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ON $(\alpha, \beta)^*$-n-DERIVATIONS AND CERTAIN n-ADDITIVE MAPPINGS IN RINGS WITH INVOLUTION

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Abstract. Let R be an associative ring with involution '$*$'. In this paper we introduce the notion of $(\alpha, \beta)^*$-n-derivation in R, where α and β are endomorphisms of R. An additive mapping $x \mapsto x^*$ of R into itself is called an involution on R if it satisfies the conditions; (i) $(x^*)^* = x$, (ii) $(xy)^* = y^*x^*$ for all $x, y \in R$. A ring R equipped with an involution '$*$' is called a $*$-ring. In the present paper it is shown that if a $*$-prime ring R admits a nonzero $(\alpha, \beta)^*$-n-derivation D such that α is surjective, then R is commutative. Some properties of certain n-additive mappings are also discussed in the setting of $*$-prime rings and semiprime $*$-rings. Further, some related properties of $(\alpha, \beta)^*$-n-derivation in semiprime $*$-ring have also been investigated. Besides, we have also constructed several examples throughout the text to justify that various restrictions imposed in the hypotheses of our theorems are not superfluous. Finally a structure theorem for $(\alpha, \beta)^*$-n-derivation in a semiprime $*$-ring has been established.

1. Introduction and preliminaries

Throughout the paper, R will represent an associative ring with involution $*$ and center Z unless otherwise stated. In addition α and β will denote its endomorphisms throughout the text unless otherwise mentioned. Ring R is called prime if $aRb = \{0\}$ implies $a = 0$ or $b = 0$. It is called semiprime if $aRa = \{0\}$ implies $a = 0$. An additive mapping $d : R \longrightarrow R$ is said to be a derivation on R if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. An additive mapping $d : R \longrightarrow R$ is said to be a (α, β)-derivation on R if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$ holds for all $x, y \in R$. An additive mapping $T : R \longrightarrow R$ is called a left (resp. right) multiplier if $T(xy) = T(x)y$ (resp. $T(xy) = xT(y)$) holds for all $x, y \in R$. An additive mapping $x \mapsto x^*$ of R into itself is called an involution on R if it satisfies the conditions; (i) $(x^*)^* = x$, (ii) $(xy)^* = y^*x^*$ for all $x, y \in R$. A ring R equipped with an involution '$*$' is called a $*$-ring (for reference see [8]). A ring R with involution '$*$' is said to be $*$-prime if $aRb = aRb^* = \{0\}$, (equivalently $aRb = a^*Rb = \{0\}$) where $a, b \in R$ implies that either $a = 0$ or $b = 0$. It is to be noted that every prime ring having an involution '$*$' is $*$-prime but the converse

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is not true in general. Of course, if R^o denotes the opposite ring of a prime ring R, then $R \times R^o$ equipped with the exchange involution $*_{ex}$, defined by $*_{ex}(x, y) = (y, x)$, is $*_{ex}$-prime but not prime. It is also to be noted that every $*$-prime ring is a semiprime ring but converse is not true. For justification consider the ring $R = \mathbb{Z}_6 \times \mathbb{Z}_6$, equipped with the exchange involution $*_{ex}$, where $\mathbb{Z}_6$ is the ring of residue classes mod 6. Here we also have $(\bar{3}, \bar{3})R(\bar{2}, \bar{4}) = (\bar{3}, \bar{3})R(\bar{2}, \bar{4})^{*_{ex}} = (\bar{0}, \bar{0})$, however $(\bar{3}, \bar{3}) \neq (\bar{0}, \bar{0})$ and $(\bar{2}, \bar{4}) \neq (\bar{0}, \bar{0})$. All the arguments just discussed in the preceding lines show that R is a semiprime ring but not $*_{ex}$-prime.

An additive mapping $d : R \longrightarrow R$ is said to be a $*$-derivation on R if $d(xy) = d(x)y^* + xd(y)$ holds for all $x, y \in R$. If R is a commutative, then $d : R \longrightarrow R$ defined by $d(x) = a(x - x^*)$, where $a \in R$, is a $*$-derivation on R (for reference see [7]). Following [1], an additive map $T : R \longrightarrow R$ is called a left (resp. right) $*$-multiplier if $T(xy) = T(x)y^*$ (resp. $T(xy) = x^*T(y)$) holds for all $x, y \in R$. Very recently Ali and Khan [2] defined symmetric $*$-biderivation, symmetric left (resp. right) $*$-bimultiplier and studied some properties of prime $*$-rings and semiprime $*$-rings, admitting symmetric $*$-biderivations and symmetric left (resp. right) $*$-bimultipliers.

Throughout the onward discussion n will represent a positive integer. Motivated by the above concepts the authors [6] introduced the notion of $*$-n-derivations and $*$-n-multipliers in $*$-rings and studied these notions in the setting of prime $*$-rings and semiprime $*$-rings. An n-additive (i.e.; additive in each argument) mapping $D : R \times R \times \cdots \times R \longrightarrow R$ is called a $*$-n-derivation of R if the relations

$$D(x_1x_1', x_2, \cdots, x_n) = D(x_1, x_2, \cdots, x_n)(x_1')^* + x_1D(x_1', x_2, \cdots, x_n)$$
$$D(x_1, x_2x_2', \cdots, x_n) = D(x_1, x_2, \cdots, x_n)(x_2')^* + x_2D(x_1, x_2', \cdots, x_n)$$
$$\vdots$$
$$D(x_1, x_2, \cdots, x_nx_n') = D(x_1, x_2, \cdots, x_n)(x_n')^* + x_nD(x_1, x_2, \cdots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \cdots, x_n, x_n' \in R$.

In the year 2009, Ali & Fosner [1] introduced the notion of $(\alpha, \beta)^*$-derivation in R and extended the concept of $*$-derivation as follows: An additive mapping $d : R \longrightarrow R$ is said to be a $(\alpha, \beta)^*$-derivation of R if $d(xy) = d(x)\alpha(y^*) + \beta(x)d(y)$ holds for all $x, y \in R$. Authors studied this notion in the setting of prime $*$-rings and semiprime $*$-rings. Latter in the same year Ashraf & Ali [4] gave the notion of symmetric $(\alpha, \beta)^*$-biderivation as follows: A mapping $d : R \times R \longrightarrow R$ is said to be symmetric if $d(x, y) = d(y, x)$ for all $x, y \in R$. A mapping $d : R \times R \longrightarrow R$ is said to be biadditive if it is additive in both arguments. A symmetric biadditive map $d : R \times R \longrightarrow R$ is said to be a symmetric $(\alpha, \beta)^*$-biderivation if $d(xy, z) = d(x, z)\alpha(y^*) + \beta(x)d(y, z)$ holds for all $x, y, z \in R$. Authors studied these notions in the setting of prime $*$-rings and semiprime $*$-rings. Motivated by the above notions, we define the notion of $(\alpha, \beta)^*$-n-derivation in $*$-rings as following.

An n-additive (i.e.; additive in each argument) mapping $D : R \times R \times \cdots \times R \longrightarrow R$ is called a $(\alpha, \beta)^*$-*n-derivation* of R if the relations

$$D(x_1x_1', x_2, \cdots, x_n) = D(x_1, x_2, \cdots, x_n)\alpha({x_1'}^*) + \beta(x_1)D(x_1', x_2, \cdots, x_n)$$
$$D(x_1, x_2x_2', \cdots, x_n) = D(x_1, x_2, \cdots, x_n)\alpha({x_2'}^*) + \beta(x_2)D(x_1, x_2', \cdots, x_n)$$

$$\vdots$$

$$D(x_1,x_2,\cdots,x_nx_n^{'})=D(x_1,x_2,\cdots,x_n)\alpha(x_n^{'*})+\beta(x_n)D(x_1,x_2,\cdots,x_n^{'})$$

hold for all $x_1,x_1^{'},x_2,x_2^{'},\cdots,x_n,x_n^{'}\in R$. For an example of $(\alpha,\beta)^*$-n-derivation, consider $\mathbb{Z}$ the ring of integers and

$$R=\left\{\begin{pmatrix}0&x&y\\0&0&z\\0&0&0\end{pmatrix}\mid x,y,z,0\in\mathbb{Z}\right\}.$$

Define $\alpha,\beta:R\longrightarrow R$ as following,

$$\alpha\left(\begin{pmatrix}0&x&y\\0&0&z\\0&0&0\end{pmatrix}\right)=\begin{pmatrix}0&x&-y\\0&0&-z\\0&0&0\end{pmatrix},$$

$$\beta\left(\begin{pmatrix}0&x&y\\0&0&z\\0&0&0\end{pmatrix}\right)=\begin{pmatrix}0&0&z\\0&0&0\\0&0&0\end{pmatrix}.$$

Next define $D:R\times R\times\cdots\times R\longrightarrow R$ and $r\mapsto r^*$ of R into itself such that

$$D\left(\begin{pmatrix}0&x_1&y_1\\0&0&z_1\\0&0&0\end{pmatrix},\begin{pmatrix}0&x_2&y_2\\0&0&z_2\\0&0&0\end{pmatrix},\cdots,\begin{pmatrix}0&x_n&y_n\\0&0&z_n\\0&0&0\end{pmatrix}\right)=$$
$$\begin{pmatrix}0&0&0\\0&0&z_1z_2\cdots z_n\\0&0&0\end{pmatrix},$$

and

$$\begin{pmatrix}0&x&y\\0&0&z\\0&0&0\end{pmatrix}^*=\begin{pmatrix}0&z&y\\0&0&x\\0&0&0\end{pmatrix}.$$

One can easily verify that '$*$' is an involution on ring R. Also it is straightforward to check that α,β are endomorphisms of R and D is a nonzero $(\alpha,\beta)^*$-n-derivation of R.

An additive map $T:R\longrightarrow R$ is called a left (resp. right) α^*-multiplier if $T(xy)=T(x)\alpha(y^*)$ (resp. $T(xy)=\alpha(x^*)T(y)$) holds for all $x,y\in R$. If T is both a left α^*-multiplier and a right α^*-multiplier, then T is called a α^*-multiplier. Recently Ashraf and Ali [4] extended the above notions in the following direction: A symmetric biadditive map $T:R\times R\longrightarrow R$ is said to be a symmetric left (resp. right) α^*-bimultiplier if $T(xy,z)=T(x,z)\alpha(y^*)$ (resp. $T(xy,z)=\alpha(x^*)T(y,z)$ holds for all $x,y,z\in R$. If T is both a left symmetric α^*-bimultiplier as well as a symmetric right α^*-bimultiplier, then T is called a symmetric α^*-bimultiplier. Authors studied these notions in the setting of prime $*$-rings and semiprime $*$-rings. Motivated by above ones and the notion of $*$-n-multipliers studied by us [6], we introduce the notion of α^*-n-multipliers as follows:

An n-additive (i.e.; additive in each argument) mapping $T : R\times R\times\cdots\times R \longrightarrow R$ is called a *left α^*-n-multiplier* of R if the relations

$$\begin{aligned}
T(x_1x_1^{'}, x_2, \cdots, x_n) &= T(x_1, x_2, \cdots, x_n)\alpha(x_1^{'*})\\
T(x_1, x_2x_2^{'}, \cdots, x_n) &= T(x_1, x_2, \cdots, x_n)\alpha(x_2^{'*})\\
&\vdots\\
T(x_1, x_2, \cdots, x_nx_n^{'}) &= T(x_1, x_2, \cdots, x_n)\alpha(x_n^{'*})
\end{aligned}$$

hold for all $x_1, x_1^{'}, x_2, x_2^{'}, \cdots, x_n, x_n^{'} \in R$. An n-additive (i.e.; additive in each argument) mapping $T : R \times R \times \cdots \times R \longrightarrow R$ is called a *right α^*-n-multiplier* of R if the relations

$$\begin{aligned}
T(x_1x_1^{'}, x_2, \cdots, x_n) &= \alpha(x_1{}^*)T(x_1^{'}, x_2, \cdots, x_n)\\
T(x_1, x_2x_2^{'}, \cdots, x_n) &= \alpha(x_2{}^*)T(x_1, x_2^{'}, \cdots, x_n)\\
&\vdots\\
T(x_1, x_2, \cdots, x_nx_n^{'}) &= \alpha(x_n{}^*)T(x_1, x_2, \cdots, x_n^{'})
\end{aligned}$$

hold for all $x_1, x_1^{'}, x_2, x_2^{'}, \cdots, x_n, x_n^{'} \in R$. Finally an n-additive (i.e.; additive in each argument) mapping $T : R\times R\times\cdots\times R \longrightarrow R$ is called a *α^*-n-multiplier* of R if it is both a left α^*-n-multiplier as well as a right α^*-n-multiplier of R. For an example of left α^*-n-multiplier, consider $\mathbb{Z}$ the ring of integers and

$$R = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} \mid x, y, z, 0 \in \mathbb{Z} \right\}.$$

Define $\alpha : R \longrightarrow R$ as following.

$$\alpha\left(\begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x & -y \\ 0 & 0 & -z \\ 0 & 0 & 0 \end{pmatrix}.$$

Next define $T : R \times R \times \cdots \times R \longrightarrow R$ and $r \mapsto r^*$ of R into itself such that

$$T\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_1x_2\cdots x_n \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$\begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & z & y \\ 0 & 0 & x \\ 0 & 0 & 0 \end{pmatrix}.$$

One can easily verify that '$*$' is an involution on ring R. Also it is straightforward to check that α is endomorphism of R and T is a nonzero left α^*-n-multiplier of R but not a right α^*-n-multiplier of R. For an example of right α^*-n-multiplier, consider the $*$-ring R and α just discussed above. Now define a map $T : R \times R \times \cdots \times R \longrightarrow R$ such that

$$T\left(\left(\begin{array}{ccc} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{array}\right), \left(\begin{array}{ccc} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{array}\right), \cdots, \left(\begin{array}{ccc} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{array}\right)\right) =$$

$$\left(\begin{array}{ccc} 0 & x_1x_2\cdots x_n & z_1z_2\cdots z_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right).$$

It can be easily verified that T is a nonzero right α^*-n-multiplier of R but not a left α^*-n-multiplier of R. For an example of α^*-n-multiplier, let Q and $\mathbb{R}[x]$ be the ring of real quaternions and the polynomial ring in x over the ring $\mathbb{R}$ of real numbers respectively. Assume $R = Q \times \mathbb{R}[x]$, the ring of cartesian product of Q and $\mathbb{R}[x]$ with regard to componentwise addition and multiplication. Let $*_1$, $*_2$ and $*$ denote the involutions of rings Q, $\mathbb{R}[x]$ and R respectively, defined by $q^{*_1} = \alpha - \beta i - \gamma j - \delta k$, where $q = \alpha + \beta i + \gamma j + \delta k \in Q$; $f(x)^{*_2} = f(x)$, where $f(x) \in \mathbb{R}[x]$ and $(q, f(x))^* = (q^{*_1}, f(x)^{*_2})$ for all $(q, f(x)) \in R$. Let us define $\alpha : R \longrightarrow R$ as $\alpha(q, f(x)) = (q, f(-x))$. Next define $T : R \times R \times \cdots \times R \longrightarrow R$ such that $T((q_1, f_1(x)), (q_2, f_2(x)), \cdots, (q_n, f_n(x))) = (0, f_1(-x)f_2(-x)\cdots f_n(-x))$. It can be easily verified T is a nonzero α^*-n-multiplier of R, where α is an onto endomorphism. The above three notions of α^*-n-multipliers are of independent interest in any arbitrary $*$-ring. Let R be a commutative $*$-ring having a nonzero left α^*-n-multiplier T. Now consider $T(x_1y, x_2, \cdots, x_n) = T(yx_1, x_2, \cdots, x_n) = T(y, x_2, \cdots, x_n)\alpha(x_1^*) = \alpha(x_1^*)T(y, x_2, \cdots, x_n)$. It is obvious to observe that T is also a nonzero right α^*-n-multiplier of R i.e.; both types of α^*-n-multipliers coincide in a commutative $*$-ring. In the next section it is shown that if a $*$-prime ring R admits a nonzero α^*-n-multiplier of either type, then R is commutative. This shows that in a $*$-prime ring nonzero α^*-n-multipliers of both types coincide. The objective of the present paper is to establish the analogues of some results earlier obtained in [2, 3, 4, 5, 6, 7] for $(\alpha, \beta)^*$-n-derivation and certain n-additive mappings even in the setting of $*$-prime rings. It is shown that if a $*$-prime ring R admits a nonzero $(\alpha, \beta)^*$-n-derivation D such that α is surjective, then R is commutative. Further, some related properties of $(\alpha, \beta)^*$-n-derivation in semiprime $*$-ring have also been investigated. Besides, we have also constructed several examples throughout the text to justify that various restrictions imposed in the hypotheses of our theorems are not redundant. In the end a structure theorem of $(\alpha, \beta)^*$-n-derivation in a semiprime $*$-ring has been also obtained. In fact, our results generalize, improve, extend and compliment several results obtained earlier on $(\alpha, \beta)^*$-derivation, symmetric $(\alpha, \beta)^*$-bi-derivation, $*$-n-derivation, $*$-multiplier and certain bi and n-additive mappings. For example Theorems 2.1, 3.1 of [4], Theorems 3.1−3.4 of [5], Theorems 3.1−3.3, 3.6−3.8 of [6], Proposition 1 of [7] etc.- to mention a few only.

2. Main results

In the year 2009 the first author with Ali [4, Corollary 2.4] proved that if a prime $*$-ring R admits a nonzero $(\alpha,\beta)^*$-derivation d such that α is onto, then R is commutative. Further, in the same paper [4, Corollary 3.4] its analogue for symmetric $(\alpha,\beta)^*$-bi-derivation in the setting of prime $*$-rings was also established. We have proved that the restriction of symmetry of $(\alpha,\beta)^*$-bi-derivation used by the authors is redundant. It has also been shown that these results hold even in $*$-prime rings. In fact, in the setting of $*$-prime rings, we obtained the following result for a nonzero $(\alpha,\beta)^*$-n-derivation D, where α is onto.

Theorem 2.1. *Let R be a $*$-prime ring. If it admits a nonzero $(\alpha,\beta)^*$-n-derivation D such that α is surjective then R is commutative.*

Proof. By hypothesis we have, for all $x_1, y, z, x_2, \cdots, x_n \in R$

$$\begin{aligned}
D((x_1y)z, x_2, \cdots, x_n) &= D(x_1y, x_2, \cdots, x_n)\alpha(z^*) + \beta(x_1y)D(z, x_2, \cdots, x_n) \\
&= \{D(x_1, x_2, \cdots, x_n)\alpha(y^*) + \beta(x_1)D(y, x_2, \cdots, x_n)\} \\
&\quad \alpha(z^*) + \beta(x_1)\beta(y)D(z, x_2, \cdots, x_n) \\
&= D(x_1, x_2, \cdots, x_n)\alpha(y^*)\alpha(z^*) \\
&\quad +\beta(x_1)D(y, x_2, \cdots, x_n)\alpha(z^*) \\
&\quad +\beta(x_1)\beta(y)D(z, x_2, \cdots, x_n).
\end{aligned}$$

Also

$$\begin{aligned}
D(x_1(yz), x_2, \cdots, x_n) &= D(x_1, x_2, \cdots, x_n)\alpha((yz)^*) + \beta(x_1)D(yz, x_2, \cdots, x_n) \\
&= D(x_1, x_2, \cdots, x_n)\alpha(z^*y^*) + \beta(x_1)\{D(y, x_2, \cdots, x_n) \\
&\quad \alpha(z^*) + \beta(y)D(z, x_2, \cdots, x_n)\} \\
&= D(x_1, x_2, \cdots, x_n)\alpha(z^*)\alpha(y^*) \\
&\quad +\beta(x_1)D(y, x_2, \cdots, x_n)\alpha(z^*) \\
&\quad +\beta(x_1)\beta(y)D(z, x_2, \cdots, x_n).
\end{aligned}$$

Combining the above two relations, we get $D(x_1, x_2, \cdots, x_n)\alpha(y^*)\alpha(z^*) = D(x_1, x_2, \cdots, x_n)\alpha(z^*)\alpha(y^*)$ for all $x_1, x_2, \cdots, x_n$, $y, z \in R$. Putting y^* and z^* in the places of y and z respectively and using the fact that α is surjective, we find that

$$D(x_1, x_2, \cdots, x_n)uv = D(x_1, x_2, \cdots, x_n)vu \tag{2.1}$$

for all $u, v \in R$. Now replacing u by ur where $r \in R$, in the relation (2.1) and using it again we arrive at $D(x_1, x_2, \cdots, x_n)urv = D(x_1, x_2, \cdots, x_n)uvr$ i.e.; $D(x_1, x_2, \cdots, x_n)R[r, v] = \{0\}$. This also gives us $D(x_1, x_2, \cdots, x_n)R[r, v]^* = \{0\}$. Since $D \neq 0$, $*$-primeness of R implies that $rv = vr$ for all $v, r \in R$. Therefore, we conclude that R is commutative. □

Corollary 2.2. *Let R be a $*$-prime ring and α be onto. If it admits a nonzero left α^*-n-multiplier T, then R is commutative.*

Corollary 2.3 ([6, Theorem 3.1])**.** *Let R be a prime $*$-ring. If it admits a nonzero $*$-n-derivation D, then R is commutative.*

Following example demonstrates that the $*$-primeness in the hypothesis of Theorem 2.1 can not be omitted.

Example 2.4. Let Q and $\mathbb{R}[x]$ be the ring of real quaternions and the polynomial ring in x over the ring $\mathbb{R}$ of real numbers respectively. Assume $R = Q \times \mathbb{R}[x]$ is the ring of cartesian product of Q and $\mathbb{R}[x]$ with regard to componentwise addition and multiplication. Let $*_1$, $*_2$ and $*$ denote the involutions of rings Q, $\mathbb{R}[x]$ and R respectively, defined by $q^{*_1} = \alpha - \beta i - \gamma j - \delta k$, where $q = \alpha + \beta i + \gamma j + \delta k \in Q$; $f(x)^{*_2} = f(x)$, where $f(x) \in \mathbb{R}[x]$ and $(q, f(x))^* = (q^{*_1}, f(x)^{*_2})$ for all $(q, f(x)) \in R$. Let us define $\alpha, \beta : R \longrightarrow R$ as $\alpha(q, f(x)) = (q, f(-x))$ and $\beta(q, f(x)) = (q, 0)$. Next define $D : R \times R \times \cdots \times R \longrightarrow R$ such that $D((q_1, f_1(x)), (q_2, f_2(x)), \cdots, (q_n, f_n(x))) = (0, f_1(-x)f_2(-x)\cdots f_n(-x))$. It can be easily verified that R is a semiprime ring but not a $*$-prime ring, admitting D as a nonzero $(\alpha,\beta)^*$-n-derivation of R, where α and β are endomorphisms of R and α is onto. However R is not commutative.

Theorem 2.5. *Let R be a $*$-prime ring and α be onto. If $T : R^n \longrightarrow R$ is a nonzero n-additive mapping such that $T(x_1y, x_2, \cdots, x_n) = T(x_1, x_2, \cdots, x_n)\alpha(y^*)$ for all $x_1, y, x_2, \cdots, x_n \in R$, then R is commutative.*

Proof. By hypothesis, for all $x_1, y, z, x_2, \cdots, x_n \in R$ we have $T(x_1(yz), x_2, \cdots, x_n) = T(x_1, x_2, \cdots, x_n)\alpha(z^*)\alpha(y^*)$. On the other hand we also have $T((x_1y)z, x_2, \cdots, x_n) = T(x_1, x_2, \cdots, x_n)\alpha(y^*)\alpha(z^*)$. Combining the preceding two relations we have $T(x_1, x_2, \cdots, x_n)\alpha(z^*)\alpha(y^*) = T(x_1, x_2, \cdots, x_n)\alpha(y^*)$ $\alpha(z^*)$. Replacing y, z by y^* and z^* respectively and using the fact that α is onto, we arrive at

$$T(x_1, x_2, \cdots, x_n)vu = T(x_1, x_2, \cdots, x_n)uv \tag{2.2}$$

for all $u, v \in R$. Now putting vr for v where $r \in R$, in the relation (2.2) and using it again we have $T(x_1, x_2, \cdots, x_n)v[r, u] = 0$ i.e.; $T(x_1, x_2, \cdots, x_n)R[r, u] = \{0\}$. It also gives us $T(x_1, x_2, \cdots, x_n)R[r, u]^* = \{0\}$. Since $T \neq 0$, hence $*$-primeness of R yields $[r, u] = 0$ for all $r, u \in R$. Finally, we conclude that the ring R is commutative. □

Corollary 2.6. *Let R be a $*$-prime ring and α be onto. If it admits a nonzero left α^*-n-multiplier T, then R is commutative.*

Corollary 2.7 ([6, Theorem 3.2])**.** *Let R be a prime ring with involution '$*$'. If $T : R^n \longrightarrow R$ is a nonzero n-additive mapping such that $T(x_1y, x_2, \cdots, x_n) = T(x_1, x_2, \cdots, x_n)y^*$ for all $x_1, y, x_2, \cdots, x_n \in R$, then R is commutative.*

Theorem 2.8. *Let R be a $*$-prime ring and α be surjective. If $T : R^n \longrightarrow R$ is a nonzero n-additive mapping such that $T(yx_1, x_2, \cdots, x_n) = \alpha(y^*)T(x_1, x_2, \cdots, x_n)$ for all $x_1, y, x_2, \cdots, x_n \in R$, then R is commutative.*

Proof. Computing $T((yz)x_1, x_2, \cdots, x_n)$ and $T(y(zx_1), x_2, \cdots, x_n)$ where $x_1, y, z, x_2, \cdots, x_n \in R$, in two different ways according to the given hypothesis and using the similar technique as used to prove Theorem 2.5, one can easily get the required result. □

Corollary 2.9. *Let R be a $*$-prime ring and α be onto. If it admits a nonzero right α^*-n-multiplier T, then R is commutative.*

Corollary 2.10 ([6, Theorem 3.3])**.** *Let R be a prime $*$-ring. If $T : R^n \longrightarrow R$ is a nonzero n-additive mapping such that $T(yx_1, x_2, \cdots, x_n) = y^*T(x_1, x_2, \cdots, x_n)$ for all $x_1, y, x_2, \cdots, x_n \in R$, then R is commutative.*

The following example shows that the restriction of $*$-primeness imposed on the hypotheses of Theorems 2.5 and 2.8 is not superfluous.

Example 2.11. Let Q and C be the ring of real quaternions and complex numbers respectively. Assume $R = Q \times C$ is the ring of cartesian product of Q and C with regard to componentwise addition and multiplication. Let $*_1$, $*_2$ and $*$ denote the involutions of rings Q, C and R respectively, defined by $q^{*_1} = \alpha - \beta i - \gamma j - \delta k$, where $q = \alpha + \beta i + \gamma j + \delta k \in Q$; $z^{*_2} = z$, where $z = x + iy \in C$ and $(q, z)^* = (q^{*_1}, z^{*_2})$ for all $(q, z) \in R$. Let us define $\alpha : R \longrightarrow R$ by $\alpha(q, z) = (q, \bar{z})$, where $\bar{z} = x - iy$. Next define $T : R \times R \times \cdots \times R \longrightarrow R$ such that $T((q_1, z_1), (q_2, z_2), \cdots, (q_n, z_n)) = (0, \bar{z}_1\bar{z}_2 \cdots \bar{z}_n)$. It can be easily verified that R is a semiprime ring but not a $*$-prime ring and has α as a surjective endomorphism. Further it can be also shown that T is a nonzero n-additive mapping of R such that $T((q_1, z_1)(q_1', z_1'), (q_2, z_2), \cdots, (q_n, z_n)) = T((q_1, z_1), (q_2, z_2), \cdots, (q_n, z_n))$ $\alpha((q_1', z_1')^*)$ and $T((q_1, z_1)(q_1', z_1'), (q_2, z_2), \cdots, (q_n, z_n)) = \alpha((q_1, z_1)^*)T((q_1', z_1'),$ $(q_2, z_2), \cdots, (q_n, z_n))$ hold for all $(q_1, z_1), (q_1', z_1'), (q_2, z_2), \cdots, (q_n, z_n) \in R$. However R is not commutative.

Remark 2.12. The above Theorems 2.5 & 2.8 also hold if the relations in the hypotheses hold in any argument.

In the year 2009 the first author with Ali [4, Theorem 2.1] proved that if a semiprime $*$-ring R admits a $(\alpha, \beta)^*$-derivation d such that α is onto, then $d(R) \subseteq Z$. Further, in the same paper [4, Theorem 3.1] they also proved its analogue for symmetric $(\alpha, \beta)^*$-bi-derivation in the setting of semiprime $*$-rings. We have proved that the restriction of symmetry of $(\alpha, \beta)^*$-bi-derivation used in [4, Theorem 3.1] is redundant. In fact, in the setting of semiprime $*$-rings, we obtained the following result for a $(\alpha, \beta)^*$-n-derivation D, where α is onto.

Theorem 2.13. *Let R be a semiprime $*$-ring. If R admits a $(\alpha, \beta)^*$-n-derivation D and α is surjective, then $D(R, R, \cdots, R) \subseteq Z$.*

Proof. Since R is a $*$-ring having a $(\alpha, \beta)^*$-n-derivation D and α is onto, we have relation (2.1). Now putting $uD(x_1, x_2, \cdots, x_n)$ in place of u in the relation (2.1) and using it again we get $D(x_1, x_2, \cdots, x_n)u[D(x_1, x_2, \cdots, x_n), v] = 0$ for all $x_1, x_2, \cdots, x_n; u, v \in R$. This relation provides us

$$vD(x_1, x_2, \cdots, x_n)u[D(x_1, x_2, \cdots, x_n), v] = 0. \tag{2.3}$$

Replacing u by vu in the relation $D(x_1, x_2, \cdots, x_n)u[D(x_1, x_2, \cdots, x_n), v] = 0$ we obtain that

$$D(x_1, x_2, \cdots, x_n)vu[D(x_1, x_2, \cdots, x_n), v] = 0. \tag{2.4}$$

Now comparing the identities (2.3) and (2.4) we arrive at
$D(x_1,x_2,\cdots,x_n)vu[D(x_1,x_2,\cdots,x_n),v]$
$= vD(x_1,x_2,\cdots,x_n)u[D(x_1,x_2,\cdots,x_n),v]$
i.e.; $[D(x_1,x_2,\cdots,x_n),v]u[D(x_1,x_2,\cdots,x_n),v] = 0$. This relation provides us $[D(x_1,x_2,\cdots,x_n),v]R[D(x_1,x_2,\cdots,x_n),v] = \{0\}$. Now semiprimeness of R yields that $[D(x_1,x_2,\cdots,x_n),v] = 0$ i.e; $D(R,R,\cdots,R) \subseteq Z$. □

Corollary 2.14. *Let R be a semiprime ring with involution '$*$' and α be onto. If it admits a left α^*-n-multiplier T, then $T(R,R,\cdots,R) \subseteq Z$.*

Corollary 2.15 ([6, Theorem 3.6])**.** *Let R be a semiprime $*$-ring, admitting a $*$-n-derivation D. Then $D(R,R,\cdots,R) \subseteq Z$.*

Theorem 2.16. *Let R be a semiprime ring with involution '$*$' and α be onto. If R admits an n-additive mapping $T : R^n \longrightarrow R$ such that $T(x_1y,x_2,\cdots,x_n) = T(x_1,x_2,\cdots,x_n)\alpha(y^*)$ for all $x_1,x_2,\cdots,x_n,y \in R$, then $T(R,R,\cdots,R) \subseteq Z$.*

Proof. By hypothesis R is a $*$-ring having an n-additive mapping $T : R^n \longrightarrow R$ such that $T(x_1y,x_2,\cdots,x_n) = T(x_1,x_2,\cdots,x_n)\alpha(y^*)$ for all $x_1,x_2,\cdots,x_n,y \in R$, hence we have relation (2.2). Now substituting $uT(x_1,x_2,\cdots,x_n)$ in place of u in the relation (2.2) and using it again we arrive at

$$T(x_1,x_2,\cdots,x_n)u[T(x_1,x_2,\cdots,x_n),v] = 0$$

for all $x_1,x_2,\cdots,x_n;u,v \in R$. This relation provides us

$$vT(x_1,x_2,\cdots,x_n)u[T(x_1,x_2,\cdots,x_n),v] = 0. \tag{2.5}$$

Replacing u by vu in the relation $T(x_1,x_2,\cdots,x_n)u[T(x_1,x_2,\cdots,x_n),v] = 0$ we obtain that

$$T(x_1,x_2,\cdots,x_n)vu[T(x_1,x_2,\cdots,x_n),v] = 0. \tag{2.6}$$

Now comparing the identities (2.5) and (2.6) we arrive at
$T(x_1,x_2,\cdots,x_n)vu[T(x_1,x_2,\cdots,x_n),v]$
$=vT(x_1,x_2,\cdots,x_n)u[T(x_1,x_2,\cdots,x_n),v]$
i.e.; $[T(x_1,x_2,\cdots,x_n),v]u[T(x_1,x_2,\cdots,x_n),v] = 0$ for all $x_1,x_2,\cdots,x_n;u,v \in R$. This implies that $[T(x_1,x_2,\cdots,x_n),v]R[T(x_1,x_2,\cdots,x_n),v] = \{0\}$. Now semiprimeness of R yields that $[T(x_1,x_2,\cdots,x_n),v] = 0$ i.e; $T(R,R,\cdots,R) \subseteq Z$. □

Corollary 2.17. *Let R be a semiprime ring with involution '$*$' and α be onto. If it admits a left α^*-n-multiplier T, then $T(R,R,\cdots,R) \subseteq Z$.*

Corollary 2.18 ([6, Theorem 3.7])**.** *Let R be a semiprime ring with involution '$*$'. If R admits an n-additive mapping $T : R^n \longrightarrow R$ such that*

$$T(x_1y,x_2,\cdots,x_n) = T(x_1,x_2,\cdots,x_n)y^*$$

for all $x_1,x_2,\cdots,x_n,y \in R$. Then $T(R,R,\cdots,R) \subseteq Z$.

The following example demonstrates that the condition of semiprimeness in the hypotheses of Theorems 2.13 & 2.16 is crucial.

Example 2.19. Consider $\mathbb{Z}$ the ring of integers and

$$R = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} \mid x, y, z, 0 \in \mathbb{Z} \right\}.$$

Define $\alpha, \beta : R \longrightarrow R$ as following.

$$\alpha\left(\begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x & -y \\ 0 & 0 & -z \\ 0 & 0 & 0 \end{pmatrix},$$

$$\beta\left(\begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Next define $D : R \times R \times \cdots \times R \longrightarrow R$ and $r \mapsto r^*$ of R into itself such that

$$D\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & z_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & z_2 \\ 0 & 0 & 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & z_n \\ 0 & 0 & 0 \end{pmatrix}\right) =$$
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & z_1 z_2 \cdots z_n \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$\begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & z & y \\ 0 & 0 & x \\ 0 & 0 & 0 \end{pmatrix}.$$

One can easily verify that '$*$' is an involution on ring R, which is not semiprime. Also it is straightforward to check that α,β are endomorphisms of R and α is onto. Further it can be easily proved that (i) D is a $(\alpha, \beta)^*$-n-derivation of R and (ii) D is an n-additive mapping from R^n to R such that $D(r_1 r, r_2, \cdots, r_n) = D(r_1, r_2, \cdots, r_n)\alpha(r^*)$ for all $r_1, r_2, \cdots, r_n, r \in R$. However, $D(R, R, \cdots, R) \nsubseteq Z$.

Theorem 2.20. *Let R be a semiprime $*$-ring and α be onto. If R admits an n-additive mapping $T : R^n \longrightarrow R$ such that*

$$T(yx_1, x_2, \cdots, x_n) = \alpha(y^*)T(x_1, x_2, \cdots, x_n)$$

for all $x_1, x_2, \cdots, x_n; y \in R$. Then $T(R, R, \cdots, R) \subseteq Z$.

Proof. Using similar arguments with necessary variations as used to prove Theorem 2.16, one can easily obtain the proof. □

Corollary 2.21. *Let R be a semiprime ring with involution '$*$' and α be onto. If it admits a right α^*-n-multiplier T, then $T(R, R, \cdots, R) \subseteq Z$.*

Corollary 2.22 ([6, Theorem 3.8])**.** *Let R be a semiprime $*$-ring. If R admits an n-additive mapping $T : R^n \longrightarrow R$ such that*

$$T(yx_1, x_2, \cdots, x_n) = y^*T(x_1, x_2, \cdots, x_n)$$

for all $x_1, x_2, \cdots, x_n; y \in R$. Then $T(R, R, \cdots, R) \subseteq Z$.

The following example demonstrates that the condition of semiprimeness in the hypothesis of Theorem 2.20 is crucial.

Example 2.23. Consider $\mathbb{Z}$ the ring of integers and

$$R = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix} \mid x, y, z, 0 \in \mathbb{Z} \right\}.$$

Define $\alpha : R \longrightarrow R$ such that

$$\alpha\left(\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & 0 \\ -x & 0 & 0 \\ -y & z & 0 \end{pmatrix}.$$

Next define $T : R \times R \times \cdots \times R \longrightarrow R$ and $r \mapsto r^*$ of R into itself such that

$$T\left(\begin{pmatrix} 0 & 0 & 0 \\ x_1 & 0 & 0 \\ y_1 & z_1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ x_2 & 0 & 0 \\ y_2 & z_2 & 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 & 0 & 0 \\ x_n & 0 & 0 \\ y_n & z_n & 0 \end{pmatrix}\right) =$$
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & z_1 z_2 \cdots z_n & 0 \end{pmatrix},$$

and

$$\begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ y & z & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & 0 & 0 \\ z & 0 & 0 \\ y & x & 0 \end{pmatrix}.$$

One can easily show that '$*$' is an involution on ring R, which is not semiprime. Also it is straightforward to check that α is an onto endomorphism of R. Further it can be easily proved that T is an n-additive mapping from R^n to R such that $T(rr_1, r_2, \cdots, r_n) = \alpha(r^*)T(r_1, r_2, \cdots, r_n)$ for all $r_1, r_2, \cdots, r_n, r \in R$. However $T(R, R, \cdots, R) \nsubseteq Z$.

Remark 2.24. The above Theorems 2.16 & 2.20 also hold if the relations in the hypotheses hold in any argument.

Theorem 2.25. *Let R be a semiprime $*$-ring and α, β be endomorphisms of R. If R admits a $(\alpha,\beta)^*$-derivation d such that α is onto. Suppose I is a nonzero ideal of R such that it is invariant under $*$, d, α and β i.e.; $I^* \subseteq I$, $d(I) \subseteq I$, $\alpha(I) \subset I$, $\beta(I) \subset I$. Then d induces a $(\hat{\alpha}, \hat{\beta})^{\hat{*}}$-$n$-derivation D on the quotient ring R/I where $\hat{*}$ is an involution on quotient ring R/I induced by the involution $*$ of R, where as $\hat{\alpha}$ and $\hat{\beta}$ are endomorphisms of quotient ring R/I induced by endomorphisms α and β of R respectively.*

Proof. Define a map $x+I \mapsto (x+I)^{\hat{*}}$ of R/I into itself such that $(x+I)^{\hat{*}} = x^*+I$ for all $(x+I) \in R/I$. Let $x+I = y+I$. This implies that $x - y \in I$. Hence by hypothesis $(x-y)^* \in I$ i.e.; $x^* - y^* \in I$. Therefore $x^* + I = y^* + I$ i.e.; $(x+I)^{\hat{*}} = (y+I)^{\hat{*}}$. Thus $\hat{*}$ is a well defined map on quotient ring R/I. By using addition and product of quotient ring R/I we see that (i) $\{(x+I)+(y+I)\}^{\hat{*}} = ((x+y)+I)^{\hat{*}} = (x+y)^*+I = (x^*+y^*)+I = (x^*+I)+(y^*+I) = (x+I)^{\hat{*}}+(y+I)^{\hat{*}}$,

(ii) $((x+I)^{\hat{*}})^{\hat{*}} = (x^*+I)^{\hat{*}} = (x^*)^*+I = x+I$ and (iii) $\{(x+I)(y+I)\}^{\hat{*}} = (xy+I)^{\hat{*}} = (xy)^*+I = y^*x^*+I = (y^*+I)(x^*+I) = (y+I)^{\hat{*}}(x+I)^{\hat{*}}$ for all $(x+I),(y+I) \in R/I$. All previous facts $(i),(ii)$ and (iii) insure that $\hat{*}$ is an involution on quotient ring R/I. Next we define $\hat{\alpha},\hat{\beta} : R/I \longrightarrow R/I$ by $\hat{\alpha}(x+I) = \alpha(x)+I$ and $\hat{\beta}(x+I) = \beta(x)+I$ respectively. In the similar lines as above and using the hypothesis it can be easily verified that $\hat{\alpha}$ and $\hat{\beta}$ are well defined maps on R/I and endomorphisms as well. Now define $D : R/I \times R/I \times \cdots \times R/I \longrightarrow R/I$ as below $D((x_1+I),(x_2+I),\cdots,(x_n+I)) = d(x_1)d(x_2)\cdots d(x_n)+I$. Let $((x_1+I),(x_2+I),\cdots,(x_n+I)) = ((y_1+I),(y_2+I),\cdots,(y_n+I))$. This implies that $(x_1-y_1) \in I,(x_2-y_2) \in I,\cdots,(x_n-y_n) \in I$. By hypothesis $d(x_1)+I = d(y_1)+I, d(x_2)+I = d(y_2)+I,\cdots,d(x_n)+I = d(y_n)+I$ i.e.; $(d(x_1)+I)(d(x_2)+I)\cdots(d(x_n)+I) = (d(y_1)+I)(d(y_2)+I)\cdots(d(y_n)+I)$. Now we obtain that $d(x_1)d(x_2)\cdots d(x_n)+I = d(y_1)d(y_2)\cdots d(y_n)+I$ i.e,; $D((x_1+I),(x_2+I),\cdots,(x_n+I)) = D((y_1+I),(y_2+I),\cdots,(y_n+I))$. Thus D is a well defined map. Consider

$$\begin{aligned}
&D((x_1+I)+(x_1'+I),(x_2+I),\cdots,(x_n+I))\\
&\quad = D((x_1+x_1'+I),(x_2+I),\cdots,(x_n+I))\\
&\quad = d(x_1+x_1')d(x_2)\cdots d(x_n)+I\\
&\quad = (d(x_1)d(x_2)\cdots d(x_n)+I)+(d(x_1')d(x_2)\cdots d(x_n)+I)\\
&\quad = D((x_1+I),(x_2+I),\cdots,(x_n+I))+D((x_1'+I),(x_2+I),\cdots,(x_n+I)).
\end{aligned}$$

The previous relation insures that D is additive in the first argument. Similarly one can show that D is additive in all arguments. Thus D is an n-additive map. Consider $D((x_1+I)(x_1'+I),(x_2+I),\cdots,(x_n+I))$

$$\begin{aligned}
&\quad = D((x_1x_1'+I),(x_2+I),\cdots,(x_n+I))\\
&\quad = d(x_1x_1')d(x_2)\cdots d(x_n)+I\\
&\quad = \{(d(x_1)\alpha((x_1')^*)+\beta(x_1)d(x_1'))d(x_2)\cdots d(x_n)\}+I.
\end{aligned}$$

Now using Theorem 2.13, the above relation takes the form

$$\begin{aligned}
&D((x_1+I)(x_1'+I),(x_2+I),\cdots,(x_n+I))\\
&= \{(d(x_1)d(x_2)\cdots d(x_n)\alpha((x_1')^*)+\beta(x_1)d(x_1')d(x_2)\cdots d(x_n))\}+I\\
&= \{(d(x_1)d(x_2)\cdots d(x_n)+I)(\alpha((x_1')^*)+I)\}+\\
&\quad \{(\beta(x_1)+I)(d(x_1')d(x_2)\cdots d(x_n)+I)\}\\
&= D((x_1+I),(x_2+I),\cdots,(x_n+I))\hat{\alpha}((x_1')^*+I)+\\
&\quad \hat{\beta}(x_1+I)D((x_1'+I),(x_2+I),\cdots,(x_n+I))\\
&= D((x_1+I),(x_2+I),\cdots,(x_n+I))\hat{\alpha}((x_1'+I)^{\hat{*}})+\\
&\quad \hat{\beta}(x_1+I)D((x_1'+I),(x_2+I),\cdots,(x_n+I)).
\end{aligned}$$

Similarly we can prove that the previous relation holds in each argument. Finally we conclude that D is a $(\hat{\alpha},\hat{\beta})^{\hat{*}}$-$n$-derivation on quotient ring R/I. □

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