., Tehranian, A., & Rasouli, H. (2024). Computing the strong metric dimension for co-maximal ideal graphs of commutative rings. Journal of Algebra and Its Applications, 23(03), 2450048.
26. Slater, P. J. (1975). Leaves of trees. Congr. Numer, 14(549-559), 37.
27. Tillquist, R. C., Frongillo, R. M., & Lladser, M. E. (2023). Getting the lay of the land in discrete space: A survey of metric dimension and its applications. SIAM Review, 65(4), 919-962. https://doi.org/10.1137/21M1409512
28. Yi, E. (2013). On strong metric dimension of graphs and their complements. Acta Mathematica Sinica, English Series, 29(8), 1479-1492. https://doi.org/10.1007/s10114-013-2365-z

29. Zhai, L., Ma, X., Shao, Y., & Zhong, G. (2022). Metric and strong metric dimension in commuting graphs of finite groups. Communications in Algebra, 51(3), 1000-1010. https://doi.org/10.1080/00927872.2022.2118761

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