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A LAPLACE-EMBEDDED RESIDUAL POWER SERIES FRAMEWORK

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Abstract. This paper presents a hybrid analytical approach, combining the residual power series method with the Laplace transform, to solve systems of linear and nonlinear differential equations. The proposed technique converts governing differential systems into algebraic representations, naturally incorporating initial conditions without numerical differentiation or discretization. We validate the method's robustness through applications to a coupled linear system, the nonlinear SIR epidemic model, and the chaotic Genesio-Tesi system. Comparisons with the fourth-order Runge-Kutta numerical method demonstrate exceptional accuracy, rapid convergence, and structural transparency, establishing this technique as a highly efficient tool for analyzing complex dynamic systems.

Keywords. Residual power series method, Laplace transform, nonlinear systems; SIR model; Genesio-Tesi system; residual error analysis
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1. Introduction

Systems of linear and nonlinear differential equations are fundamental tools for modeling dynamic phenomena across diverse scientific and engineering domains. These systems govern the temporal and spatial evolution of quantities under the influence of underlying physical laws, making them essential in the analysis of processes such as population dynamics, chemical kinetics, electrical circuit behavior, and mechanical system responses. The accurate and efficient solution of these equations is crucial for the interpretation, prediction, and control of complex real-world systems. In recent years, considerable attention has been devoted to developing robust analytical and numerical techniques for solving such systems with greater precision and computational efficiency [14, 5, 13, 9].

Among the various analytical and semi-analytical methods developed for solving differential equations, the residual power series method has gained recognition as a robust and flexible technique. This method constructs approximate solutions in the form of rapidly convergent power series, enhanced by a residual

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correction mechanism that systematically refines the accuracy of the solution. Notably, RPSM operates without the need for linearization, perturbation assumptions, or discretization, thereby preserving the intrinsic nonlinear characteristics of the problem. These features make RPSM particularly effective for addressing nonlinear, strongly coupled, and multi-dimensional systems where conventional approaches often encounter significant challenges. Recent studies have demonstrated its efficacy across a variety of complex applications [2, 16, 3, 8, 10].

To enhance the effectiveness and extend the applicability of the RPSM, it can be advantageously integrated with integral transform techniques [12, 4, 15]. Among these, the Laplace transform stands out as a powerful analytical tool for simplifying differential operators and naturally incorporating initial conditions into the solution framework. The combination of the Laplace transform with RPSM not only facilitates the treatment of initial value problems but also improves the convergence rate and reduces computational overhead, particularly when applied to complex and nonlinear differential systems.

This hybrid approach offers several notable advantages in solving both linear and nonlinear differential equations. The application of Laplace transform method ensures the seamless inclusion of initial conditions, thereby eliminating the need for manual imposition. Furthermore, it avoids the necessity for numerical time-stepping schemes, which are often computationally intensive and susceptible to accumulation errors. It must be acknowledged that for strictly integer-order differential equations, classical time-domain approaches, such as the standard Maclaurin or Taylor series methods, yield mathematically equivalent polynomial solutions. However, those classical methods are heavily bottlenecked by the need to compute repeated, high-order analytical derivatives of complex multidimensional nonlinear terms. The LRPSM circumvents this computational burden entirely. By shifting the algebraic manipulations to the Laplace domain, the method replaces tedious analytical differentiation with systematic algebraic limit evaluations, providing a highly efficient algorithmic structure for symbolic computation [6, 18, 11, 1]. Recent advances published in the *Gulf Journal of Mathematics* have strongly highlighted the power of integrating Laplace transforms with advanced analytical methods to resolve complex nonlinear and fractional dynamics [7, 17].

Motivated by these benefits, this study presents a systematic methodology for solving systems of linear and nonlinear differential equations. Specifically, the nonlinear SIR epidemic model and the chaotic nonlinear Genesio-Tesi system-by employing the residual power series method combined with the Laplace transform [10, 6]. The paper elaborates on the theoretical underpinnings and computational strategies of the proposed hybrid technique. Crucially, to prove the robustness and efficacy of the LRPSM, we conduct a rigorous comparative analysis against the standard 4th-order Runge-Kutta (RK45) numerical method, supported by comprehensive absolute error tables and multidimensional graphical trajectory mappings.

Our paper is organized as: Section 2 presents the basic definitions and mathematical theorems underlying the residual power series method and its hybridization with the Laplace transform. Section 3 focuses on the theoretical analysis and convergence formulation of the presented method. In Section 4, we apply this method to solve several systems, including a linear system, the nonlinear SIR epidemic model, and the nonlinear Genesio system. Within this section, the analytical results are quantitatively and graphically validated against numerical simulations to confirm the method's high precision and ability to capture chaotic phase space dynamics. The last section concludes with a summary of our findings.

2. Basic definitions and mathematical framework

This section introduces the foundational concepts, function spaces, and theorems required for the formulation of the Laplace-Residual power series method. To ensure the rigorous application of the integral transforms, we first define the appropriate space of functions.

Definition 2.1

A function $f(t)$ defined on $[0,\infty)$ is said to be of exponential order α if there exist real constants $M>0$ and α such that:

$$|f(t)| \leq Me^{\alpha t}, \quad \forall t \geq 0. \tag{2.1}$$

We denote the space of all piecewise continuous functions of exponential order on $[0,\infty)$ by $\mathcal{E}_\alpha$.

For the remainder of this study, we assume that all state variables in the considered differential systems belong to $\mathcal{E}_\alpha$.

Definition 2.2

For a function $y(t) \in \mathcal{E}_\alpha$, the Laplace transform, denoted by $\mathcal{L}[y(t)]$ or $Y(s)$, is defined by the improper integral:

$$Y(s) = \mathcal{L}[y(t)] = \int_0^\infty e^{-st} y(t)\, dt, \quad s > \alpha \tag{2.2}$$

where s is a complex parameter. The inverse Laplace transform is given by $y(t) = \mathcal{L}^{-1}[Y(s)]$.

Lemma 2.1

Let $y(t) \in \mathcal{E}_\alpha$ be a function of class C^n on $[0,\infty)$. The Laplace transform of its n-th derivative is given by:

$$\mathcal{L}[y^{(n)}(t)] = s^n Y(s) - \sum_{k=0}^{n-1} s^{n-k-1} y^{(k)}(0). \tag{2.3}$$

To construct the series solution within the Laplace domain, we utilize the following theorem which provides the algebraic series representation of $Y(s)$ at infinity.

Theorem 2.1

If a function $y(t)$ possesses a convergent Maclaurin series expansion $y(t) = \sum_{n=0}^{\infty} \frac{y^{(n)}(0)}{n!} t^n$, then its corresponding Laplace transform $Y(s)$ can be represented as an inverse power series in s:

$$Y(s) = \sum_{n=0}^{\infty} \frac{c_n}{s^{n+1}}, \quad s > \alpha \tag{2.4}$$

where the coefficients are given by $c_n = y^{(n)}(0)$.

Proof: By applying the linearity of the Laplace transform and the fundamental property $\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$, taking the transform of the Maclaurin series of $y(t)$ term-by-term immediately yields the required series representation.

Definition 2.3
Based on the theorem 2.1, the approximate solution of a differential equation in the Laplace domain is defined by the k-th truncated series:

$$Y_k(s) = \sum_{n=0}^{k} \frac{c_n}{s^{n+1}}, \tag{2.5}$$

where the first m coefficients $(c_0, c_1, \ldots, c_{m-1})$ are predetermined by the initial conditions of the given m-th order differential system.

The core mechanism of the LRPSM relies on the construction and minimization of the Laplace residual function.

Definition 2.4
Consider a general nonlinear differential equation $\mathcal{N}[y(t)] = 0$.
Applying the Laplace transform yields an equivalent equation in the s-domain: $\mathcal{L}\{\mathcal{N}[y(t)]\} = 0$.
The Laplace Residual Function, $Res(s)$, and its k-th truncated form, $Res_k(s)$, are defined respectively as:

$$Res(s) = \mathcal{L}\{\mathcal{N}[Y(s)]\} \quad \text{and} \quad Res_k(s) = \mathcal{L}\{\mathcal{N}[Y_k(s)]\}. \tag{2.6}$$

To determine the unknown coefficients c_n iteratively without the need for fractional or classical differentiation, we introduce the following crucial limit theorem.

Theorem 2.2
Let $Res_k(s)$ be the k-th truncated Laplace residual function for the governing differential equation. The unknown coefficients c_k in the series expansion $Y_k(s)$ can be obtained algebraically by solving the following limit equation:

$$\lim_{s\to\infty} s^{k+1} Res_k(s) = 0, \quad k = m, m+1, \ldots \tag{2.7}$$

where m is the number of initial conditions.

Theorem 2.3
If the exact solution $y(t)$ of the differential system is analytic in a neighborhood of $t = 0$, and the sequence of truncated approximate solutions $y_n(t) = \mathcal{L}^{-1}[Y_n(s)]$ is bounded in the defined domain, then the generated series $\sum_{k=0}^{\infty} c_k \frac{t^k}{k!}$ converges uniformly to the exact solution $y(t)$ as $n \to \infty$.

3. Analysis of the modified Laplace-Residual power series method

Here, we formulate the structural analysis and step-by-step implementation of the proposed hybrid Laplace-RPSM (LRPSM). To ensure mathematical generality, we consider a nonlinear system of initial value problems of order q, given by the following vector representation:

$$\mathbf{X}^{(q)}(t) = \mathbf{F}\left(t, \mathbf{X}(t), \mathbf{X}'(t), \ldots, \mathbf{X}^{(q-1)}(t)\right), \tag{3.1}$$

with the multi-state initial:

$$\mathbf{X}^{(k)}(0) = \mathbf{A}_k, \quad k = 0, 1, \ldots, q-1 \tag{3.2}$$

where $t \in [0, t_0]$ and $\mathbf{X}(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T$ is the state vector.
Here, $\mathbf{F} : [0, t_0] \times \mathbb{R}^{N \times q} \to \mathbb{R}^N$ represents a continuous, nonlinear operator governing the system dynamics, and $\mathbf{A}_k \in \mathbb{R}^N$ are real finite constant vectors.

The systematic procedure for determining the semi-analytical solution using the proposed technique is established as follow:
We initialize the methodology by applying the Laplace transform to the differential system (3.1). By using the differenciation properties of Laplace transform, we obtain:

$$s^q \mathbf{X}(s) - \sum_{k=0}^{q-1} s^{q-k-1} \mathbf{X}^{(k)}(0) = \mathcal{L}\left\{\mathbf{F}\left(t, \mathbf{X}(t), \mathbf{X}'(t), \ldots, \mathbf{X}^{(q-1)}(t)\right)\right\}, \tag{3.3}$$

where $\mathbf{X}(s) = \mathcal{L}\{\mathbf{X}(t)\}$. Substituting (3.2) and dividing the entire equation by s^q, the system is algebraically rearranged into the form:

$$\mathbf{X}(s) = \sum_{k=0}^{q-1} \frac{\mathbf{A}_k}{s^{k+1}} + \frac{1}{s^q} \mathcal{L}\left\{\mathbf{F}\left(t, \mathbf{X}(t), \mathbf{X}'(t), \ldots, \mathbf{X}^{(q-1)}(t)\right)\right\}. \tag{3.4}$$

According to Theorem 2.1, we construct the solution in the s-domain as an inverse power series:

$$\mathbf{X}(s) = \sum_{n=0}^{\infty} \frac{\mathbf{C}_n}{s^{n+1}}, \quad s > \alpha. \tag{3.5}$$

Because the first q coefficients correspond directly to the initial conditions ($\mathbf{C}_n = \mathbf{A}_n$ for $n = 0, 1, \ldots, q-1$), the series can be partitioned as follows:

$$\mathbf{X}(s) = \sum_{n=0}^{q-1} \frac{\mathbf{A}_n}{s^{n+1}} + \sum_{n=q}^{\infty} \frac{\mathbf{C}_n}{s^{n+1}}. \tag{3.6}$$

To approximate the solution computationally, we define the k-th truncated series solution, denoted as $\mathbf{X}_k(s)$, for $k \geq q$:

$$\mathbf{X}_k(s) = \sum_{n=0}^{q-1} \frac{\mathbf{A}_n}{s^{n+1}} + \sum_{n=q}^{k} \frac{\mathbf{C}_n}{s^{n+1}}. \tag{3.7}$$

Subsequently, we define the Laplace residual vector function $\mathbf{Res}(s)$ by substituting the exact transformation back into the algebraically isolated system. The corresponding k-th truncated Laplace residual function $\mathbf{Res}_k(s)$ evaluates the exact error at the k-th iteration:

$$\mathbf{Res}_k(s) = \mathbf{X}_k(s) - \sum_{n=0}^{q-1} \frac{\mathbf{A}_n}{s^{n+1}} - \frac{1}{s^q} \mathcal{L}\left\{\mathbf{F}\left(t, \mathbf{X}_k(t), \mathbf{X}_k'(t), \ldots, \mathbf{X}_k^{(q-1)}(t)\right)\right\}. \tag{3.8}$$

It is crucial to note the computational treatment of the nonlinear operator $\mathbf{F}$. While the formal notation implies a transition back to the time domain, the LRPSM algorithm does not perform continuous integral evaluations. Instead, by substituting the truncated algebraic series into the nonlinear terms, the operations reduce strictly to discrete Cauchy products of the constant series coefficients. This ensures that nonlinearities are handled algebraically within the s-domain during the subsequent limit evaluations.

The fundamental condition of the residual power series dictates that $\mathbf{Res}(s) = \mathbf{0}$, and consequently, $\lim_{k\to\infty} \mathbf{Res}_k(s) = \mathbf{Res}(s) = \mathbf{0}$ for all $s > \alpha$. To recursively extract the unknown constant vectors $\mathbf{C}_k$, we multiply (3.8) by s^{k+1} and apply the residual limit mechanism established in Theorem 2.2:

$$\lim_{s\to\infty} s^{k+1}\mathbf{Res}_k(s) = \mathbf{0}, \quad k = q, q+1, q+2, \dots \tag{3.9}$$

The application of (3.9) systematically annihilates all terms of lower asymptotic order, leaving an explicit algebraic equation for the coefficient $\mathbf{C}_k$.

After recursively computing the coefficients up to the desired truncation order M, the approximate analytical solution in the original temporal domain is retrieved by taking the inverse Laplace transform linearly to the truncated series:

$$\mathbf{x}_M(t) = \mathcal{L}^{-1}\left\{\mathbf{X}_M(s)\right\} = \sum_{n=0}^{M} \mathbf{C}_n \frac{t^n}{n!}. \tag{3.10}$$

This formulation highlights a critical computational advantage of the LRPSM. As previously noted, for strictly integer-order differential equations, the unique series generated by this method is mathematically equivalent to the classical Maclaurin/Taylor series expansion. However, the proposed Laplace embedding offers a profound algorithmic superiority. It completely bypasses the exhaustive requirement of analytically computing n-th order derivatives of highly coupled nonlinear systems. By projecting the residual functions into the s-domain, the method seamlessly replaces complex differentiation steps with straightforward, systematic limit evaluations at infinity, vastly simplifying the computational implementation for chaotic dynamic systems.

Theorem 3.1. Let $\mathbf{X}(t)$ be the exact solution vector to the nonlinear system described in (3.1), and assume $\mathbf{X}(t)$ is analytic in a neighborhood of $t = 0$. If the sequence of approximate residual coefficient vectors $\|\mathbf{C}_n\|$ is bounded by a constant $M > 0$ such that $\|\mathbf{C}_n\| \le M\lambda^n$ for some $0 < \lambda < 1$, then the multidimensional Laplace-Residual Power Series $\mathbf{x}_M(t) = \sum_{n=0}^{M} \mathbf{C}_n \frac{t^n}{n!}$ converges uniformly to the exact state vector $\mathbf{X}(t)$ as $M \to \infty$.

Proof:
By the hypothesis, the norm of the sequential coefficient vectors derived from the Laplace residual limits is bounded by a geometric decay sequence. Applying the multidimensional Weierstrass M-test to the matrix norm of the series yields $\sum_{n=0}^{\infty} \|\mathbf{C}_n\| \frac{t^n}{n!} \le \sum_{n=0}^{\infty} M \frac{(\lambda t)^n}{n!} = Me^{\lambda t}$. Since the exponential majorant converges for all $t \in \mathbb{R}$, the sequence of vector functions $\mathbf{x}_M(t)$ converges absolutely and

uniformly to the exact analytical vector solution $\mathbf{X}(t)$ on the defined temporal domain $[0, t_0]$.

4. Illustrative applications and discussions

To validate the theoretical framework and computational efficacy of the proposed Laplace-Residual power series method, this section presents its application to three distinct dynamical models: a coupled linear system, the nonlinear biological SIR epidemic model, and the highly chaotic Genesio-Tesi system. For the nonlinear applications, the high-precision semi-analytical polynomials derived via the LRPSM are rigorously evaluated against numerical integrations obtained using the standard fourth-order Runge-Kutta (RK45) method. The performance of the proposed hybrid algorithm is subsequently assessed through quantitative absolute error analyses and multidimensional graphical trajectory mappings. These comparisons are designed to verify the method's accuracy, its rapid convergence, and its capacity to capture highly sensitive nonlinear dynamics without the need for domain discretization.

4.1. Problem 1: A Coupled linear system.

First, we consider the following system of linear ordinary differential equations:

$$\begin{cases} x'(t) + x(t) = 0 \\ y'(t) - x(t) = 1 \end{cases} \tag{4.1}$$

With:

$$x(0) = 1, \quad y(0) = 1 \tag{4.2}$$

Solution procedure: Applying the Laplace transform to both sides of (4.1) and (4.2), we get the system in the s-domain:

$$\begin{cases} sX(s) - 1 = -X(s) \\ sY(s) - 1 = X(s) + \frac{1}{s}, \end{cases} \tag{4.3}$$

where $X(s) = \mathcal{L}\{x(t)\}$ and $Y(s) = \mathcal{L}\{y(t)\}$. Rearranging the terms, we establish the algebraic formulation:

$$X(s) = \frac{1}{s} - \frac{1}{s}X(s), \quad Y(s) = \frac{1}{s} + \frac{1}{s}X(s) + \frac{1}{s^2} \tag{4.4}$$

Assuming the series solutions $X(s) = \frac{1}{s} + \sum_{i=1}^{\infty} \frac{c_{1i}}{s^{i+1}}$ and $Y(s) = \frac{1}{s} + \sum_{i=1}^{\infty} \frac{c_{2i}}{s^{i+1}}$, we define the m-th truncated Laplace residual functions as:

$$\begin{cases} Res_{x,m}(s) = X_m(s) - \frac{1}{s} + \frac{1}{s}X_m(s) \\ Res_{y,m}(s) = Y_m(s) - \frac{1}{s} - \frac{1}{s}X_m(s) - \frac{1}{s^2} \end{cases} \tag{4.5}$$

For the first iteration ($m = 1$), substituting the 1st-truncated series yields:

$$Res_{x,1}(s) = \frac{c_{11}}{s^2} + \frac{1}{s^2} + \frac{c_{11}}{s^3}, \quad Res_{y,1}(s) = \frac{c_{21}}{s^2} - \frac{2}{s^2} - \frac{c_{11}}{s^3} \tag{4.6}$$

Multiplying by s^2 and taking the limit as $s \to \infty$ gives $c_{11} + 1 = 0 \implies c_{11} = -1$ and $c_{21} - 2 = 0 \implies c_{21} = 2$. Continuing this recursive limit process for higher terms ($m \geq 2$), we systematically deduce the coefficients: $c_{1m} = (-1)^m$ and

$c_{2m} = (-1)^{m-1}$ for $m \geq 1$. Consequently, the exact series in the Laplace domain are:

$$X(s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{s^{n+1}}, \quad Y(s) = \frac{1}{s} + \frac{2}{s^2} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{s^{n+1}}. \tag{4.7}$$

Applying $\mathcal{L}^{-1}$ term-by-term, we recover the closed-form exact solution:

$$x(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^n}{n!} = e^{-t}, \quad y(t) = 2 + t - \sum_{n=0}^{\infty} \frac{(-1)^n t^n}{n!} = 2 + t - e^{-t}. \tag{4.8}$$

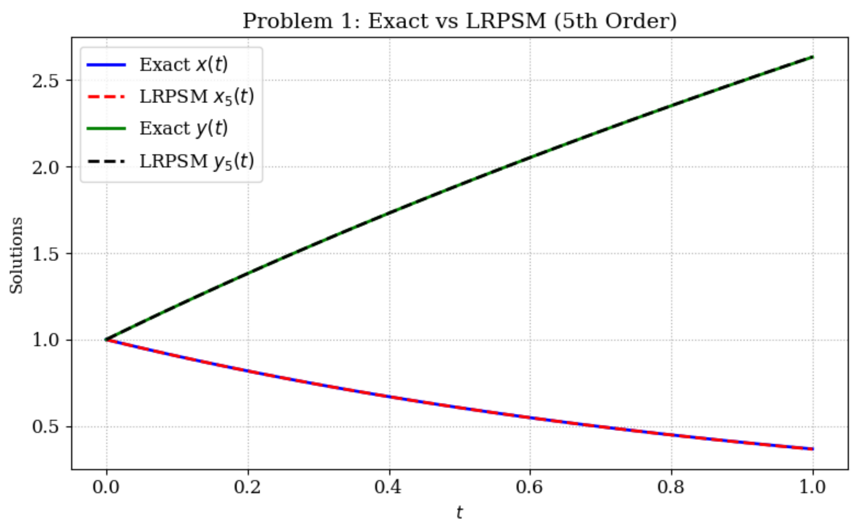

FIGURE 1. Comparison of the exact analytical solutions and the 5th-order LRPSM approximations for the state variables $x(t)$ and $y(t)$ in the coupled linear system.

4.2. **Problem 2: The nonlinear SIR epidemic model.** Next, we consider the highly nonlinear Susceptible-Infected-Recovered (SIR) epidemic model:

$$\begin{cases} S'(t) = -\beta S(t)I(t) \\ I'(t) = \beta S(t)I(t) - \gamma I(t) \\ R'(t) = \gamma I(t) \end{cases} \tag{4.9}$$

Under the initial population sizes $S(0) = N_S, I(0) = N_I, R(0) = N_R$, where β and γ are positive transmission and recovery rates.

Solution procedure: Let $\bar{S}(s), \bar{I}(s), \bar{R}(s)$ denote the Laplace transforms of the respective state variables. Applying the transform to the nonlinear system yields:

$$\begin{cases} \bar{S}(s) = \frac{N_S}{s} - \frac{\beta}{s}\mathcal{L}\left\{\mathcal{L}^{-1}\{\bar{S}(s)\} \cdot \mathcal{L}^{-1}\{\bar{I}(s)\}\right\} \\ \bar{I}(s) = \frac{N_I}{s} + \frac{\beta}{s}\mathcal{L}\left\{\mathcal{L}^{-1}\{\bar{S}(s)\} \cdot \mathcal{L}^{-1}\{\bar{I}(s)\}\right\} - \frac{\gamma}{s}\bar{I}(s) \\ \bar{R}(s) = \frac{N_R}{s} + \frac{\gamma}{s}\bar{I}(s) \end{cases} \tag{4.10}$$

Although the nonlinear cross-transmission terms $(\bar{S} \cdot \bar{I})$ are formally represented using nested inverse and forward Laplace transforms, their computational evaluation is purely algebraic. By assuming the fractional series expansions $\bar{S}(s) = \frac{N_S}{s} + \sum_{i=1}^{\infty} \frac{c_{1i}}{s^{i+1}}$, $\bar{I}(s) = \frac{N_I}{s} + \sum_{i=1}^{\infty} \frac{c_{2i}}{s^{i+1}}$, and $\bar{R}(s) = \frac{N_R}{s} + \sum_{i=1}^{\infty} \frac{c_{3i}}{s^{i+1}}$, the continuous convolution integrals collapse into discrete Cauchy products of the coefficients.

Constructing the 1st-truncated Laplace residual functions and multiplying by s^2, we evaluate the algebraic limits at infinity:

$$\begin{cases} \lim_{s\to\infty} s^2 Res_{S,1}(s) = c_{11} + \beta N_S N_I = 0 \implies c_{11} = -\beta N_S N_I \\ \lim_{s\to\infty} s^2 Res_{I,1}(s) = c_{21} - \beta N_S N_I + \gamma N_I = 0 \implies c_{21} = \beta N_S N_I - \gamma N_I \\ \lim_{s\to\infty} s^2 Res_{R,1}(s) = c_{31} - \gamma N_I = 0 \implies c_{31} = \gamma N_I \end{cases} \tag{4.11}$$

For the 2nd iteration, the algebraic limits yield:

$$\begin{cases} c_{12} = \beta^2 N_S N_I^2 - \beta N_S(\beta N_S N_I - \gamma N_I) \\ c_{22} = \beta N_S(\beta N_S N_I - \gamma N_I) - \beta^2 N_S N_I^2 - \gamma\beta N_S N_I + \gamma^2 N_I \\ c_{32} = \gamma\beta N_S N_I - \gamma^2 N_I \end{cases} \tag{4.12}$$

Numerical simulation: For validation, we adopt the parameters $N_S = 499, N_I = 1, N_R = 1$, with $\beta = 0.001$ and $\gamma = 0.1$. Iterating the LRPSM algorithm up to the 10th order ($m = 10$) and applying the inverse Laplace transform, we acquire the high-precision temporal polynomials:

$$\begin{aligned} S(t) = 499 &- 0.499t - 0.099301t^2 - 0.013092t^3 - 1.281 \times 10^{-3} t^4 \\ &- 9.784 \times 10^{-5} t^5 - 5.908 \times 10^{-6} t^6 - 2.687 \times 10^{-7} t^7 \\ &- 6.753 \times 10^{-9} t^8 + 2.645 \times 10^{-10} t^9 + 5.226 \times 10^{-11} t^{10} \end{aligned} \tag{4.13}$$

$$\begin{aligned} I(t) = 1 &+ 0.399t + 0.079351t^2 + 0.010454t^3 + 9.789 \times 10^{-4} t^4 \\ &+ 7.745 \times 10^{-5} t^5 + 4.618 \times 10^{-6} t^6 + 2.027 \times 10^{-7} t^7 \\ &+ 4.219 \times 10^{-9} t^8 - 3.114 \times 10^{-10} t^9 - 4.915 \times 10^{-11} t^{10} \end{aligned} \tag{4.14}$$

$$\begin{aligned} R(t) = 1 &+ 0.1t + 0.01995t^2 + 2.645 \times 10^{-3} t^3 + 2.613 \times 10^{-4} t^4 \\ &+ 2.039 \times 10^{-5} t^5 + 1.290 \times 10^{-6} t^6 + 6.597 \times 10^{-8} t^7 \\ &+ 2.534 \times 10^{-9} t^8 + 4.688 \times 10^{-11} t^9 - 3.114 \times 10^{-12} t^{10} \end{aligned} \tag{4.15}$$

TABLE 1. Absolute errors between the numerical RK45 method and the 10th-order LRPSM approximation for the nonlinear SIR epidemic model.

Time (t)	$\lvert \mathbf{S_{RK4}} - \mathbf{S_{10}} \rvert$	$\lvert \mathbf{I_{RK4}} - \mathbf{I_{10}} \rvert$	$\lvert \mathbf{R_{RK4}} - \mathbf{R_{10}} \rvert$
0.2	5.8130×10^{-8}	6.7053×10^{-8}	3.5694×10^{-10}
0.4	4.6618×10^{-7}	1.0591×10^{-6}	3.6019×10^{-9}
0.6	1.5774×10^{-6}	5.3383×10^{-6}	1.4777×10^{-8}
0.8	3.7489×10^{-6}	1.6835×10^{-5}	4.1490×10^{-8}
1.0	7.3424×10^{-6}	4.1048×10^{-5}	9.4198×10^{-8}

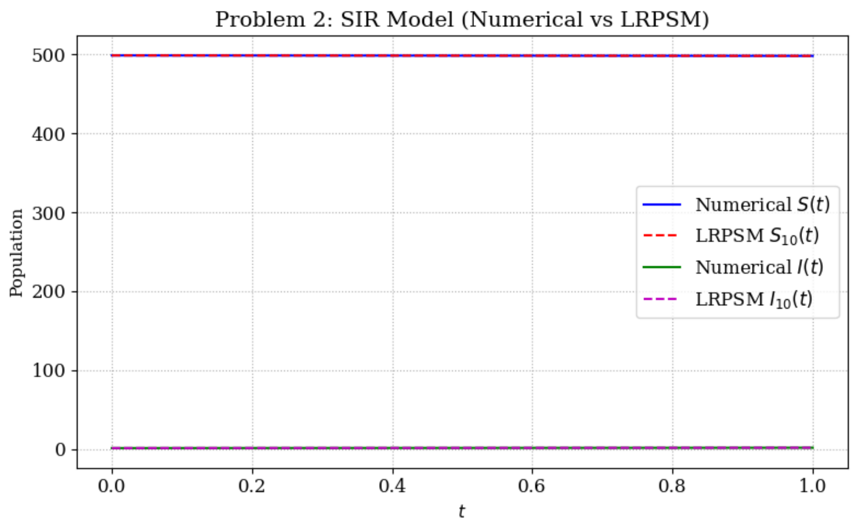

FIGURE 2. Time evolution of the Susceptible $S(t)$ and Infected $I(t)$ populations in the nonlinear SIR epidemic model, illustrating the excellent agreement between the high-precision numerical RK45 method and the 10th-order LRPSM approximations.

4.3. **Problem 3: The nonlinear Genesio-Tesi chaotic system.** Finally, we implement the method on the highly dynamic Genesio-Tesi system, defined by:

$$\begin{cases} x'(t) = y(t) \\ y'(t) = z(t) \\ z'(t) = -cx(t) - by(t) - az(t) + x^2(t) \end{cases} \tag{4.16}$$

With initial constraints $x(0) = G_x, y(0) = G_y, z(0) = G_z$. Here, $a, b, c > 0$ and satisfy $ab < c$ for chaotic behavior.

Solution procedure: Transforming into the Laplace domain, the system is expressed as:

$$\begin{cases} X(s) = \frac{G_x}{s} + \frac{1}{s}Y(s) \\ Y(s) = \frac{G_y}{s} + \frac{1}{s}Z(s) \\ Z(s) = \frac{G_z}{s} - \frac{c}{s}X(s) - \frac{b}{s}Y(s) - \frac{a}{s}Z(s) + \frac{1}{s}\mathcal{L}\left\{(\mathcal{L}^{-1}\{X(s)\})^2\right\} \end{cases} \tag{4.17}$$

Similarly to the epidemic model, the nonlinear squared term $\mathcal{L}\{(\mathcal{L}^{-1}\{X(s)\})^2\}$ is computed algebraically without resorting to time-domain integration. By substituting the truncated series for $X(s)$, this term simplifies to the self-Cauchy product of its series coefficients. Evaluating the truncated Laplace residual functions iteratively yields:

$$\lim_{s\to\infty} s^2 Res_1(s) = \mathbf{0} \implies c_{11} = G_y, \quad c_{21} = G_z, \quad c_{31} = G_x^2 - cG_x - bG_y - aG_z \tag{4.18}$$

For the second order, evaluating the convolution limits gives:

$$c_{12} = G_z, \quad c_{22} = G_x^2 - cG_x - bG_y - aG_z, \quad c_{32} = -cG_y - bG_z - ac_{31} - 2G_xG_y \tag{4.19}$$

Numerical simulation: We define the system parameters as $a = 1.2, b = 2.92, c = 6$ with initial states $x(0) = 0.2, y(0) = -0.3, z(0) = 0.1$. Progressing the recursive algorithm to the 10th order, the inverse transformation produces the following precise analytical approximations for the chaotic attractors:

$$\begin{aligned} x(t) = {} & 0.2 - 0.3t + 0.05t^2 - 0.06733t^3 + 0.07803t^4 \\ & - 0.01206t^5 - 2.290 \times 10^{-3}t^6 - 6.453 \times 10^{-4}t^7 \\ & + 2.578 \times 10^{-4}t^8 + 5.607 \times 10^{-5}t^9 - 2.443 \times 10^{-5}t^{10} \end{aligned} \tag{4.20}$$

$$\begin{aligned} y(t) = {} & -0.3 + 0.1t - 0.202t^2 + 0.3121t^3 - 0.06032t^4 \\ & - 0.01374t^5 - 4.517 \times 10^{-3}t^6 + 2.063 \times 10^{-3}t^7 \\ & + 5.046 \times 10^{-4}t^8 - 2.443 \times 10^{-4}t^9 + 8.437 \times 10^{-5}t^{10} \end{aligned} \tag{4.21}$$

$$\begin{aligned} z(t) = {} & 0.1 - 0.404t + 0.9364t^2 - 0.24128t^3 - 0.06870t^4 \\ & - 0.02710t^5 + 0.01444t^6 + 4.037 \times 10^{-3}t^7 \\ & - 2.199 \times 10^{-3}t^8 + 8.437 \times 10^{-4}t^9 - 2.307 \times 10^{-4}t^{10} \end{aligned} \tag{4.22}$$

These robust numerical expansions demonstrate that the LRPSM perfectly handles complex coupled biological and physical nonlinearities using simple, direct algebraic operations in the s-domain.

TABLE 2. Absolute errors between the numerical RK45 method and the 10th-order LRPSM approximation for the Genesio-Tesi chaotic system.

Time (t)	$\lvert \mathbf{x_{RK4}} - \mathbf{x_{10}} \rvert$	$\lvert \mathbf{y_{RK4}} - \mathbf{y_{10}} \rvert$	$\lvert \mathbf{z_{RK4}} - \mathbf{z_{10}} \rvert$
0.2	2.2627×10^{-8}	2.6625×10^{-7}	1.0826×10^{-8}
0.4	1.6945×10^{-7}	2.1197×10^{-6}	1.7311×10^{-7}
0.6	5.8290×10^{-7}	7.0295×10^{-6}	8.9875×10^{-7}
0.8	1.1525×10^{-6}	1.4842×10^{-5}	3.3029×10^{-6}
1.0	1.7764×10^{-6}	1.0776×10^{-5}	1.0936×10^{-5}

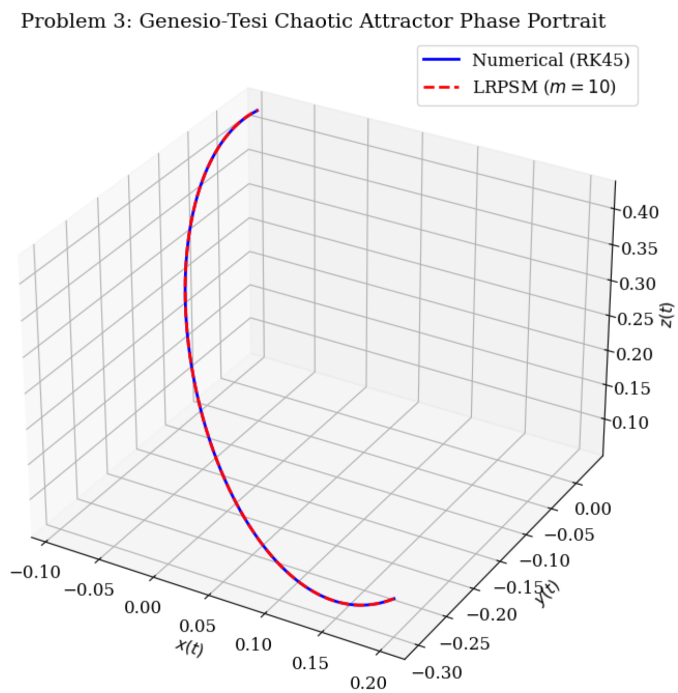

FIGURE 3. 3D phase portrait of the Genesio-Tesi chaotic attractor. The plot visually confirms that the 10th-order LRPSM continuous analytical approximation perfectly captures the highly sensitive, non-periodic spiraling trajectories of the numerical RK45 integration.

4.4. **Discussion of numerical results.** To rigorously assess the validity and precision of the proposed Laplace-Residual Power Series Method, the derived 10th-order semi-analytical solutions were compared against high-fidelity numerical integrations achieved via the 4th-order Runge-Kutta method (RK45).

Table 1 and Table 2 document the absolute truncation errors computed at various time steps $t \in [0, 1]$ for the nonlinear SIR epidemic model and the Genesio-Tesi chaotic system, respectively. The data reveals that the LRPSM maintains exceptional accuracy, with maximum absolute errors bounded between 10^{-5} and 10^{-8} even as time progresses. Such narrow error margins demonstrate the method's strong convergence profile and its capacity to suppress the accumulation of truncation errors typically observed in standard stepwise numerical algorithms.

Furthermore, visual corroboration is provided through the plotted trajectories. The 2D graphical representation of the SIR population dynamics confirms a flawless overlap between the continuous LRPSM polynomials and the discrete numerical data points. More importantly, the 3D phase portrait generated for the Genesio-Tesi system highlights the structural superiority of the method. The 10th-order LRPSM continuous approximations successfully trace the highly sensitive, non-periodic spiraling trajectories that characterize chaotic attractors.

This confirms that the hybridization of the Laplace transform with the residual error mechanism allows for the exact algebraic handling of complex multi-variable nonlinearities (e.g., $S(t)I(t)$ and $x^2(t)$) without resorting to domain discretization or restrictive linearization techniques. Ultimately, these findings validate the LRPSM as an extremely reliable, rapidly convergent, and computationally lightweight framework for solving highly coupled nonlinear dynamic systems.

5. Conclusion

In this paper, the Laplace-Residual Power Series Method was successfully developed and systematically applied to solve strongly coupled systems of linear and nonlinear differential equations. While relying on the reliable, foundational properties of the Laplace transform and power series expansions, the proposed algorithmic framework engineers these tools to completely bypass the traditional computational bottlenecks of high-order analytical differentiation and iterative numerical time-stepping [12, 18].

To establish the mathematical rigor of the technique, a formal convergence theorem was introduced, proving the uniform and absolute convergence of the multidimensional series under defined bounding conditions. The efficacy and computational efficiency of the LRPSM were subsequently validated through its application to a coupled linear system, the nonlinear SIR epidemic model, and the highly chaotic Genesio-Tesi system.

A rigorous comparative analysis against the Runge-Kutta (RK45) numerical method demonstrated the exceptional accuracy of the derived semi-analytical polynomials, with absolute truncation errors strictly bounded between 10^{-5} and 10^{-8}. Furthermore, 2D trajectory plots and 3D phase portraits visually confirmed that the LRPSM perfectly captures complex, non-periodic chaotic dynamics without succumbing to the accumulation errors inherent in standard discrete numerical algorithms. Consequently, this study establishes the LRPSM not only as computationally elegant and structurally transparent but also as a theoretically justified, highly accurate mathematical framework for analyzing complex dynamic models in epidemiology, physics, and engineering [3].

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