

CHARACTERIZATION OF MAGNIFYING ELEMENTS IN ALMOST MONOMORPHISM AND ALMOST EPIMORPHISM SEMIGROUPS

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ABSTRACT. This research examines magnifying elements within the almost monomorphism semigroup and the almost epimorphism semigroup. The objective of this study is to characterize both left and right magnifying elements in the almost monomorphism and almost epimorphism semigroups by determining their necessary and sufficient conditions.

Keywords. left magnifying element, right magnifying element, linear transformation semigroup.

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1. INTRODUCTION AND PRELIMINARIES

Ljapin [7] first established the notion of magnifying elements within the context of semigroup theory in 1963. Formally, considering a semigroup S , an element $a \in S$ is classified as *left magnifying* if $aM = S$ holds for some proper subset M of S . Dually, the term *right magnifying element* refers to an element $a \in S$ such that $Ma = S$ for some proper subset M of S . Following this concept, several algebraic properties and characterizations of semigroups containing magnifying elements were investigated by Catino and Migliorini [1], and Gutan [5].

The study of magnifying elements transitioned from abstract semigroups to transformation semigroups. Magill [8] explored the behavior of these elements within transformation semigroups. Since then, researchers have generalized these concepts across various specific types of transformation semigroups. For instance, magnifying elements have been characterized in semigroups of transformations with restricted ranges [2], semigroups of transformations with invariant sets [4], and other generalizations of the full transformation semigroups [6]. Alongside magnifying elements, other fundamental algebraic properties of these generalized semigroups continue to attract attention. For instance, Srimora et. al. [10] recently investigated the regularity of transformation semigroups with invariant sets and restricted by an equivalence relation.

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Researchers began adapting the concepts from functions to linear transformations. For a vector space V , let $L(V)$ denote the full linear transformation semigroup. In 2018, Chinram and Baupradist [3], established the characterizations of magnifying elements within linear transformation semigroups that possess invariant subspaces. In 2020, Prakitsri [9] provided a study on magnifying elements within linear transformation semigroups where nullity or co-rank is infinite.

Along this paper, mappings will be written on the right, that is, for a vector $v \in V$ and a linear transformation $\alpha \in L(V)$, the image of v under α is denoted by $v\alpha$. For any $\alpha \in L(V)$, the *image* of α is denoted by $\text{im } \alpha = \{v\alpha \mid v \in V\}$, and the *kernel* of α is denoted by $\ker \alpha = \{v \in V \mid v\alpha = 0\}$. Both $\text{im } \alpha$ and $\ker \alpha$ are subspaces of V . The dimension of $\ker \alpha$ is called the *nullity* of α , and the dimension of the quotient space $V/\text{im } \alpha$ is called the *co-rank* of α . We use $\dim V$ to denote the dimension of V . Furthermore, let 1_V denotes the identity map on V .

Let us use the notation $V = U \oplus W$ to indicate that the vector space V decomposes into the direct sum of its subspaces U and W . It is well-known in linear algebra that for any subspace U of V , there exists a complementary subspace W of V such that $V = U \oplus W$, and in this case, $W \cong V/U$. Hence, $\dim(W) = \dim(V/U)$.

For a vector space V , let

$$\begin{aligned} AM(V) &= \{\alpha \in L(V) \mid \dim(\ker \alpha) < \infty\}, \\ AE(V) &= \{\alpha \in L(V) \mid \dim(V/\text{im } \alpha) < \infty\}. \end{aligned}$$

Both are subsemigroups of $L(V)$. The semigroup $AM(V)$ is called *almost monomorphism semigroup* on V . Dually, the semigroup $AE(V)$ is called *almost epimorphism semigroup* on V . Observe that if $\dim V$ is finite, then $AM(V) = AE(V) = L(V)$.

It was established by Magill [8] that the presence of magnifying elements within $L(V)$ is strictly equivalent to V being infinite-dimensional. Left and right magnifying element in $L(V)$ were also characterized by [8], as follows. Note that the conditions for left and right are swapped here to align with our right-mapping convention.

Theorem 1.1. [8] *Let V be an infinite-dimensional vector space and let $\alpha \in L(V)$. Then the following holds.*

- (i) α is left magnifying in $L(V)$ if and only if α is injective and not surjective.
- (ii) α is right magnifying in $L(V)$ if and only if α is surjective and not injective.

Note that we often write the notation

$$\alpha = \begin{pmatrix} B_1 & v \\ 0 & v \end{pmatrix}_{v \in B \setminus B_1}$$

to describe a linear map α defined by $u\alpha = 0$ for all $u \in B_1$ and $v\alpha = v$ for all $v \in B \setminus B_1$, where B is a basis for V extended from B_1 .

This paper aims to characterize left and right magnifying elements in the semigroups $AM(V)$ and $AE(V)$.

2. MAIN RESULTS

In this section, we let V be an infinite-dimensional vector space.

2.1. The Semigroup $AM(V)$. We first show that, unlike $L(V)$, the condition of $\alpha \in AM(V)$ such that α is injective but not surjective is not enough to be a left magnifying element in $AM(V)$ as in the following example.

Example 2.1. Let $V = \mathbb{R}[x]$ and define $\alpha : V \rightarrow V$ by $p(x)\alpha = p(x^2)$. Then α is injective and $V/\text{im } \alpha \cong \text{span}\{x, x^3, x^5, \dots\}$, so $\dim(V/\text{im } \alpha)$ is infinite. Assume that α is a left magnifying element in $AM(V)$, that is, $\alpha M = AM(V)$ for some $M \subsetneq AM(V)$. As $1_V \in AM(V)$, we obtain $\alpha\gamma = 1_V$ for some $\gamma \in M$, and hence γ is surjective. We claim that $V = \text{im } \alpha \oplus \ker \gamma$. Let $v \in V$. Then $v = v\gamma\alpha + (v - v\gamma\alpha)$. We can see that $v\gamma\alpha \in \text{im } \alpha$. Observe that

$$(v - v\gamma\alpha)\gamma = v\gamma - v\gamma\alpha\gamma = v\gamma - v\gamma 1_V = v\gamma - v\gamma = 0,$$

which means $v - v\gamma\alpha \in \ker \gamma$. Next, let $v \in \text{im } \alpha \cap \ker \gamma$. Thus $v = u\alpha$ for some $u \in V$ and $v\gamma = 0$. This implies that $0 = v\gamma = u\alpha\gamma = u1_V = u$, so $v = u\alpha = 0$. Hence $V = \text{im } \alpha \oplus \ker \gamma$. It follows that $\ker \gamma \cong V/\text{im } \alpha$, and thus $\dim(\ker \gamma) = \dim(V/\text{im } \alpha)$ which is infinite. This contradicts $\gamma \in AM(V)$. Therefore α is not a left magnifying element in $AM(V)$.

We found that $\alpha \in AM(V)$ which is injective and $0 < \dim(V/\text{im } \alpha) < \infty$ becomes a left magnifying element in $AM(V)$, see the following lemma.

Lemma 2.2. *Let $\alpha \in AM(V)$. If α is injective and $0 < \dim(V/\text{im } \alpha) < \infty$, then α is a left magnifying element in $AM(V)$.*

Proof. Suppose that α is injective and $0 < \dim(V/\text{im } \alpha) < \infty$. Assume that $\dim(V/\text{im } \alpha) = n \in \mathbb{N}$. We let B be a basis for V and $C_1 := B\alpha$. Then C_1 is a basis of $\text{im } \alpha$ since α is injective. Moreover, let $C_2 = \{w_1, w_2, \dots, w_n\}$ be a basis for a subspace W such that $V = \text{im } \alpha \oplus W$ and $W \cong V/\text{im } \alpha$. It can be seen that $C := C_1 \oplus C_2$ is a basis for V . Next, let

$$M = \{\gamma \in AM(V) \mid w_1\gamma = 0\}.$$

It is clear that M is a proper subset of $AM(V)$. Let $\beta \in AM(V)$. Observe that for every $v \in C_1$ there exists $u_v \in B$ such that $u_v\alpha = v$. Define $\gamma \in L(V)$ by

$$\gamma = \begin{pmatrix} w_i & v \\ 0 & u_v\beta \end{pmatrix}_{v \in C_1, i=1,2,\dots,n}.$$

Then $\gamma \in M$ and $v\alpha\gamma = v\beta$ for all $v \in B$, so $\alpha M = AM(V)$. Therefore, α is a left magnifying element in $AM(V)$. $\square$

Lemma 2.3. *Let $\alpha \in AM(V)$. If α is a left magnifying element in $AM(V)$, then α is injective and $0 < \dim(V/\text{im } \alpha) < \infty$.*

Proof. Suppose that α is left magnifying in $AM(V)$. Then let M be a proper subset of $AM(V)$ such that $\alpha M = AM(V)$. To show that α is an injective mapping, let $u \in \ker \alpha$. Choose $\beta = 1_V \in AM(V)$. There is $\gamma \in M$ such that $\alpha\gamma = \beta$. Thus $u = u\beta = u\alpha\gamma = 0$, so α is injective. We claim that α is not surjective. Assume that α is a surjective mapping. Hence α is bijective. From

$\alpha M = AM(V)$, we have $M = \alpha^{-1}AM(V) = AM(V)$, which is a contradiction. This leads to $0 < \dim(V/\text{im } \alpha)$. Next, we claim that $\dim(V/\text{im } \alpha) < \infty$. From $\alpha\gamma = \beta = 1_V$, we obtain that γ is surjective. Let W be a subspace of V such that $V = \text{im } \alpha \oplus W$ where $W \cong V/\text{im } \alpha$. For each $w \in W$, there is $v_w \in \text{im } \alpha$ such that $v_w\gamma = w\gamma$ since γ is surjective and $\alpha\gamma = 1_V$. Then $v_w - w \in \ker \gamma$. Define $\theta : W \rightarrow \ker \gamma$ by $w\theta = v_w - w$ for all $w \in W$. Let $w \in \ker \theta$. It follows that $v_w - w = w\theta = 0$, so $v_w = w$. Since $w \in W$, $w = v_w \in \text{im } \alpha$ and $\text{im } \alpha \cap W = \{0\}$, we get $w = v_w = 0$. Hence θ is injective, and thus $\dim(V/\text{im } \alpha) = \dim W \leq \dim(\ker \gamma) < \infty$. $\square$

Combining the results from Lemmas 2.2 and 2.3, we can now formulate the criteria for an element α in $AM(V)$ to be left magnifying as follows.

Theorem 2.4. *Let $\alpha \in AM(V)$. Then α is a left magnifying element in $AM(V)$ if and only if α is injective and $0 < \dim(V/\text{im } \alpha) < \infty$.*

The following is an example of left magnifying elements in $AM(V)$.

Example 2.5. Consider a basis B for the vector space V and $u \in B$. Since $|B| = |B \setminus \{u\}|$, let $\phi : B \rightarrow B \setminus \{u\}$ be a bijection. Define $\alpha \in AM(V)$ by $v\alpha = v\phi$ for all $v \in B$. Then α is injective and $\dim(V/\text{im } \alpha) = 1$. According to Theorem 2.4, this α serves as a left magnifying element.

Next, for a right magnifying element in $AM(V)$, the condition is similar to $L(V)$.

Theorem 2.6. *Let $\alpha \in AM(V)$. Then α is a right magnifying element in $AM(V)$ if and only if α is surjective but not injective.*

Proof. Suppose α acts as a right magnifying element within $AM(V)$. By definition, there exists $M \subsetneq AM(V)$ satisfying $M\alpha = AM(V)$. Consider $\beta = 1_V$ and let $v \in V$. Thus $v = v\beta = v\gamma\alpha$ for some $\gamma \in M$. Hence α is surjective. Assume that α is an injective mapping. It follows that $M\alpha = AM(V) = AM(V)\alpha$, so $M = AM(V)$ which is a contradiction. Therefore α is not injective.

Next, suppose that α is surjective but not injective. There exists $\delta \in L(V)$ such that $\delta\alpha = 1_V$. Observe that δ is injective. As α is not injective, we have δ is not surjective. Choose

$$M = \{\gamma \in AM(V) \mid \text{im } \gamma \subseteq \text{im } \delta\}.$$

Since $\text{im } \delta \subsetneq V$, M is a proper subset of $AM(V)$. We now let $\beta \in AM(V)$. Define $\gamma \in L(V)$ by $\gamma = \beta\delta$. We claim that $\ker \gamma = \ker \beta$. We can see that $\ker \beta \subseteq \ker \gamma$. Let $v \in \ker \gamma$. Then $0 = v\gamma = v\beta\delta$. Since δ is injective, we get $v\beta = 0$ and hence $v \in \ker \beta$. Thus $\dim(\ker \gamma) = \dim(\ker \beta) < \infty$, so $\gamma \in AM(V)$. Clearly, $\text{im } \gamma \subseteq \text{im } \delta$. Hence $\gamma \in M$. Moreover, $\gamma\alpha = \beta\delta\alpha = \beta 1_V = \beta$. We then conclude that α is a right magnifying element in $AM(V)$. $\square$

Example 2.7. Suppose B is a basis for V . By picking an arbitrary element $u \in B$, we can establish a bijective mapping ϕ from $B \setminus \{u\}$ onto B . Define $\alpha \in AM(V)$ by

$$\alpha = \begin{pmatrix} u & v \\ 0 & v\phi \end{pmatrix}_{v \in B \setminus \{u\}}.$$

Then α is surjective but lacks injectivity. Consequently, by applying Theorem 2.6, we conclude that α is a right magnifying element within $AM(V)$.

2.2. The Semigroup $AE(V)$.

Theorem 2.8. *Let $\alpha \in AE(V)$. Then α is a left magnifying element in $AE(V)$ if and only if α is injective but not surjective.*

Proof. The necessary condition can be verified using the same rationale as in Lemma 2.3.

Conversely, suppose that α is one-to-one and not onto. Then $\text{im } \alpha \subsetneq V$. Thus, we can decompose V into a direct sum $V = \text{im } \alpha \oplus W$ for some complementary subspace W such that $W \cong V/\text{im } \alpha$. Since $\alpha \in AE(V)$, we get $\dim W = \dim(V/\text{im } \alpha) < \infty$. Assume that $\dim W = n$ where $n \in \mathbb{N}$. Let $B_1 = \{w_1, w_2, \dots, w_n\}$ be a basis of W . Choose

$$M = \{\gamma \in AE(V) \mid w_1\gamma = 0\}.$$

Thus M is a proper subset of $AE(V)$. Let $\beta \in AE(V)$. Since $V = \text{im } \alpha \oplus W$, each $v \in V$ can be uniquely written in the form $v = u_v\alpha + w$ where $u_v \in V$ and $w \in W$. Define $\gamma \in L(V)$ by $(u_v\alpha + w)\gamma = u_v\beta$ for all $v \in V$. Then $v\alpha\gamma = (v\alpha + 0)\gamma = v\beta$ for all $v \in V$, so $\alpha\gamma = \beta$. We can see that $\text{im } \gamma = \text{im } \beta$, so $\dim(V/\text{im } \gamma) = \dim(V/\text{im } \beta) < \infty$. Moreover, $w_1\gamma = (0\alpha + w_1)\gamma = 0$ and hence $\gamma \in M$. Consequently, α is left magnifying within $AE(V)$. $\square$

Example 2.9. We now assume that $\dim V$ is countably infinite. Let $B = \{v_1, v_2, v_3, \dots\}$ be a basis of V . Define $\alpha \in AE(V)$ by $v_i\alpha = v_{i+1}$ for all $i \in \mathbb{N}$. Obviously, α is injective. Moreover, we can see that $\text{im } \alpha = \text{span}\{v_2, v_3, v_4, \dots\}$. This implies that $\dim(V/\text{im } \alpha) = 1$ and thus α is not surjective. From Theorem 2.8, α is left magnifying in $AE(V)$.

Also, a surjective transformation in $AE(V)$ that is not injective is not necessarily right magnifying within $AE(V)$, as shown in the following.

Example 2.10. Consider the space $V = \mathbb{R}[x]$ of all polynomials over $\mathbb{R}$. Observe that $B = \{1, x, x^2, x^3, \dots\}$ is a basis for V . Define $\alpha : V \rightarrow V$ by $x^{2n}\alpha = x^n$ and $x^{2n+1}\alpha = 0$ for all $n \in \mathbb{N} \cup \{0\}$. To show that α is surjective, let $q(x) = a_0 + a_1x + a_2x^2 + \dots + a_kx^k \in V$. Choose

$$p(x) = a_0 + a_1x^2 + a_2x^4 + \dots + a_kx^{2k} \in V.$$

Then $p(x)\alpha = q(x)$, so α is surjective. Moreover, $\{x, x^3, x^5, \dots\}$ is a basis of $\ker \alpha$ and thus $\dim(\ker \alpha)$ is infinite. Suppose that α is a right magnifying element in $AE(V)$. Then $M\alpha = AE(V)$ for some $M \subsetneq AE(V)$. Since $1_V \in AE(V)$, there exists $\gamma \in M$ such that $\gamma\alpha = 1_V$. Then $\{1, x^2, x^4, \dots\} \in \text{im } \gamma$ but $\{x, x^3, x^5, \dots\} \notin \text{im } \gamma$. Hence $\dim(V/\text{im } \gamma)$ is infinite, which is a contradiction. We can conclude that α is not a right magnifying element in $AE(V)$.

When we add the condition that α is surjective and $0 < \dim(\ker \alpha) < \infty$, we can show that it is a right magnifying element in $AE(V)$.

Lemma 2.11. *Let $\alpha \in AE(V)$. If α is surjective and $0 < \dim(\ker \alpha) < \infty$, then α is a right magnifying element in $AE(V)$.*

Proof. Suppose that α is surjective and $0 < \dim(\ker \alpha) < \infty$. Note that $V = \ker \alpha \oplus W$ for some subspace W of V . Let $\alpha|_W : W \rightarrow V$ be the restriction of α to W . Since $V = \ker \alpha \oplus W$, we obtain that $\alpha|_W$ is an isomorphism. Let $\delta = (\alpha|_W)^{-1}$. Then $\delta\alpha = 1_V$ and $\text{im } \delta = W$. Choose

$$M = \{\gamma \in AE(V) \mid \text{im } \gamma \subseteq W\}.$$

We can see that $M \subsetneq AE(V)$ since $1_V \in AE(V)$ but $\text{im } 1_V = V \not\subseteq W$. Let $\beta \in AE(V)$. Define $\gamma \in L(V)$ by $\gamma = \beta\delta$. Observe that $\text{im } \gamma = \text{im}(\beta\delta) = (\text{im } \beta)\delta \subseteq \text{im } \delta = W$, and $\gamma\alpha = \beta\delta\alpha = \beta 1_V = \beta$. To show $\gamma \in AE(V)$, we claim that $\dim(V/\text{im } \beta) = \dim(W/\text{im } \gamma)$. Define $\phi : V \rightarrow W/\text{im } \gamma$ by $v\phi = v\delta + \text{im } \gamma$ for all $v \in V$. Let $w \in W$. Since $\delta : V \rightarrow W$ is surjective, there exists $v \in V$ such that $v\delta = w$. Then $v\phi = v\delta + \text{im } \gamma = w + \text{im } \gamma$, so ϕ is surjective. Observe that $v \in \ker \phi$ if and only if $v\delta \in \text{im } \gamma$. Let $v \in \ker \phi$. Since $\text{im } \gamma = (\text{im } \beta)\delta$, we have $v\delta = u\delta$ for some $u \in \text{im } \beta$. As δ is injective, $v = u \in \text{im } \beta$. Thus $\ker \phi \subseteq \text{im } \beta$. Let $v \in \text{im } \beta$. Then $v\delta \in (\text{im } \beta)\delta = \text{im } \gamma$, and hence $v \in \ker \phi$. It follows that $\ker \phi = \text{im } \beta$. From the First Isomorphism Theorem, we know that $V/\ker \phi \cong \text{im } \phi$, so $V/\text{im } \beta \cong W/\text{im } \gamma$. Hence $\dim(V/\text{im } \beta) = \dim(W/\text{im } \gamma)$. Since $\text{im } \gamma \subseteq W \subseteq V$, we have

$$\dim(V/\text{im } \gamma) = \dim(V/W) + \dim(W/\text{im } \gamma)$$

Since $V = \ker \alpha \oplus W$, we know $\dim(V/W) = \dim(\ker \alpha)$. Thus,

$$\dim(V/\text{im } \gamma) = \dim(\ker \alpha) + \dim(V/\text{im } \beta) < \infty$$

as $0 < \dim(\ker \alpha) < \infty$ and $\beta \in AE(V)$. We can now conclude that $\gamma \in M$ and thus $M\alpha = AE(V)$. Therefore, α is a right magnifying element in $AE(V)$. $\square$

The converse is also true by the following lemma.

Lemma 2.12. *Let $\alpha \in AE(V)$. If α is a right magnifying element in $AE(V)$, then α is surjective and $0 < \dim(\ker \alpha) < \infty$.*

Proof. Let us consider a right magnifying element α in $AE(V)$. There is $M \subsetneq AE(V)$ such that $M\alpha = AE(V)$. Since $1_V \in AE(V)$, we get $\gamma\alpha = 1_V$ for some $\gamma \in M$. Then α is onto. If $\dim(\ker \alpha) = 0$, then α is a bijective and hence $M\alpha = AE(V)$ implies $M = AE(V)\alpha^{-1} = AE(V)$ which is impossible. Thus $\dim(\ker \alpha) > 0$. Next we show that $\dim(\ker \alpha) < \infty$. Note that $\dim(V/\text{im } \gamma) < \infty$ since $\gamma \in M$. From $\gamma\alpha = 1_V$, we have γ is injective. We claim that $\ker \alpha \cap \text{im } \gamma = \{0\}$. Let $v \in \ker \alpha \cap \text{im } \gamma$. Then $v\alpha = 0$ and $w\gamma = v$ for some $w \in V$. It follows that

$$w = w1_V = w\gamma\alpha = v\alpha = 0,$$

so $v = w\gamma = 0\gamma = 0$. Hence $\ker \alpha \cap \text{im } \gamma = \{0\}$. Define $\theta : \ker \alpha \rightarrow V/\text{im } \gamma$ by $v\theta = v + \text{im } \gamma$ for all $v \in \ker \alpha$. We next claim that $\ker \theta = \{0\}$. Let $v \in \ker \theta$. Then $\text{im } \gamma = v\theta = v + \text{im } \gamma$, so $v \in \text{im } \gamma$. Thus $v \in \ker \alpha \cap \text{im } \gamma = \{0\}$ and hence $v = 0$. This means that θ is injective. Therefore, $\dim(\ker \alpha) \leq \dim(V/\text{im } \gamma) < \infty$. $\square$

We then conclude another main theorem from Lemmas 2.11 and 2.12.

Theorem 2.13. *Let $\alpha \in AE(V)$. Then α is a right magnifying element in $AE(V)$ if and only if α is surjective and $0 < \dim(\ker \alpha) < \infty$.*

Example 2.14. Assume that $\dim V$ is countably infinite. Let the set $B = \{v_1, v_2, v_3, \dots\}$ be a basis for V . Define $\alpha \in AE(V)$ by

$$\alpha = \left(\begin{array}{cc} v_1 & v_i \\ 0 & v_{i-1} \end{array} \right)_{i \in \mathbb{N} \setminus \{1\}}.$$

Clearly, α is surjective and $\dim(\ker \alpha) = 1$. Hence, from Theorem 2.13, α is right magnifying in $AE(V)$.

3. CONCLUSION

This paper presents the characterizations of both left and right magnifying elements within the semigroups $AM(V)$ and $AE(V)$, which are summarized as follows. For $\alpha \in AM(V)$,

- (i) α is a left magnifying element in $AM(V)$ if and only if α is injective and $0 < \dim(V/\text{im } \alpha) < \infty$,
- (ii) α is a right magnifying element in $AM(V)$ if and only if α is surjective but not injective.

For α in $AE(V)$,

- (i) α is a left magnifying element in $AE(V)$ if and only if α is injective but not surjective,
- (ii) α is a right magnifying element in $AE(V)$ if and only if α is surjective and $0 < \dim(\ker \alpha) < \infty$.

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