

ON CORE-DOMINATED CLOSED SETS IN NEUTROSOPHIC TOPOLOGY

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ABSTRACT. Neutrosophic topology has introduced various generalized closed sets to model uncertainty; however, many existing approaches fail to ensure the dominance of true membership over indeterminacy and falsity. To address this limitation, we propose a new class called *core-dominated neutrosophic closed sets*, which combine a strict dominance condition with structural constraints based on closure and interior operators. We investigate their fundamental properties and relationships with existing closed sets, and introduce the notions of CDN-continuity, CDN-closed maps, and CDN-homeomorphisms. It is shown that these concepts extend classical ones, while the converses do not generally hold, as demonstrated by counterexamples.

Keywords. Core-dominated frameworks, neutrosophic continuous maps, CDN-continuous maps, CDN-homeomorphisms

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1. INTRODUCTION

Fuzzy sets were first proposed by Zadeh [1] in 1965 as a way to deal with ambiguous, imprecise, and uncertain data. Afterwards, in 1986, Atanassov [2] developed this idea by putting forth intuitionistic fuzzy sets, which were later widely acknowledged as a vital instrument in engineering, mathematics [3], and healthcare [4]. By creating the concept of intuitionistic fuzzy topological spaces utilizing intuitionistic fuzzy sets, Coker [5] improved this topic in 1997. Improving on this, intuitionistic fuzzy λ -closed sets were presented in 2011 [6].

In this paper, we investigate this idea in the context of neutrosophic topological spaces (NTS, for abbreviation) in order to describe complex real-world information. Smarandache [7, 8] developed the idea of neutrosophic sets (NSS, for abbreviation), which are defined by truth, uncertainty, and falsity values. Researchers have made substantial contributions to N-theory over time, resulting in the creation of N-graphs [9], N-groups [10], N-rings [11], and N-vector spaces [12].

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With scholars constantly investigating their various uses, the research of neutrosophic sets is still developing. The establishment of N-logic with its utilization was aided by [13, 14]. Also see related works [15, 16]

Salama and Alblawi [17] introduced the notions of neutrosophic closed (NC) and neutrosophic open (NO) sets in 2012 when they developed NTSs via neutrosophic sets. Later, continuous and closed maps in NTSs were studied by Salama et al. [18] in 2014. Additionally, in 2016 [19] proposed several features of N- ω -CS. The notion of N-semi-OS and N-pre-OS in NTSs was first presented in 2017 [20]. Since then, a number of scholars have suggested and investigated different kinds of open and closed sets in NTSs.

Since the significance of neutrosophic sets is quickly expanding across many applications, the investigation of weaker and stronger kinds of both closed and open sets is essential to neutrosophic topology (NT). As a result, it continues to be a major area of study for investigators worldwide. Both Dhavaseelan and Jafari [21] suggested and studied the features of N-g-CS in 2018. Jayanthi [22] introduced and examined N- α -g-CS in the same year. In 2020, Ramesh [23] investigated N-g- α -closed maps, while [24] introduced N-g- α -closed sets. Rajeshwaran and Chandramathi [25] established N-g-semi-pre-CS in 2021, and in 2024 they expanded on their studies through the creation of N-g-semi-pre-continuous maps. Moreover, in the same year [26] studied mappings in N-p δ s-irresolute with N- α δ s-irresolute topological spaces. Finally, as a variant of closed and semi-closed sets within NTSs, the notion of closed λ -type sets was presented [27].

In this work, we propose a new class of neutrosophic closed sets, referred to as core-controlled neutrosophic sets (CDN-closed sets), which build upon existing concepts by introducing a dominance constraint on the membership values. This constraint ensures that the membership value dominates over uncertainty and falsity, thereby offering a more accurate representation of real-world situations where uncertainty plays a central role. We explore the fundamental characteristics of CDN-closed sets and their relationships with other neutrosophic closed sets, such as λ -closed sets, providing deeper insights into their application in neutrosophic topology.

Furthermore, this paper introduces and investigates the concepts of CDN-continuous maps, CDN-closed maps, and CDN-homeomorphisms within the framework of core-controlled neutrosophic sets. We establish key results regarding the composition, evaluation, and invariance properties of these mappings. The study also reveals that while CDN-continuity and CDN-homeomorphisms imply corresponding concepts, the converse does not always hold, which is demonstrated through counterexamples.

The rest of the paper is organized as follows: Section 2 presents the preliminary concepts and fundamental definitions needed for the study. Section 3 introduces the new class of core-controlled neutrosophic sets (CDN-closed sets), exploring their basic properties and their relation to λ -closed sets. Section 4 defines CDN-continuous maps, CDN-closed maps, and CDN-homeomorphisms, highlighting their key characteristics and relationships with N λ -continuous maps, N λ -closed maps, and N λ -homeomorphisms. Finally, Section 5 concludes the paper by summarizing the main findings and suggesting potential avenues for future research.

2. PRELIMINARIES

In this part, we give some definitions needed for our work.

Definition 2.1 (Arokiarani et al. [20]). Let $\mathcal{P} \neq \emptyset$. A neutrosophic point in $\mathcal{P}$, denoted by $\text{NSp}_{(e,j,c)}$, is defined as follows:

$$a_{(e,j,c)}(a) = \begin{cases} (e, j, c), & \text{if } a = a_j, \\ (0, 0, 1), & \text{if } a \neq a_j. \end{cases}$$

The membership, indeterminacy, and non-membership values of $a_{(e,j,c)}$ are represented by $e, j, c \in [0, 1]$, respectively, with

$$0 < e + j + c \leq 3,$$

where $a_j \in \mathcal{P}$ is the support of $a_{(e,j,c)}$.

Definition 2.2. Every neutrosophic set $\mathcal{K}$ in a neutrosophic topological space $(\mathcal{P}, \mathcal{S})$ is called:

- (1) Ng-closed set (Ng-CS) [21] if

$$\text{N-cl}(\mathcal{K}) \subseteq \mathcal{M}$$

whenever $\mathcal{K} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a neutrosophic open set (NOS).

- (2) Ng-semi-pre-closed set (Ng-SP-CS) [25] if

$$\text{N-spcl}(\mathcal{K}) \subseteq \mathcal{M}$$

whenever $\mathcal{K} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a neutrosophic open set (NOS).

- (3) $\text{N}\omega$ -closed set ($\text{N}\omega$ -CS) [19] if

$$\text{N-cl}(\mathcal{K}) \subseteq \mathcal{M}$$

whenever $\mathcal{K} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a neutrosophic semi-open set.

- (4) $\text{N-}\lambda$ -closed set ($\text{N-}\lambda$ -CS) [26] if

$$\text{N-cl}(\mathcal{M}) \subseteq \mathcal{K}$$

whenever $\mathcal{K} \subseteq \mathcal{M}$ for every neutrosophic open set $\mathcal{M}$.

Definition 2.3 (Atanassov [2]). Assume that $h : (\mathcal{P}, \mathcal{S}) \rightarrow (\mathcal{O}, \mathcal{T})$ is a mapping.

- (1) Let

$$\mathcal{K} = \{\langle p, J_{\mathcal{K}}(p), H_{\mathcal{K}}(p), B_{\mathcal{K}}(p) \rangle : p \in \mathcal{P}\}$$

be a neutrosophic set in $(\mathcal{P}, \mathcal{S})$. Then $h(\mathcal{K})$ is a neutrosophic set in $(\mathcal{O}, \mathcal{T})$ defined by:

$$J_{h(\mathcal{K})}(o) = \begin{cases} \sup_{p \in h^{-1}(o)} J_{\mathcal{K}}(p), & \text{if } h^{-1}(o) \neq \emptyset, \\ 0, & \text{if } h^{-1}(o) = \emptyset, \end{cases}$$

$$H_{h(\mathcal{K})}(o) = \begin{cases} \sup_{p \in h^{-1}(o)} H_{\mathcal{K}}(p), & \text{if } h^{-1}(o) \neq \emptyset, \\ 0, & \text{if } h^{-1}(o) = \emptyset, \end{cases}$$

$$B_{h(\mathcal{K})}(o) = \begin{cases} \inf_{p \in h^{-1}(o)} B_{\mathcal{K}}(p), & \text{if } h^{-1}(o) \neq \emptyset, \\ 0, & \text{if } h^{-1}(o) = \emptyset, \end{cases}$$

for all $o \in \mathcal{O}$, where

$$h^{-1}(o) = \{p \in \mathcal{P} : h(p) = o\}.$$

(2) Let

$$\mathcal{H} = \{\langle o, J_{\mathcal{H}}(o), H_{\mathcal{H}}(o), B_{\mathcal{H}}(o) \rangle : o \in \mathcal{O}\}$$

be a neutrosophic set in $(\mathcal{O}, \mathcal{T})$. Then the inverse image $h^{-1}(\mathcal{H})$ is a neutrosophic set in $(\mathcal{P}, \mathcal{S})$ defined by:

$$J_{h^{-1}(\mathcal{H})}(p) = J_{\mathcal{H}}(h(p)), \quad \forall p \in \mathcal{P},$$

$$H_{h^{-1}(\mathcal{H})}(p) = H_{\mathcal{H}}(h(p)), \quad \forall p \in \mathcal{P},$$

$$B_{h^{-1}(\mathcal{H})}(p) = B_{\mathcal{H}}(h(p)), \quad \forall p \in \mathcal{P}.$$

Definition 2.4. Let $h : \mathcal{P} \rightarrow \mathcal{O}$ be a mapping. Then h is called:

(1) N-continuous (N-Cont) [18] if

$$h^{-1}(\mathcal{K}) \text{ is a NCS in } \mathcal{P}, \quad \forall \text{ NCS } \mathcal{K} \text{ in } \mathcal{O}.$$

(2) N-g-continuous (Ng-Cont) [21] if

$$h^{-1}(\mathcal{K}) \text{ is a Ng-CS in } \mathcal{P}, \quad \forall \text{ NCS } \mathcal{K} \text{ in } \mathcal{O}.$$

(3) N-g-semi-pre-continuous (Ng-SP-Cont) [25] if

$$h^{-1}(\mathcal{K}) \text{ is a Ng-SP-CS in } \mathcal{P}, \quad \forall \text{ NCS } \mathcal{K} \text{ in } \mathcal{O}.$$

(4) N- λ -continuous (N- λ -Cont) [26] if

$$h^{-1}(\mathcal{K}) \text{ is a N-}\lambda\text{-CS in } \mathcal{P}, \quad \forall \text{ NCS } \mathcal{K} \text{ in } \mathcal{O}.$$

Definition 2.5. Let $h : \mathcal{P} \rightarrow \mathcal{O}$ be a mapping. Then h is called:

(1) N-closed (N-C) (resp., N-open (N-O)) map [20] if

$$h(\mathcal{K}) \text{ is a NCS (resp., NOS) in } \mathcal{O}, \quad \forall \text{ NCS (resp., NOS) } \mathcal{K} \text{ in } \mathcal{P}.$$

(2) N-semi-closed map (N-S-C) [20] if

$$h(\mathcal{K}) \text{ is a N-semi-CS in } \mathcal{O}, \quad \forall \text{ N-semi-closed set } \mathcal{K} \text{ in } \mathcal{P}.$$

(3) N-g-closed map (N-g-C) [23] if

$$h(\mathcal{K}) \text{ is a Ng-CS in } \mathcal{O}, \quad \forall \text{ NCS } \mathcal{K} \text{ in } \mathcal{P}.$$

(4) N- λ -closed map (N- λ -C) [26] if

$$h(\mathcal{K}) \text{ is a N-}\lambda\text{-CS in } \mathcal{O}, \quad \forall \text{ NCS } \mathcal{K} \text{ in } \mathcal{P}.$$

3. NEW CLASS OF NEUTROSOPHIC SETS

In this section, we present a new and more powerful type of neutrosophic sets, known as core-controlled neutrosophic sets, which extend existing concepts by introducing a semantic dominance condition on membership values. We also examine their behavior under set-theoretic operations as well as their relationship with N - λ -closed sets [26].

Definition 3.1. Let $(\mathcal{P}, \mathcal{S})$ be a neutrosophic topological space (NTS) and let $\mathcal{K} \subseteq \mathcal{P}$. Then a set $\mathcal{K}$ is said to be a core-controlled neutrosophic set (CDN-CS) if it satisfies:

$$N\text{-cl}(\mathcal{K}) \subseteq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{K}))),$$

and for all $p \in \mathcal{P}$,

$$J_{\mathcal{K}}(p) > H_{\mathcal{K}}(p) + B_{\mathcal{K}}(p).$$

Remark 3.2. The above condition imposes a reverse-stability constraint ensuring that the iterated operator

$$N\text{-cl}(N\text{-int}(N\text{-cl}(\cdot)))$$

does not exceed the set K . This property is essential for deriving valid inclusion results in subsequent theorems.

Example 3.3. Let $\mathcal{P} = \{p_1, p_2\}$ and $\mathcal{S} = \{\tilde{0}, \tilde{1}, \mathcal{K}\}$ be a neutrosophic topology on $\mathcal{P}$, where

$$\mathcal{K} = \{\langle p_1, 0.7, 0.3, 0.3 \rangle, \langle p_2, 0.6, 0.4, 0.3 \rangle\}.$$

Let $\mathcal{H}$ be a neutrosophic set defined by

$$\mathcal{H} = \{\langle p_1, 0.9, 0.4, 0.4 \rangle, \langle p_2, 0.7, 0.2, 0.2 \rangle\}.$$

Then the dominance condition holds since

$$J_{\mathcal{H}}(p_1) = 0.9 > 0.4 + 0.4 = 0.8, \quad J_{\mathcal{H}}(p_2) = 0.7 > 0.2 + 0.2 = 0.4.$$

Moreover,

$$N\text{-cl}(\mathcal{H}) = \tilde{1} \subseteq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{H}))) = N\text{-cl}(\tilde{1}) = \tilde{1}.$$

Therefore, $\mathcal{H}$ is a core-controlled neutrosophic closed set (CDN-CS) in $(\mathcal{P}, \mathcal{S})$.

Theorem 3.4 (Characterization of CDN-CS). *Let $(\mathcal{P}, \mathcal{S})$ be a neutrosophic topological space and let $\mathcal{K} \subseteq \mathcal{P}$. Then $\mathcal{K}$ is a core-dominated neutrosophic closed set (CDN-CS) if and only if the following two conditions hold:*

- (i) $N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{K}))) \subseteq \mathcal{K}$,
- (ii) $J_{\mathcal{K}}(p) > H_{\mathcal{K}}(p) + B_{\mathcal{K}}(p), \quad \forall p \in \mathcal{P}$.

Proof. ($\Rightarrow$) Assume that $\mathcal{K}$ is a CDN-CS. By Definition 3.1, a CDN-CS is precisely a neutrosophic set satisfying

$$N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{K}))) \subseteq \mathcal{K}$$

and

$$J_{\mathcal{K}}(p) > H_{\mathcal{K}}(p) + B_{\mathcal{K}}(p), \quad \forall p \in \mathcal{P}.$$

Hence, conditions (i) and (ii) hold.

($\Leftarrow$) Conversely, suppose that $\mathcal{K}$ satisfies conditions (i) and (ii). Then, by Definition 3.1, these two conditions are exactly the defining properties of a CDN-CS. Therefore, $\mathcal{K}$ is a CDN-CS. $\square$

As demonstrated in the following instance, the opposite of the aforementioned theorem may not be valid.

Example 3.5. Let $\mathcal{P} = \{p_1, p_2\}$ and $\mathcal{S} = \{\emptyset_N, \mathcal{P}_N, \mathcal{K}\}$ be a neutrosophic topology on $\mathcal{P}$, where

$$\mathcal{K} = \{\langle p_1, 0.6, 0.4, 0.3 \rangle, \langle p_2, 0.7, 0.4, 0.6 \rangle\}.$$

Consider the neutrosophic set

$$\mathcal{H} = \{\langle p_1, 0.7, 0.5, 0.4 \rangle, \langle p_2, 0.6, 0.3, 0.4 \rangle\}.$$

Then $\mathcal{H}$ represents an N- λ -closed set, but it is not a CDN-CS, since

$$H_{\mathcal{H}}(p_1) = 0.7 \not\geq 0.9 \quad \text{and} \quad H_{\mathcal{H}}(p_2) = 0.6 \not\geq 0.7.$$

Lemma 3.6. Let Y be a neutrosophic open set and $A \subseteq \mathcal{P}$. If $Y \subseteq \text{N-cl}(A)$, then $Y \subseteq \text{N-int}(\text{N-cl}(A))$.

Proof. Since $\text{N-int}(\text{N-cl}(A))$ is the largest neutrosophic open subset of $\text{N-cl}(A)$, and Y is open with $Y \subseteq \text{N-cl}(A)$, it follows that

$$Y \subseteq \text{N-int}(\text{N-cl}(A)).$$

$\square$

Theorem 3.7. Let $(\mathcal{P}, \mathcal{S})$ be a neutrosophic topological space and let $\mathcal{K} \subseteq \mathcal{P}$. Assume that

$$\text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K}))) = \mathcal{K}.$$

Then $\mathcal{K}$ is an N- λ -closed set.

Proof. Let Y be a neutrosophic open set such that $Y \subseteq \mathcal{K}$. By monotonicity of the closure operator, we have

$$\text{N-cl}(Y) \subseteq \text{N-cl}(\mathcal{K}).$$

Since $Y \subseteq \mathcal{K}$ and $\mathcal{K} = \text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K})))$, we obtain

$$Y \subseteq \text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K}))).$$

Because Y is neutrosophic open, applying Lemma 3.6 yields

$$Y \subseteq \text{N-int}(\text{N-cl}(\mathcal{K})).$$

Taking neutrosophic closures on both sides, we get

$$\text{N-cl}(Y) \subseteq \text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K}))).$$

Using the assumption, we have

$$\text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K}))) = \mathcal{K},$$

hence

$$\text{N-cl}(Y) \subseteq \mathcal{K}.$$

Therefore, $\mathcal{K}$ satisfies the definition of an N- λ -closed set. $\square$

Corollary 3.8. *The class of N - λ -closed sets has a proper subclass consisting of core-controlled neutrosophic closed sets (CDN-CS).*

Proof. From Theorem 3.3, every CDN-CS is an N - λ -closed set. Hence, the result follows immediately. $\square$

Remark 3.9. Let $(\mathcal{P}, \mathcal{S})$ be a neutrosophic topological space (NTS) and $\mathcal{K} \subseteq \mathcal{P}$. Then we have the following non-implications:

$$\text{CDN-CS} \not\Rightarrow \text{Ng-CS}, \quad \text{CDN-CS} \not\Rightarrow \text{Ng-SP-CS}, \quad \text{CDN-CS} \not\Rightarrow \text{Nw-CS}.$$

As demonstrated in the following examples, the converse of the aforementioned theorem may not hold.

Example 3.10. Let $\mathcal{P} = \{p_1, p_2\}$ and let $\mathcal{S} = \{\emptyset_N, P_N, K\}$ be a neutrosophic topology on $\mathcal{P}$. The following cases illustrate the relationships between CDN-CSs and other generalized neutrosophic closed sets.

(i) Let

$$K = \{\langle p_1, 0.9, 0.2, 0.2 \rangle, \langle p_2, 0.9, 0.2, 0.2 \rangle\},$$

and define

$$\mathcal{H} = \{\langle p_1, 0.8, 0.3, 0.3 \rangle, \langle p_2, 0.8, 0.3, 0.3 \rangle\}.$$

Then $\mathcal{H}$ is a CDN-CS but not an Ng-CS, since $N\text{-cl}(\mathcal{H}) \subsetneq K$. Moreover, $\mathcal{H}$ is neither an Ng-SP-CS nor an Nw-CS.

(ii) Let

$$K = \{\langle p_1, 0.5, 0.3, 0.4 \rangle, \langle p_2, 0.6, 0.2, 0.5 \rangle\}.$$

Suppose $\mathcal{H}$ is an Ng-CS. Then $\mathcal{H}$ is not a CDN-CS, since $H_{\mathcal{H}}(p_1) = 0.6 \not\geq 0.7$ and $H_{\mathcal{H}}(p_2) = 0.7 \not\geq 1.0$.

(iii) Let

$$K = \{\langle p_1, 0.55, 0.4, 0.45 \rangle, \langle p_2, 0.65, 0.73, 0.6 \rangle\}.$$

Then $\mathcal{H}$ is an Ng-SP-CS but not a CDN-CS, since $H_{\mathcal{H}}(p_1) = 0.55 \not\geq 0.85$ and $H_{\mathcal{H}}(p_2) = 0.65 \not\geq 0.79$.

(iv) Let

$$K = \{\langle p_1, 0.47, 0.35, 0.35 \rangle, \langle p_2, 0.54, 0.25, 0.30 \rangle\}.$$

Then $\mathcal{H}$ is an Nw-CS but not a CDN-CS, since $H_{\mathcal{H}}(p_1) = 0.47 \not\geq 0.70$ and $H_{\mathcal{H}}(p_2) = 0.54 \not\geq 0.65$.

Theorem 3.11. *Let $(\mathcal{P}, \mathcal{S})$ be a neutrosophic topological space. If $\mathcal{K} \subseteq \mathcal{P}$ is both a CDN-CS and an N - λ -closed set, then $\mathcal{K}$ is an Ng-CS.*

Proof. Let $\mathcal{Y}$ be a neutrosophic open set such that $\mathcal{K} \subseteq \mathcal{Y}$. Since $\mathcal{K}$ is an N - λ -closed set, it satisfies the defining property that

$$N\text{-cl}(\mathcal{U}) \subseteq \mathcal{K} \quad \text{for every neutrosophic open set } \mathcal{U} \subseteq \mathcal{K}.$$

Now, because $\mathcal{K} \subseteq \mathcal{Y}$ and $\mathcal{Y}$ is neutrosophic open, by the monotonicity of the closure operator we have

$$N\text{-cl}(\mathcal{K}) \subseteq N\text{-cl}(\mathcal{Y}).$$

Moreover, it is always true that

$$\mathcal{K} \subseteq N\text{-cl}(\mathcal{K}).$$

Therefore,

$$\mathcal{K} \subseteq \text{N-cl}(\mathcal{K}) \subseteq \text{N-cl}(\mathcal{Y}),$$

which implies that

$$\mathcal{K} \subseteq \text{N-cl}(\mathcal{Y}).$$

Consequently, $\mathcal{K}$ satisfies the definition of an Ng-CS. $\square$

Theorem 3.12 (Conditional Intersection Property). *Let $\mathcal{K}$ and $\mathcal{H}$ be two CDN-CSs in a neutrosophic topological space $(\mathcal{P}, \mathcal{S})$. If*

$$\text{N-cl}(\mathcal{K} \cap \mathcal{H}) = \text{N-cl}(\mathcal{K}) \cap \text{N-cl}(\mathcal{H}),$$

then $\mathcal{K} \cap \mathcal{H}$ is a CDN-CS.

Proof. Assume that

$$\text{N-cl}(\mathcal{K} \cap \mathcal{H}) = \text{N-cl}(\mathcal{K}) \cap \text{N-cl}(\mathcal{H}).$$

Thus,

$$\text{N-int}(\text{N-cl}(\mathcal{K} \cap \mathcal{H})) = \text{N-int}(\text{N-cl}(\mathcal{K}) \cap \text{N-cl}(\mathcal{H})).$$

So, by the monotonicity of the interior operator, we have

$$\text{N-int}(\text{N-cl}(\mathcal{K}) \cap \text{N-cl}(\mathcal{H})) \subseteq \text{N-int}(\text{N-cl}(\mathcal{K})) \cap \text{N-int}(\text{N-cl}(\mathcal{H})).$$

Applying the closure operator to both sides yields

$$\text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K} \cap \mathcal{H}))) \subseteq \text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K}))) \cap \text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{H}))).$$

Since $\mathcal{K}$ and $\mathcal{H}$ are CDN-CSs, it follows that

$$\text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K}))) \subseteq \mathcal{K}, \quad \text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{H}))) \subseteq \mathcal{H}.$$

Hence,

$$\text{N-cl}(\text{N-int}(\text{N-cl}(\mathcal{K} \cap \mathcal{H}))) \subseteq \mathcal{K} \cap \mathcal{H}.$$

Therefore, $\mathcal{K} \cap \mathcal{H}$ satisfies the defining condition of a CDN-CS, and thus is a CDN-CS. $\square$

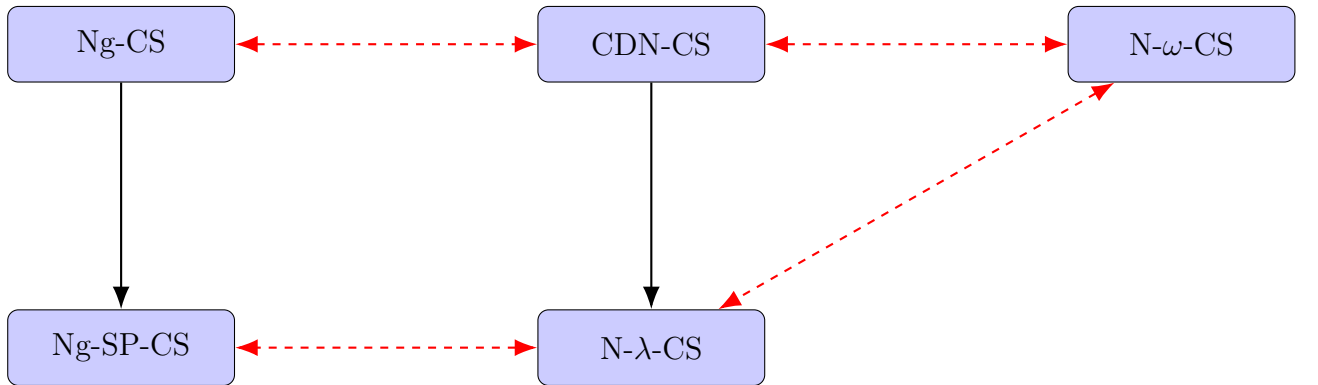


FIGURE 1. Relationships between neutrosophic closed sets. Solid arrows: implication; dashed red arrows: NOT.

Definition 3.13 (CDN-CS Interior and Closure). Assume that $(\mathcal{P}, s)$ is a neutrosophic topological space (NTS) and let

$$\mathcal{K} = \{\langle p, J_{\mathcal{H}}(p), H_{\mathcal{H}}(p), B_{\mathcal{H}}(p) \rangle : p \in \mathcal{P}\} \subseteq \mathcal{P}.$$

Then, the *CDN-CS-interior* and *CDN-CS-closure* of $\mathcal{K}$ are defined as follows:

$$\text{CDN-CS-int}(\mathcal{K}) = \bigcup \{\mathcal{F} : \mathcal{F} \text{ is a CDN-OS in } \mathcal{P} \text{ and } \mathcal{F} \subseteq \mathcal{K}\},$$

$$\text{CDN-CS-cl}(\mathcal{K}) = \bigcap \{\mathcal{G} : \mathcal{G} \text{ is a CDN-CS in } \mathcal{P} \text{ and } \mathcal{K} \subseteq \mathcal{G}\}.$$

Theorem 3.14. *The intersection of two CDN-CS is also CDN-CS.*

Proof. Assume $(\mathcal{P}, s)$ is a NTS and let $\mathcal{K}$ and $\mathcal{H}$ be two CDN-CSs of $\mathcal{P}$. Via Definition 3.1, we have:

$$N\text{-cl}(\mathcal{K}) \subseteq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{K}))) \quad \text{and} \quad N\text{-cl}(\mathcal{H}) \subseteq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{H}))),$$

with $\forall p \in \mathcal{P}$, $J_{\mathcal{H}}(p) > H_{\mathcal{H}}(p) + B_{\mathcal{H}}(p)$ and $\forall p \in \mathcal{P}$, $J_{\mathcal{K}}(p) > H_{\mathcal{K}}(p) + B_{\mathcal{K}}(p)$.

Due to the monotone nature of the neutrosophic closing operator, we get:

$$N\text{-cl}(\mathcal{K} \cap \mathcal{H}) \subseteq N\text{-cl}(\mathcal{K}) \cap N\text{-cl}(\mathcal{H}).$$

Hence,

$$N\text{-cl}(\mathcal{K}) \cap N\text{-cl}(\mathcal{H}) \subseteq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{K}))) \cap N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{H}))).$$

Since

$$N\text{-int}(N\text{-cl}(\mathcal{K})) \cap N\text{-int}(N\text{-cl}(\mathcal{H})) \subseteq N\text{-int}(N\text{-cl}(\mathcal{K} \cap \mathcal{H})),$$

it follows that

$$N\text{-cl}(\mathcal{K} \cap \mathcal{H}) \subseteq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{K} \cap \mathcal{H}))),$$

so the fundamental requirement for Definition 3.1 is satisfied for $\mathcal{K} \cap \mathcal{H}$.

Moreover, the neutrosophic components satisfy:

$$\begin{aligned} J_{\mathcal{H} \cap \mathcal{K}}(p) &= \max\{J_{\mathcal{H}}(p), J_{\mathcal{K}}(p)\}, \\ H_{\mathcal{H} \cap \mathcal{K}}(p) &= \max\{H_{\mathcal{H}}(p), H_{\mathcal{K}}(p)\}, \\ B_{\mathcal{H} \cap \mathcal{K}}(p) &= \min\{B_{\mathcal{H}}(p), B_{\mathcal{K}}(p)\}. \end{aligned}$$

Since $J_{\mathcal{H}}(p) > H_{\mathcal{H}}(p) + B_{\mathcal{H}}(p)$ and $J_{\mathcal{K}}(p) > H_{\mathcal{K}}(p) + B_{\mathcal{K}}(p)$ for all $p \in \mathcal{P}$, we conclude that

$$J_{\mathcal{H} \cap \mathcal{K}}(p) > H_{\mathcal{H} \cap \mathcal{K}}(p) + B_{\mathcal{H} \cap \mathcal{K}}(p), \quad \forall p \in \mathcal{P}.$$

Thus, $\mathcal{K} \cap \mathcal{H}$ satisfies both conditions of Definition 3.1, making it a CDN-CS. $\square$

Remark 3.15. The union of two CDN-CSs need not be a CDN-CS. The following example illustrates this situation.

Example 3.16. Assume

$$\mathcal{P} = \{p_1, p_2\}, \quad s = \{\emptyset_N, P_N, \mathcal{K}\}$$

be a NT defined on $\mathcal{P}$, where

$$\mathcal{K} = \{\langle p_1, 0.7, 0.3, 0.3 \rangle, \langle p_2, 0, 1, 1 \rangle\}.$$

Let $\mathcal{H} = \{\langle p_1, 0.7, 0.5, 0.4 \rangle, \langle p_2, 0.6, 0.3, 0.4 \rangle\}$ represent a nontrivial NOS. Consider two NSs:

$$\mathcal{H} = \{\langle p_1, 0.6, 0.2, 0.2 \rangle, \langle p_2, 0, 1, 1 \rangle\}, \quad \mathcal{M} = \{\langle p_1, 0, 1, 1 \rangle, \langle p_2, 0.6, 0.2, 0.2 \rangle\}.$$

One verifies that $\mathcal{G}$ represents the just nontrivial NOS including p_1 , while $\mathcal{K}$ is exclusively supported at p_1 . Then, we have

$$N\text{-cl}(\mathcal{H}) = \mathcal{H} \cup \mathcal{K}^c = P_N, \quad N\text{-cl}(\mathcal{M}) = P_N.$$

Thus,

$$N\text{-int}(N\text{-cl}(\mathcal{H})) = N\text{-int}(P_N) = P_N, \quad N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{H}))) = N\text{-cl}(P_N) = P_N,$$

and similarly

$$N\text{-int}(N\text{-cl}(\mathcal{M})) = N\text{-int}(P_N) = P_N, \quad N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{M}))) = N\text{-cl}(P_N) = P_N.$$

Now, consider

$$\mathcal{H} \cup \mathcal{M} = \{\langle p_1, 0.6, 0.2, 0.2 \rangle, \langle p_2, 0.6, 0.2, 0.2 \rangle\}.$$

We get

$$N\text{-cl}(\mathcal{H} \cup \mathcal{M}) \subsetneq N\text{-cl}(N\text{-int}(N\text{-cl}(\mathcal{H} \cup \mathcal{M}))).$$

Hence, the union $\mathcal{H} \cup \mathcal{M}$ does not satisfy Definition 3.1 and therefore is not a CDN-CS.

4. THE HOMEOMORPHISMS OF CDN-CSs

In this section, we demonstrate some properties and counterexamples of CDN-CS-continuous, closed maps, and homeomorphisms, along with some related characteristics.

Let $(\mathcal{P}, s)$ and $(\mathcal{O}, t)$ be NTSSs. It is presumed that every function considered is a mapping between these spaces.

Definition 4.1. Assume $(\mathcal{P}, s)$ and $(\mathcal{O}, t)$ are two NTSSs. A map

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

is said to be *continuous of core-dominated neutrosophic* (abbreviated as CDN-Cont) if for every $K \subseteq \mathcal{O}$, the inverse image $h^{-1}(K)$ represents a CDN-CS in $\mathcal{P}$.

Example 4.2. Assume $(\mathcal{P}, s)$ and $(\mathcal{O}, t)$ are two NTSSs, and let two neutrosophic topologies be

$$s = \{\emptyset_{\mathcal{N}}, \mathcal{P}_{\mathcal{N}}, K\} \quad \text{and} \quad t = \{\emptyset_{\mathcal{N}}, \mathcal{O}_{\mathcal{N}}, M\}$$

on $\mathcal{P}$ and $\mathcal{O}$ respectively, where

$$K = \{\langle p_1, 0.60, 0.40, 0.10 \rangle, \langle p_2, 0.70, 0.30, 0.20 \rangle\},$$

$$M = \{\langle q_1, 0.60, 0.40, 0.10 \rangle, \langle q_2, 0.70, 0.30, 0.25 \rangle\}.$$

Consider the map

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

defined by $h(p_1) = q_1$ and $h(p_2) = q_2$. Then h represents a CDN-Cont map because

$$\mathcal{N}\text{-cl}(h^{-1}(M)) \subseteq \mathcal{N}\text{-cl}(\mathcal{N}\text{-int}(\mathcal{N}\text{-cl}(h^{-1}(M)))) = \mathcal{P}_{\mathcal{N}}, \quad \forall p \in \mathcal{P}.$$

Moreover, for all $p \in \mathcal{P}$,

$$J_{h^{-1}(M)}(p) > H_{h^{-1}(M)}(p) + B_{h^{-1}(M)}(p),$$

i.e., $0.60 > 0.50$ and $0.70 > 0.55$. Consequently, $h^{-1}(M)$ represents a CDN-CS in $\mathcal{P}$, and therefore h is CDN-Cont.

Theorem 4.3. *Every CDN-Cont map represents an $N\lambda$ -Cont map.*

Proof. Assume $h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$ represents a CDN-Cont map and let $K \subseteq \mathcal{O}$ be any $N\lambda$ -CS.

Since each CDN-CS represents a λ -closed set, a subset of the collection of λ -closed sets represents the class of CDN-closed sets. In particular, whenever $G \subseteq K$ represents a CDN-CS, then G is λ -closed.

By the continuity of h , the inverse image $h^{-1}(G)$ represents a CDN-CS in $\mathcal{P}$, hence it is λ -closed in $\mathcal{P}$.

Moreover, since the λ -closure operator is monotone, the inverse images preserve inclusions. Consequently, $h^{-1}(K)$ is λ -closed in $\mathcal{P}$. Therefore, h represents an $N\lambda$ -Cont map. $\square$

The inverse of the previous theorem might not be true, as shown in the example that follows.

Example 4.4. Assume $(\mathcal{P}, s)$ and $(\mathcal{O}, t)$ be two NTSs and let two neutrosophic topologies

$$s = \{\emptyset_N, \mathcal{P}_N, K\} \quad \text{and} \quad t = \{\emptyset_N, \mathcal{O}_N, M\}$$

on $\mathcal{P}$ and $\mathcal{O}$ respectively, with

$$K = \{\langle p_1, 0.50, 0.40, 0.30 \rangle, \langle p_2, 0.50, 0.25, 0.40 \rangle\},$$

$$M = \{\langle q_1, 0.70, 0.45, 0.35 \rangle, \langle q_2, 0.50, 0.25, 0.40 \rangle\}.$$

Thus, a map

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t) \quad \text{s.t.} \quad h(p_1) = q_1, \quad h(p_2) = q_2$$

represents an $N\lambda$ -Cont map.

However, it is not a CDN-Cont map since

$$0.70 \not\geq 0.80 \quad \text{and} \quad 0.50 \not\geq 0.65.$$

Consequently, $h^{-1}(M)$ does not represent a CDN-CS in $\mathcal{P}$.

Corollary 4.5. *A valid subclass of the category of $N\lambda$ -Cont maps represents the category of CDN-Cont maps.*

Proof. From Theorem 4.3, all CDN-Cont maps are $N\lambda$ -Cont. Nevertheless, one can construct $N\lambda$ -Cont maps whose inverse images retain $N\lambda$ -CSs but fail to satisfy the additional dominance condition required for CDN-CSs, since there exist $N\lambda$ -CSs which are not CDN-CS. Therefore, this inclusion is proper. $\square$

Theorem 4.6. *The composition of two CDN-Cont maps is a CDN-Cont map.*

Proof. Suppose

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t) \quad \text{and} \quad v : (\mathcal{O}, t) \longrightarrow (\mathcal{J}, r)$$

are two CDN-Cont maps. Let $F \subseteq \mathcal{J}$ be any CDN-CS.

Since v is CDN-Cont, $v^{-1}(F)$ represents a CDN-CS in $\mathcal{O}$. Since h is CDN-Cont,

$$h^{-1}(v^{-1}(F)) = (v \circ h)^{-1}(F)$$

represents a CDN-CS in $\mathcal{P}$.

Therefore, the composition $v \circ h$ represents a CDN-Cont map. $\square$

Corollary 4.7. *The class of CDN-Cont maps is closed under composition.*

Proof. Immediate from Theorem 4.6. $\square$

Theorem 4.8. *The identity mapping*

$$\text{id}_{\mathcal{P}} : (\mathcal{P}, s) \longrightarrow (\mathcal{P}, s)$$

is a CDN-Cont map.

Proof. For any CDN-CS $K \subseteq \mathcal{P}$, we have

$$\text{id}_{\mathcal{P}}^{-1}(K) = K,$$

which represents a CDN-CS. Hence, $\text{id}_{\mathcal{P}}$ is a CDN-Cont map. $\square$

Theorem 4.9. *Let*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

be a CDN-Cont map and let $K \subseteq \mathcal{P}$ be a CDN-CS. Then the restriction

$$h|_K : (K, s|_K) \longrightarrow (\mathcal{O}, t)$$

represents a CDN-Cont map.

Proof. Let $M \subseteq \mathcal{O}$ be any CDN-CS. Then

$$(h|_K)^{-1}(M) = K \cap h^{-1}(M).$$

Since $h^{-1}(M)$ is CDN-CS in $\mathcal{P}$ and the intersection of two CDN-CSs is CDN-CS (Theorem 3.13), it follows that $(h|_K)^{-1}(M)$ is CDN-CS in K . Therefore, $h|_K$ is CDN-Cont. $\square$

Definition 4.10. A map

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

is called *CDN-Closed* (resp., *CDN-Open*) if for each CDN-CS (resp., CDN-OS) $K \subseteq \mathcal{P}$, the image $h(K)$ is a CDN-CS (resp., CDN-OS) in $\mathcal{O}$.

Theorem 4.11. *Every CDN-Closed map is an $N\lambda$ -Closed map.*

Proof. Let $h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$ be a CDN-Closed map and let $K \subseteq \mathcal{P}$ be a CDN-CS. Then $h(K)$ is CDN-CS in $\mathcal{O}$, hence $N\lambda$ -CS in $\mathcal{O}$. Therefore, h maps CDN-CSs to $N\lambda$ -CSs, satisfying the definition of an $N\lambda$ -Closed map.

The converse is not necessarily true, as shown in Example 4.4, where a map is $N\lambda$ -Closed but not CDN-Closed. $\square$

Theorem 4.12. *Let*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t), \quad v : (\mathcal{O}, t) \longrightarrow (\mathcal{J}, r)$$

be CDN-Closed maps. Then the composition

$$v \circ h : (\mathcal{P}, s) \longrightarrow (\mathcal{J}, r)$$

is CDN-Closed.

Proof. Let $K \subseteq \mathcal{P}$ be any CDN-CS. Since h is CDN-Closed, $h(K)$ is CDN-CS in $\mathcal{O}$. Since v is CDN-Closed, $v(h(K))$ is CDN-CS in $\mathcal{J}$. Therefore, $(v \circ h)(K)$ is CDN-CS for each K , showing that $v \circ h$ is CDN-Closed. $\square$

Theorem 4.13. *Let*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

be a bijective map. Then h is CDN-Closed if and only if h is CDN-Open.

Proof. Firstly, assume that h is both bijective and CDN-Closed. Let $M \subseteq \mathcal{P}$ be a CDN-OS. Then M^c is a CDN-CS in $\mathcal{P}$. By the CDN-closedness of h ,

$$h(M^c) \text{ is a CDN-CS in } \mathcal{O}.$$

Since h is bijective,

$$h(M^c) = \mathcal{O} \setminus h(M),$$

so $h(M^c)$ being CDN-CS implies that $h(M)$ is CDN-OS. Therefore, h is CDN-Open.

Conversely, assume that h is bijective and CDN-Open. Let $N \subseteq \mathcal{P}$ be a CDN-CS. Then N^c is CDN-OS, so

$$h(N^c) \text{ is CDN-OS in } \mathcal{O}.$$

By bijectivity,

$$h(N^c) = (h(N))^c,$$

and $(h(N))^c$ being CDN-OS implies $h(N)$ is CDN-CS. Hence, h is CDN-Closed. $\square$

Definition 4.14. A bijective map

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

is called a *CDN-homeomorphism* (CDN-Home) if both h and h^{-1} are CDN-Cont.

Theorem 4.15. *Let*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

be a bijective and CDN-Closed map. Then

$$h^{-1} : (\mathcal{O}, t) \longrightarrow (\mathcal{P}, s)$$

is CDN-Cont.

Proof. Let $M \subseteq \mathcal{P}$ be any CDN-CS. Since h is CDN-Closed,

$$h(M) \text{ is a CDN-CS in } \mathcal{O}.$$

Observe that

$$(h^{-1})^{-1}(M) = h(M),$$

so the inverse image under h^{-1} of each CDN-CS in $\mathcal{P}$ is a CDN-CS in $\mathcal{O}$. Therefore, h^{-1} is CDN-Cont. $\square$

Theorem 4.16. *A bijection*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

is a CDN-Home if and only if h is both CDN-Cont and CDN-Closed.

Proof. (Forward) Suppose h is a CDN-Home. Let $M \subseteq \mathcal{P}$ be a CDN-CS. Since h^{-1} is CDN-Cont, the inverse image of any CDN-CS under h^{-1} is a CDN-CS in $\mathcal{O}$. Using M and observing that

$$(h^{-1})^{-1}(M) = h(M),$$

we conclude that $h(M)$ is CDN-CS in $\mathcal{O}$. Therefore, h is CDN-Closed. By assumption, h is also CDN-Cont. Hence, h is both CDN-Cont and CDN-Closed.

(Backward) Conversely, suppose h is bijective, CDN-Cont, and CDN-Closed. By Theorem 4.15, bijectivity and CDN-Closedness imply that h^{-1} is CDN-Cont. Consequently, both h and h^{-1} are CDN-Cont, which shows that h is a CDN-Home. $\square$

Theorem 4.17. *Let*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t) \quad \text{and} \quad v : (\mathcal{O}, t) \longrightarrow (\mathcal{J}, r)$$

be CDN-Home maps. Then

$$v \circ h : (\mathcal{P}, s) \longrightarrow (\mathcal{J}, r)$$

is also a CDN-Home.

Proof. Since h and v are bijections, $v \circ h$ is bijective. Both h and v are CDN-Cont, and using the standard inverse-image property,

$$(v \circ h)^{-1}(N) = h^{-1}(v^{-1}(N)).$$

Furthermore, $(v \circ h)^{-1} = h^{-1} \circ v^{-1}$, and since h^{-1} and v^{-1} are CDN-Cont, their composition is CDN-Cont. Therefore, both $v \circ h$ and its inverse are CDN-Cont, and $v \circ h$ is a CDN-Home. $\square$

Corollary 4.18. *If*

$$h : (\mathcal{P}, s) \longrightarrow (\mathcal{O}, t)$$

is a CDN-Home, then for each $M \subseteq \mathcal{P}$,

$$M \text{ is CDN-CS in } \mathcal{P} \iff h(M) \text{ is CDN-CS in } \mathcal{O}.$$

Proof. Assume M is CDN-CS in $\mathcal{P}$. Since h is CDN-Closed (Theorem 4.16), $h(M)$ is CDN-CS in $\mathcal{O}$. Conversely, if $h(M)$ is CDN-CS in $\mathcal{O}$, then

$$M = h^{-1}(h(M))$$

is CDN-CS in $\mathcal{P}$ because h^{-1} is CDN-Cont. $\square$

Theorem 4.19. *Every CDN-Home map is also an $N\lambda$ -Home map.*

Proof. Assume h is a CDN-Home. Then both h and h^{-1} are CDN-Cont, so their inverse images preserve CDN-CS sets. Since every CDN-CS is an $N\lambda$ -CS, the inverse images under h and h^{-1} also preserve $N\lambda$ -CS sets. Therefore, h and h^{-1} are $N\lambda$ -Cont, and h is an $N\lambda$ -Home.

Remark: The converse does not hold in general; Example 4.4 shows a map that is an $N\lambda$ -Home but not a CDN-Home, since h fails to be CDN-Cont. $\square$

5. CONCLUSIONS

This paper introduces a novel and robust class of closed sets in neutrosophic topological spaces, called *Core-Dominated Neutrosophic Closed Sets (CDN-CSs)*. The framework ensures that the truth-membership dominates uncertainty and falsity, combining closure and interior operators with dominance conditions on membership functions. We establish fundamental properties showing that CDN-CSs provide a more refined categorization than traditional neutrosophic λ -closed sets (λ -CSs).

Through set-theoretic analysis, we show that CDN-CSs remain stable under finite intersections and effectively capture uncertainty in a controlled manner. We also introduce related concepts such as CDN-continuous maps, CDN-closed maps, and CDN-homeomorphisms, proving key theorems on their composition, characterization, and invariance under specific mappings. Counterexamples illustrate that this framework is strictly stronger than previous approaches.

These findings extend the conceptual landscape of neutrosophic topology, offering a semantically accurate and practical perspective on closedness and continuity in uncertain environments. Future work may explore applications of CDN-CSs in real-world systems, particularly in artificial intelligence, decision-making, and data analysis, and investigate their relationships with other advanced topological structures to enhance modeling of complex phenomena.

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