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ROBUST EVENT-TRIGGERED CONTROL FOR NETWORKED FUZZY SYSTEMS

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Abstract. This paper presents a Neural Gradient Descent-based Event-Triggered (NGDET) control design for Networked Control Systems (NCS) based on Takagi-Sugeno (T-S) fuzzy systems affected by measurement noise, external disturbances, and network-induced delays. The main objective is to improve communication efficiency while preserving closed-loop stability and robust performance. To this end, an adaptive event-triggering mechanism is proposed, where the triggering threshold is updated online through a neural gradient descent technique according to the system behavior and communication network constraints. Then, by constructing an appropriate Lyapunov–Krasovskii Functional (LKF), relaxed Linear Matrix Inequality (LMI) conditions are developed for the design of a networked controller involving a sampled data Parallel Distributed Compensation (PDC) approach ensuring asymptotic stability and robustness of NCS closed-loop dynamic. The simulation results demonstrate its effectiveness by enlarging the admissible delay bound, reducing packet transmissions, and achieving less conservative performance compared to existing methods from the literature, while preserving robustness against disturbances and measurement noise.
Keywords: Networked control systems, Takagi–Sugeno fuzzy systems, Event-triggered control, Neural gradient descent, measurement noise.

1. Introduction

Networked control systems have attracted considerable attention due to their low cost, flexibility, and suitability for distributed complex systems. Yet, network-induced delays, packet dropouts, bandwidth constraints, and data congestion may degrade performance and even lead to instability in the closed-loop system. In practice, disturbances, uncertainties, and measurement noise further increase the control complexity. To address these challenges, extensive efforts have focused on stability analysis and controller design for NCSs while reducing communication burden and preserving closed-loop performance [5, 21].

In the NCSs framework, time-triggered mechanisms are widely used due to their periodic structure, simplicity, and easy implementation [20]. However, they often

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generate redundant transmissions, which reduce the efficiency of limited communication resources and can cause congestion, packet losses, time-varying delays, and a higher communication burden [3]. Therefore, classical time-triggered strategies become less efficient since data are transmitted regardless of actual system needs [7]. To overcome these limitations, event-triggered mechanisms have been introduced as an alternative, transmitting data only when a predefined condition is satisfied, thereby reducing communication load while preserving control performance [18].

However, conventional event-triggered mechanisms, particularly those based on fixed thresholds, may exhibit limited flexibility and weak adaptability under system uncertainties and network constraints.

In this context, recent research has focused on designing advanced and more efficient event-triggered mechanisms to enhance flexibility and robustness by adapting the triggering condition in real time according to system behavior and communication network constraints [5, 16].

Consequently, recent research has focused on developing more efficient control strategies to reduce unnecessary transmissions while preserving the desired performance levels of NCSs, using various techniques such as adaptive event-triggered mechanism with the triggering threshold dynamically updated [4], fuzzy weighted memory event-triggered mechanism [11], fuzzy event-triggered H_∞ control [10, 12]. Mixed-triggered filter design for fuzzy Markovian jump systems [6]. More recently, dynamic and memory-based triggering mechanisms [22], data-driven event-triggered control mechanism [16], neural network-based adaptive event-triggered control [2, 17], and more recently, non-fragile mixed event-triggered state estimation approaches [15].

It is noted that most of the aforementioned studies have focused on the networked control of linear systems. However, many practical systems exhibit inherent nonlinear behaviors, which may limit the applicability of such approaches. Therefore, considerable research efforts have been devoted to extending event-triggered NCS framework to nonlinear systems to ensure stability and maintain satisfactory control performance [5, 6, 21].

Among the nonlinear control approaches, T–S fuzzy systems have received increasing attention due to their simple structure and effectiveness in representing nonlinear dynamics. These models describe complex systems through a set of fuzzy If–Then rules, where each rule defines a local linear subsystem combined through membership functions. This structure enables nonlinear systems to be approximated by a convex combination of linear models, allowing the use of classical linear control techniques for nonlinear control design.

In T–S model-based NCSs, LKF are commonly used to derive LMI-based conditions for designing PDC sampled-data controllers. Therefore, recent advances in event-triggered control have addressed event-based asynchronous communication delays [5], dynamic event-triggered with non-uniform sampling [14], adaptive memory-based H_∞ control [13, 21] and mixed-triggered Markovian jump [6].

In practical T–S-based NCSs, measurement noise and uncertainties may affect system dynamics, leading to performance degradation. To improve reliability and guarantee stability in the presence of measurement noise and uncertainties,

many T–S model-based networked and non-fragile control approaches have been developed for NCSs [1, 7, 9].
Motivated by the above-mentioned studies, this paper proposes the design of an adaptive NGDET control for the robust stabilization of T–S fuzzy networked control systems affected by communication delays, measurement noise, and external perturbations. The proposed approach aims to enhance closed-loop stability and robustness while reducing unnecessary network communications. The main contributions of this paper can be summarized as follows:

- A novel NGDET control strategy is proposed for T-S-based NCSs in order to reduce unnecessary data transmissions while preserving the stability and robustness of the closed-loop NCS.
- By integrating the NGDET mechanism with a PDC controller, the triggering threshold is adaptively adjusted online according to the system states and error evolution, which improves flexibility compared with the conventional static event-triggered mechanism in the literature.
- The proposed adaptive NGDET mechanism significantly decreases communication load and network bandwidth utilization by transmitting data only when necessary, thereby reducing redundant packet releases over the communication network.
- The developed NGDET controller improves the robustness of the NCS against external disturbances, measurement noise, and network-induced delays. In addition, the triggering frequency is automatically increased only during critical operating conditions and reduced during steady-state operation, leading to an efficient trade-off between communication efficiency and control performance.
- To derive less conservative LMI-based design conditions that allow for a larger admissible range of network-induced delays and event-triggered packet transmission intervals, achieved through an appropriate selection of LKF and the use of suitable relaxation lemmas.

The rest of the paper is structured as follows. Section 2 presents the necessary preliminaries and problem formulation, including the considered T–S fuzzy NCS framework and the proposed NGDET mechanism. Section 3 focuses on the theoretical developments and derives LMI-based sufficient conditions for stability analysis and controller synthesis. Finally, section 4 provides a numeric example to demonstrate the effectiveness of the proposed NGDET mechanism and to highlight how the developed LMI conditions are less conservative compared to previous methods reported in the recent literature.

Notations 1. In the following, stars $*$ in matrices denote block transpose quantities. For a matrix M of appropriate dimension, one denotes $\mathcal{H}_e(M) = M + M^T$. A finite set of r positive integers is denoted $\mathcal{I}_r = (1, ..., r)$, r is the number of rules. Also, $\forall j \in \{1, \ldots, 8\}$, we denote the block entry matrices $e_j = \left[0_{n\times(j-1)n}, I_{n\times n}, 0_{n\times(8n-j)n}\right] \in \mathbb{R}^{8n\times n}$, for example: $e_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}$. Furthermore, to simplify the presentation and reduce notational complexity, the following notation is adopted for fuzzy matrix summations: for: $(i, j) \in \mathbb{I}_r^2$.

$M_h = \sum_{i=1}^{r} h_i z(t) M_i, M_{\bar{h}} = \sum_{i=1}^{r} h_i z(t - \eta(t)) M_i$ and $M_{h\bar{h}} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i z(t) h_j z(t - \eta(t)) M_{ij}$

2. Preliminaries and problem statement

Consider the class of continuous-time T-S models , subject to external disturbances and measurement noise, which can be expressed as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\big(A_i x(t) + B_i u(t) + B_{wi} w(t)\big), \forall i \in \mathbb{I}_r \\ y(t) = \sum_{i=1}^{r} h_i(z(t))\big(C_i x(t) + E_i f_s(t)\big) \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector , $y(t) \in \mathbb{R}^q$ is the output vector, $\omega(t) \in \mathbb{R}^p$ is a vector of external disturbances belonging to $L_2[0, +\infty)$, $f_s(t) \in \mathbb{R}^\kappa$ is the measurement noise, $z(t) \in \mathbb{R}^\mu$ is a vector of premise variables, $0 \leq h_i(z(t)) \leq 1$ are membership functions satisfying the convex sum property $\sum_{i=1}^{r} h_i(z(t)) = 1$, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{q\times n}$, $E_i \in \mathbb{R}^{q\times \kappa}$ and $B_{wi} \in \mathbb{R}^{n\times p}$ are known constant matrices defining the vertices of the T-S model. Note that the measurement noise $f_s(t)$, representing disturbances or errors arising from sensors, is a stochastic signal modeled by a Bernoulli random process. Its energy satisfies the following constraint:

$$\int_{-\infty}^{+\infty} f_s^T(t) f_s(t)\, dt \leq \int_{-\infty}^{+\infty} y^T(t) Q_f y(t)\, dt \tag{2.2}$$

where Q_f is a symmetric positive definite matrix that weights the output signal.

Assumption 2.1. The sensors operate based on a time-triggered mechanism with a fixed sampling period h, generating a sampling sequence $\mathcal{S} = \{s_\kappa = \kappa h | \kappa \in \mathbb{N}\}$.

Assumption 2.2. The data are transmitted over the network as individual packets without packet loss.

Let k denotes the index of the current sampling instant, with τ_k^{sc} and τ_k^{ca} denote the communication delays from the sensor-to-controller and controller-to-actuator channels, respectively. Consequently, the overall network-induced delay is given by $\tau_k = \tau_k^{sc} + \tau_k^{ca} \in [\tau_m,\ \tau_M]$ (see Figure 1).

To improve communication efficiency in NCS, we propose a NGDET mechanism as an adaptive alternative to conventional event-triggered mechanisms, static or dynamic. The objective is to reduce redundant transmissions and thus save bandwidth while preserving control performance. The key idea is that, when the system operates near equilibrium, frequent packet releases become unnecessary. The NGDET mechanism optimizes the transmission instant t_{k+1}, in real time, ensuring an effective balance between network load and closed-loop stability. The next transmission instant t_{k+1} is given by the following condition:

$$t_{k+1} = t_k + \min_{l\in N} \Big\{ lh | \varepsilon^T(t_k)\Omega\varepsilon(t_k) > \rho(t) x^T(t_k)\Omega x(t_k) \Big\}. \tag{2.3}$$

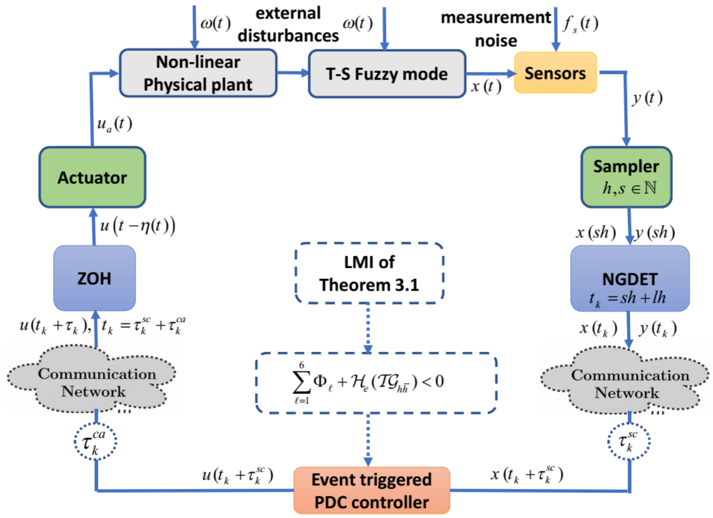

FIGURE 1. Networked control system with NGDET control schem.

where $\varepsilon_k(t) = x(t_k) - x(t_k + lh)$ denotes the output error, $\Omega > 0$ is an event-triggering weighting matrix to be designed and $\rho(t)$ is an adaptive NGDET threshold function, given by:

$$\rho(t) = \rho_m + (\rho_M - \rho_m)\left(-W\|\varepsilon(t_k)\|_2 + b\right) \tag{2.4}$$

where W and b represent the adaptive neural weight and bias of a single-neuron approximator that are updated through the gradient descent algorithm and: $[\rho_m, \rho_M] \in [0, 1]$ are the minimum and maximum threshold values, respectively.

Remark 2.3. Unlike conventional static or dynamic threshold functions, the proposed NGDET strategy provides an intelligent self-adjustment mechanism for the event-triggering condition. The algorithm updates the neural parameters W and b through a reward-driven gradient descent process. The reward function reduces unnecessary transmissions while maintaining accurate control, leading to an sigmoid-based adaptive between the error norm $\|\varepsilon(t_k)\|_2$ and the triggering threshold. As learning progresses, the controller autonomously refines the threshold function $\rho(t)$ decreasing its value when the system operates near equilibrium to reduce unnecessary transmissions, and increasing it when large deviations occur to trigger more updates of data and so that new control actions are sent at the right time t_k to maintain stability. This gradient-descent-based adaptive mechanism allows the threshold to evolve in real time according to the system state behavior, which allows to reduce conservatism, improve robustness against disturbances, and give a better trade-off between communication efficiency and closed-loop stability, even when external disturbances and measurement noise occur.

In the context of NCS represented in Figure 1, the Zero-Order Hold (ZOH) maintains the control signal constant during the interval $\mathbb{I}_z = [t_k + \tau_k, t_{k+1} + \tau_{k+1})$. Following the framework in [19], this interval can be further partitioned into several sub-intervals $\mathbb{I}_z = \bigcup_{l=0}^{\bar{v}} \mathbb{I}_l$, where each sub-interval is defined as $\mathbb{I}_l =$

$[t_k + lh + max(\tau_k), t_k + lh + h + max(\tau_k))$, where, $max(\tau_k k \in \mathbb{N})$ denotes the maximum sampling delay, and $\bar{v} \in \mathbb{N}$ satisfies the condition $t_k + \bar{v}h + max(\tau_k) < t_{k+1} + \tau_{k+1} \le t_k + \bar{v}h + h + max(\tau_k)$. Moreover $\forall t \in \mathbb{I}_z$, we define $\eta(t) = t - t_k - lh$, which satisfies $0 \le min(\tau_k) = \tau_m \le \tau_k \le \eta(t) \le max(\tau_k) + h = \tau_M$ evolving as a piecewise continuous sawtooth profile and satisfies $\dot{\eta}(t) = 1$. As a result, we can write $x(t_k + lh) = x(t - \eta(t))$.
Within this framework, we define a NGDET controller as follows:

$$u(t) = \sum_{i=1}^{r} h_i(z(t_k))\big(K_i x(t_k)\big) = \sum_{i=1}^{r} h_i(z(t-\eta(t)))K_i\Big(x(t-\eta(t)) + \varepsilon(t_k)\Big) \tag{2.5}$$

where $K_i \in \mathbb{R}^{m\times n}(i \in \mathbb{I}_r)$ represents the feedback gain matrix to be designed.
From notation 1 then (2.1) and (2.5), yields the closed-loop dynamics with external disturbances and measurement noise as follows:

$$\begin{cases} \dot{x}(t) = A_h x(t) + B_h K_{\bar{h}}\big(x(t-\eta(t)) + \varepsilon(t_k)\big) + B_{wh} w(t) \\ y(t) = C_h x(t) + E_h f_s(t) \end{cases} \tag{2.6}$$

Problem Statement: This study aims to establish LMI-based conditions for the synthesis of the NGDET controller gain matrices K_j ($j \in \mathbb{I}_r$), along with the design of the T-S-based event-triggering weighting matrix Ω, such that the NCS closed-loop dynamics (2.6) subject to measurement noise (2.2) is robustly asymptotically stable. In addition, for nonzero disturbances $w(t)$, the following H_∞ disturbance attenuation condition holds:

$$\int_0^\infty \big(x^T(s)Q_d x(s) - \gamma^2 w^T(s)w(s)\big)ds < 0 \tag{2.7}$$

with Q_d is the symmetric positive definite matrix.

3. Main result

This section presents LMI-based conditions for designing a robust NGDET controller (2.5) for the T-S fuzzy system (2.1). The objective is to determine the PDC state-feedback gains $K_j(j \in \mathbb{I}_r)$ and the event-triggering weighting matrix Ω such that the NCS closed-loop dynamic (2.6) is asymptotically stable in the presence of external disturbances, measurement noise, and bounded network-induced delays. Moreover, the system must satisfy a prescribed H_∞ performance level while reducing unnecessary data transmissions through an adaptive NGDET mechanism.

Theorem 3.1. *For given positive scalars h, τ_m, τ_M, ρ_m, ρ_M, ϵ_1, ϵ_2 and given fault matrices E_i and Q_f, $\forall i \in \mathbb{I}_r$ and $\forall j \in \mathbb{I}_r$, the NCS closed-loop dynamic (2.1) is robustly asymptotically stabilized by the NGDET controller (2.5), subject to an H_∞ disturbance attenuation level $\gamma > 0$ and with regard to the release instants defined by the NGDET mechanism (2.3), if there exist the real matrices $X > 0$, $\bar{P} > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_d > 0$, $\bar{R}_1 > 0$, $\bar{R}_2 > 0$, $\bar{\Omega} > 0$, $\bar{S} > 0$, $\bar{M}$, and*

F_j with appropriate dimensions such that the conditions of lemma 5 in [12] and $\begin{bmatrix} \bar{R}_2 & \bar{S} \\ * & \bar{R}_2 \end{bmatrix} > 0$ *are satisfied with:*

$$\Lambda_{ij} = \begin{bmatrix} \Gamma_{ij} & \tau_m\bar{\Phi}_3^2 + \frac{1}{2}\bar{\Phi}_3^1 + (\tau_M - \tau_m)U & e_1 X^T C_i^T \\ * & \bar{\Phi}_3^2 - \mathcal{H}_e(U) & 0 \\ * & * & -Q_f^{-1} \end{bmatrix} \tag{3.1}$$

where

$$\begin{aligned} \Gamma_{ij} =& \tau_m^2\bar{\Phi}_3^2 + \tau_m\bar{\Phi}_3^1 + \bar{\Phi}_3^0 + \bar{\Phi}_1 + \bar{\Phi}_2 + \bar{\Phi}_4 + e_1\bar{Q}_d e_1^T - \gamma^2 e_8 e_8^T - e_7 e_7^T + \mathcal{H}_e(\mathcal{X}_{ij}) \\ &+ e_7 E_i^T Q_f E_i e_7^T + \mathcal{H}_e(e_7 E_i^T Q_f C_i X e_1^T), \end{aligned} \tag{3.2}$$

$$\mathcal{X}_{ij} = \begin{bmatrix} A_i X & 0 & B_i F_j & 0 & -X & B_i F_j & 0 & B_{wi} \end{bmatrix}, \tag{3.3}$$

$$\Phi_3^0 = (\tau_M + \tau_m)\Delta_1^T \bar{M}\Delta_1 - \tau_M\tau_m\mathcal{H}_e(\Delta_1^T\bar{M}\Delta_2), \tag{3.4}$$

$$\Phi_3^1 = -2\Delta_1^T\bar{M}\Delta_1 + (\tau_M + \tau_m)\mathcal{H}_e(\Delta_1^T\bar{M}\Delta_2), \tag{3.5}$$

$$\Phi_3^2 = -\mathcal{H}_e(\Delta_1^T\bar{M}\Delta_2), \tag{3.6}$$

$$\Phi_1 = \mathcal{H}_e(e_1\bar{P}e_5^T), \tag{3.7}$$

$$\begin{aligned} \Phi_2 =& e_1\bar{Q}_1 e_1^T - e_2\bar{Q}_1 e_2^T - e_4\bar{Q}_2 e_4^T + e_5\big(\tau_m^2\bar{R}_1 + (\tau_M - \tau_m)^2\bar{R}_2\big)e_5^T \\ &-(e_2 - e_1)\bar{R}_1(e_2 - e_1)^T - e_2\bar{R}_2 e_2^T + e_3(-2\bar{R}_2 + \bar{S} + \bar{S}^T)e_3^T - e_4\bar{R}_2 e_4^T \\ &+\mathcal{H}_e\big(e_2(\bar{R}_2 - \bar{S})e_3^T + e_3(\bar{R}_2 - \bar{S})e_4^T + e_2\bar{S}e_4^T\big), \end{aligned} \tag{3.8}$$

$$\Phi_4 = \rho(t)(e_3 + e_6)\bar{\Omega}(e_3^T + e_6^T) - e_6\bar{\Omega}e_6^T, \tag{3.9}$$

$\Delta_1 = \big[e_1 \quad (e_3 + e_6)\big]^T$ *and* $\Delta_2 = \big[e_5 \quad 0\big]^T$.
In this case, the PDC state-feedback gains of (2.5) are computed as $K_j = F_j X^{-1}$.

Proof. Let us consider the following LKF candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{3.10}$$

where

$$V_1(t) = x^T(t)Px(t), \tag{3.11}$$

$$\begin{aligned} V_2(t) =& \int_{t-\tau_m}^{t} x^T(s)Q_1 x(s)ds + \int_{t-\tau_M}^{t-\eta(t)} x^T(s)Q_2 x(s)ds \\ &+\tau_m\int_{-\tau_m}^{0}\int_{t+v}^{t}\dot{x}^T(s)R_1\dot{x}(s)dsdv + (\tau_M - \tau_m)\int_{-\tau_M}^{-\tau_m}\int_{t+v}^{t}\dot{x}^T(s)R_2\dot{x}(s)dsdv, \end{aligned} \tag{3.12}$$

$$V_3(t) = (\tau_m - \eta(t))(\eta(t)) - \tau_M)\Theta^T(t)M\Theta(t) \text{ with } \Theta(t) = \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T, \tag{3.13}$$

The LKF (3.10) is positive if P, Q_1, Q_2, R_1, R_2 and M are all positive definite matrices. Consequently, the NCS closed-loop (2.6) is asymptotically stable if:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) < 0 \tag{3.14}$$

Let us define:

$$\zeta(t) = \big[x^T(t) \;\; x^T(t-\tau_m) \;\; x^T(t-\eta(t)) \;\; x^T(t-\tau_M) \;\; \dot{x}^T(t) \;\; \varepsilon^T(t_k) \;\; f_s^T(t) \;\; w^T(t)\big]^T \tag{3.15}$$

The derivative of $V_1(t)$ yields:

$$\dot{V}_1(t) = \mathcal{H}_e\big(x^T(t)P\dot{x}(t)\big) = \zeta^T(t)\Phi_1\zeta(t) \tag{3.16}$$

with $\Phi_1 = \mathcal{H}_e(e_1 P e_5^T)$.

for the second term of (3.10) we have:

$$\begin{aligned}\dot{V}_2(t) =& x^T(t)Q_1x(t) - x^T(t-\tau_m)Q_1x(t-\tau_m) - x^T(t-\tau_M)Q_2x(t-\tau_M)\\ &+\tau_m^2\dot{x}^T(t)R_1\dot{x}(t) + (\tau_M-\tau_m)^2\dot{x}^T(t)R_2\dot{x}(t) - \tau_m\int_{t-\tau_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\\ &-(\tau_M-\tau_m)\int_{t-\tau_M}^{t-\tau_m}\dot{x}^T(s)R_2\dot{x}(s)ds\end{aligned} \tag{3.17}$$

By applying Jensen's inequality on the first integral term of (3.17), we have:

$$-\tau_m\int_{t-\tau_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \leq \zeta^T(t)\begin{bmatrix} e_1^T\\ e_2^T\end{bmatrix}^T\begin{bmatrix} -R_1 & R_1\\ * & -R_1\end{bmatrix}\begin{bmatrix} e_1^T\\ e_2^T\end{bmatrix}\zeta(t) \tag{3.18}$$

Similarly, by applying extended Jensen's lemma [8]) on the second integral term of (3.17) with $\begin{bmatrix} R_2 & S\\ * & R_2\end{bmatrix} \geq 0$, we have:

$$\begin{aligned}&-(\tau_M-\tau_m)\int_{t-\tau_M}^{t-\tau_m}\dot{x}^T(s)R_2\dot{x}(s)ds \leq\\ &\qquad -\zeta^T(t)\begin{bmatrix} e_2^T\\ e_3^T\\ e_4^T\end{bmatrix}^T\begin{bmatrix} -R_2 & R_2-S & S\\ * & -2R_2+\mathcal{H}_e(S) & -S+R_2\\ * & * & -R_2\end{bmatrix}\begin{bmatrix} e_2^T\\ e_3^T\\ e_4^T\end{bmatrix}\zeta(t)\end{aligned} \tag{3.19}$$

Therefore, from (3.17), (3.18), and (3.19), we obtain:

$$\dot{V}_2(t) \leq \zeta^T(t)\Phi_2\zeta(t) \tag{3.20}$$

with $\Phi_2 = e_1Q_1e_1^T - e_2Q_1e_2^T - e_4Q_2e_4^T + e_5\big(\tau_m^2R_1 + (\tau_M-\tau_m)^2R_2\big)e_5^T - (e_2-e_1)R_1(e_2-e_1)^T - e_2R_2e_2^T + e_3(-2R_2+S+S^T)e_3^T - e_4R_2e_4^T + \mathcal{H}_e\big(e_2(R_2-S)e_3^T + e_3(R_2-S)e_4^T + e_2Se_4^T\big)$

Next, considering the time derivative of equation (3.13), we have:

$$\begin{aligned}\dot{V}_3(t) =& (\tau_m+\tau_M-2\eta(t))\Theta^T(t)M\Theta(t)\\ &+\big((\tau_M+\tau_m)\eta(t) - \eta^2(t) - \tau_M\tau_m\big)\mathcal{H}_e\big(\Theta^T(t)M\dot{\Theta}(t)\big) = \zeta^T(t)\Phi_3\zeta(t)\end{aligned} \tag{3.21}$$

with $\Phi_3 = \eta^2(t)\,\Phi_3^2 + \eta(t)\,\Phi_3^1 + \Phi_3^0$, $\Phi_3^2 = -\mathcal{H}_e(\Delta_1^TM\Delta_2)$, $\Phi_3^1 = -2\Delta_1^TM\Delta_1 + (\tau_M+\tau_m)\mathcal{H}_e(\Delta_1^TM\Delta_2)$ and $\Phi_3^0 = (\tau_M+\tau_m)\Delta_1^TM\Delta_1 - \tau_M\tau_m\mathcal{H}_e(\Delta_1^TM\Delta_2)$, where Δ_1 and Δ_2 are defined in Theorem 3.1.

Next, from (2.6), we can write the NCS closed-loop dynamics as:

$$\begin{bmatrix} A_h & 0 & B_hK_{\bar{h}} & 0 & -I & B_hK_{\bar{h}} & 0 & B_{wh}\end{bmatrix}\zeta(t) = 0, \text{or} \quad \mathcal{G}_{h\bar{h}}\zeta(t) = 0 \tag{3.22}$$

From (3.16), (3.20), (3.21), and (3.22), applying the Finsler's lemma [12], the NCS closed-loop dynamics (2.6) is asymptotically stable if $\exists\mathcal{T} \in \mathbb{R}^{8n\times n}$ such that:

$$\zeta^T(t)\Big(\sum_{\ell=1}^{3}\Phi_\ell + \mathcal{H}_e\big(\mathcal{T}\mathcal{G}_{h\bar{h}}\big)\Big)\zeta(t) < 0 \tag{3.23}$$

Now, from the proposed NGDET mechanism (2.3), we can write:

$$0<\rho(t)\big(x^T(t-\eta(t))+\varepsilon^T(t_k)\big)\Omega\big(x(t-\eta(t))+\varepsilon(t_k)\big)-\varepsilon^T(t_k)\Omega\varepsilon(t_k)=\zeta^T(t)\Phi_4\zeta(t) \tag{3.24}$$

with $\Phi_4 = \rho(t)\big(e_3 + e_6\big)\Omega\big(e_3^T + e_6^T\big) - e_6\Omega e_6^T$
According to the measurement noise (2.2), we can write:

$$0 \le y^T(t)Q_f y(t) - f_s^T(t)f_s(t) = \zeta^T(t)\Phi_5\zeta(t) \tag{3.25}$$

with $\Phi_5 = \mathcal{H}_e(e_7E_h^TQ_fC_he_1^T) + e_7E_h^TQ_fE_he_7^T + e_1C_h^TQ_fC_he_1^T - e_7e_7^T$
Now, let us focus on the requirement of H_∞ performance regarding external disturbances ($\omega \neq 0$). From the H_∞ criterion (2.7) and under zero initial conditions, we can write;

$$\dot{V}(t) + x^T(s)Q_dx(s) - \gamma^2w^T(s)w(s) < 0 \tag{3.26}$$

From (3.23), (3.24), and (3.25), the inequality (3.26) is satisfied $\forall\zeta(t) \neq 0$ under external disturbances and measurement noise failures if:

$$\zeta^T(t)\big(\sum_{\ell=1}^{6}\Phi_\ell + \mathcal{H}_e\big(\mathcal{T}\mathcal{G}_{h\bar{h}}\big)\big)\zeta(t) < 0 \tag{3.27}$$

with $\Phi_6 = e_1Q_de_1^T - \gamma^2e_8e_8^T$

Recall that inequality (3.27) is nonlinear due to the presence of the bilinear term $\mathcal{H}_e(\mathcal{T}\mathcal{G}_{h\bar{h}})$. To handle this nonlinearity, we introduce a slack variable and apply a standard congruence transformation as follows:
Let $X \in \mathbb{R}^{n\times n}$ be a nonsingular matrix andϵ_1 and ϵ_2 be arbitrary real scalars. Then, define $\mathcal{T} = \big[I \;\; 0 \;\; \epsilon_1 I \;\; 0 \;\; \epsilon_2 I \;\; \epsilon_1 I \;\; 0 \;\; 0\big]^T X^{-T}$. Next, take the congruence of (3.27) using the block-diagonal matrix. $D_X = \mathrm{diag}\{\underbrace{X, ..., X}_{\times 6}, \mathrm{I}, \mathrm{I}\}$, Then, applying quadratic functions lemma in [11] together with the Schur complement, yields the inequality $\Lambda_{h\bar{h}}<0$ with Λ_{ij} defined (3.1), whose, introduce the change of variables $F_{\bar{h}} = K_{\bar{h}}X \in \mathbb{R}^{m\times n}$ and $\bar{\mathrm{E}} = X^T\mathrm{E}X$ where $\mathrm{E} \in \{P, Q_1, Q_2, Q_d, R_1, R_2, \Omega, M, S\}$. Finally, using lemma 5 in [12] and since $\rho(t) \in [\rho_m, \rho_M]$, the inequality $\dot{V}(t)<0$, $\forall\zeta(t)\neq 0$, is guaranteed if the conditions of Theorem 3.1 hold. □

In the next, the practical implementation of the proposed NGDET mechanism is presented below, where the adaptive NGDET threshold function $\rho(t)$ is adjusted from the error signal using the Algorithm 1.

4. Illustrative example

In this section, a numerical example is presented to show the effectiveness and advantages of the proposed NGDET control and to highlight its improvements compared with recent related results reported in the literature.

Algorithm 1 Computation of the adaptive NGDET threshold function $\rho(t)$

Require: Error signal $\varepsilon(t_k)$, bounds ρ_m and ρ_M
Ensure: Adaptive threshold $\rho(t)$

1: Initialize the neural parameters W and b.
2: **for** each error sample $e = \|\varepsilon(t_k)\|$ **do**
3: Compute the neuron activation: $z = -We + b, \quad \sigma = \frac{1}{1+\exp(-z)}$.
4: Compute the adaptive threshold: $\rho(t) = \rho_m + (\rho_M - \rho_m)\sigma$.
5: Evaluate a reward function based on the tracking error and triggering events. triggered $= (e^2(t_k) > \rho\, e^2(t_k))$, reward $= -e^2(t_k) - \beta \times$ triggered.
6: Compute the gradients: $\frac{d\sigma}{dz} = \sigma(1-\sigma)$; $d_\rho = (\rho_{\max} - \rho_{\min})\sigma(1-\sigma)$; $\frac{\partial J}{\partial W} = -2e\, d_\rho$; $\frac{\partial J}{\partial b} = -d_\rho$.
7: Update W and b using a gradient-descent learning rule. $W \leftarrow W + \alpha\frac{\partial J}{\partial W}$ reward, $b \leftarrow b + \alpha\frac{\partial J}{\partial b}$ reward.
8: **end for**
9: Use the resulting $\rho(t)$ in the event-triggering condition.
return $\rho(t)$

4.1. **Example :** Let us consider the benchmark of a tunnel diode circuit [12], where the diode is characterized by $i_D(t) = 0.002v_D(t) + 0.01v_D^3(t)$.

In this example, the state vector is defined as $[x_1(t)\ x_2(t)] = [v_D(t)\ i_L(t)]$ with $v_D(t)$ being the measurement output and $|x_1(t)| \leq 3$. The dynamic of a tunnel diode circuit can be presented through the T–S fuzzy model (2.1) with:

$$A_1 = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, A_2 = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{w1} = B_{w2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad E_1 = E_2 = 0.65 \text{ and } Q_f = 2.$$

For $z(t) = v_D(t)$ as the premise variable, the membership functions are defined as follows:

$$h_1(z(t)) = \begin{cases} 1 + \frac{1}{3}z(t), & \text{for} \quad z(t) \in [-3, 0] \\ 1 - \frac{1}{3}z(t), & \text{for} \quad z(t) \in [0, 3] \end{cases}, \; h_2(z(t)) = 1 - h_1(z(t)) \; .$$

Case 1: Nominal case (fault-free scenario) In this first study, the NCS is analyzed under ideal operating conditions, i.e., in the absence of measurement noise and external disturbances. Specifically, the fault and disturbance signals are set as follows: $f_s(t) = 0$, and $\omega(t) = 0$. This configuration corresponds to a healthy network environment without degradation or abnormal behavior.

The main objective of this nominal case study is to enlarge the maximum admissible network-induced delay MANID(τ_M), while preserving closed-loop stability and also to reduce the communication burden by lowering the packet transmission frequency over the network.

The performance of the proposed NCS framework is assessed through a comparative analysis with several existing event-triggered and networked control approaches reported in the literature [5–7, 9, 12–14, 18, 20, 21, 21].

For consistency with existing works, the identical parameter settings as in the previous studies are used, $\tau_m = 0.1$. Numerical solutions of the LMIs in Theorem 3.1 were obtained using the YALMIP/SeDuMi solver, with the parameters $\epsilon_1 = 10^{-5}$ and $\epsilon_2 = 10^{-3}$. Assuming a sampling period $h = 0.01s$ for the clock-driven sensors, the solution of Theorem 3.1 provides the PDC state-feedback gains and event triggering weighting matrix as: $K_1 = 10^{-3} \times \begin{bmatrix} 0.016 & 0.466 \end{bmatrix}$, $K_2 = 10^{-3} \times \begin{bmatrix} -0.001 & -0.023 \end{bmatrix}$ and $\Omega = 10^{-3} \times \begin{bmatrix} 0.010 & -0.004 \\ -0.004 & 0.355 \end{bmatrix}$.

In addition, the achieved MANID(τ_M) is 450ms. This value is significantly higher than those reported in related studies. Table 1 presents a comparative analysis

TABLE 1. Comparison of the MANID(τ_M), TPN and TR

Methods	$m_{aub}(\bar{\tau})$	**TPN**	**TR**
[7] $t \in [0, 2s]$	–	101	50.6%
[14] $t \in [0, 20s]$	0.20	108	35.64%
[20] $t \in [0, 20s]$	–	588	29.4%
[21] $t \in [0, 20s]$	–	25	25%
[6] $t \in [0, 30s]$	0.21	73	24.3%
[13] $t \in [0, 10s]$	0.20	86	17.2%
[18] $t \in [0, 20s]$	0.10	60	15%
[9] $t \in [0, 20s]$	0.24	56	14%
[5] $t \in [0, 40s]$	–	43	10.75%
[12] $t \in [0, 5s]$	0.41	41	8.2%
Theorem 3.1, $t \in [0, 5s]$			
$\lambda = 3.3$	$450\,ms$	41	8.2%
$\lambda = 3.5$	–	40	8%
$\lambda = 4$	–	38	7.6%
$\lambda = 4.6$	–	36	7.2%

of the total Number of Transmitted Packets (TPN) and the Transmission Rate (TR). In the present study, both TPN and TR are evaluated for multiple values of the discount factor λ in the NGDET threshold function $\rho(t)$. The results show that increasing λ reduces TPN and TR, thereby lowering the communication burden while enlarging the delay bound. These findings highlight the robustness and improved performance of the proposed NGDET control compared to existing methods under adverse conditions.

Case 2: Robust control design : In this second study, the robustness of the proposed NGDET control against external disturbance attenuation and measurement noise is evaluated under realistic adverse operating conditions.

By solving the LMIs of Theorem 3.1 with $\tau_m = 0.1$, $\epsilon_1 = 0.0078$ and $\epsilon_2 = 0.0445$ and $h = 0.01s$, we obtained MANID(τ_M) = 180ms and the H_∞ performance level of $\gamma = 0.6$. The resulting PDC state-feedback gains are given by: $K_1 = \begin{bmatrix} 0.0253 & 0.4655 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0.0322 & 0.5048 \end{bmatrix}$ and event-triggering weighting matrix $\Omega = \begin{bmatrix} 0.3140 & 1.3749 \\ 1.3749 & 17.1474 \end{bmatrix}$.

Numerical simulations are performed with the initial condition $x(0) = \begin{bmatrix} 1 & 0.5 \end{bmatrix}^T$

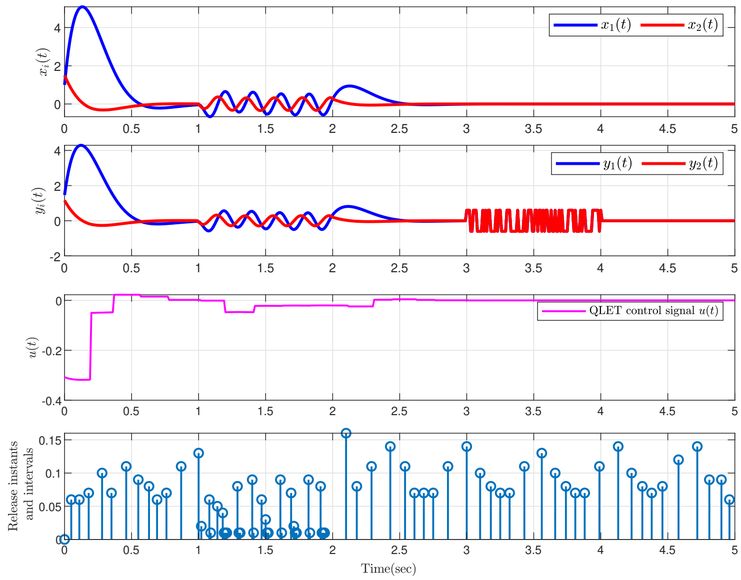

FIGURE 2. States trajectory $x_i(t)$, output $y_i(t)$, NGDET control signal $u(t)$, triggering instants t_k and corresponding intervals.

and the external disturbance signal $\omega(t) = 10sin(0.3t)$ which is injected during the interval $[1, 2s]$ and $\omega(t) = 0$ otherwise. In addition, the measurement noise signal $f_s(t)$ is modeled as a Bernoulli random process and injected into the measurement channel over the interval $[3, 4s]$ and $f_s(t) = 0$ elsewhere. In this simulation scenario, the adaptive NGDET threshold function $\rho(t)$ is bounded by $\rho_m = 0.1$ and $\rho_M = 0.8$. Figure 2 illustrates the NCS closed-loop responses under the proposed NGDET control strategy. Despite external disturbances and measurement noise, the state of the system remains bounded and rapidly converges to the origin, confirming the robustness of the proposed NGDET control strategy.

More specifically, during the disturbance interval $t \in [1, 2s]$, the triggering events become more frequent, and the intervals between events decrease, indicating that additional control updates are generated to preserve closed-loop performance. As the disturbances disappear and the system approaches equilibrium, the triggering frequency is reduced and larger intervals are observed between successive events. This adaptive behavior demonstrates the effectiveness of the proposed NGDET controller in rejecting perturbations while efficiently reducing unnecessary transmissions.

Figure 3 further illustrates the measurement noise signal, the adaptive NGDET threshold function $\rho(t)$, the neural weight $W(t)$, and the neural bias $b(t)$.

In this scenario, only 73 data packets out of 500 sampled data packets are transmitted to the ZOH over the interval $[0, 5s]$, corresponding to a TR of 14.6%. Consequently, 85.4% of the communication resources are saved while maintaining the stability of the NCS closed-loop, even in the presence of external disturbances and measurement noise. These results clearly demonstrate the efficiency and the robustness of the proposed NGDET control in reducing communication load while preserving closed-loop performances.

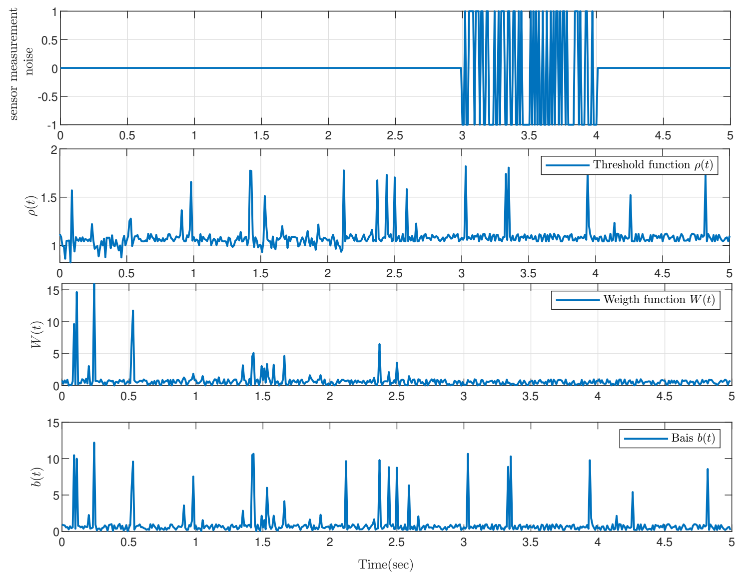

FIGURE 3. Measurement noise signal $f_s(t)$, adaptive neural gradient descent threshold function $\rho(t)$, neural weight $W(t)$ and the bias $b(t)$ (case 2).

5. Conclusion

In this paper, a robust event-triggered control strategy has been developed for networked T-S fuzzy systems subject to external disturbances and measurement noise. By integrating a neural gradient descent mechanism with the PDC controller via NCS, the triggering threshold is adaptively adjusted online, thereby reducing redundant transmissions by activating data exchange only when required while preserving the desired closed-loop performance of NCS. Based on the convenient choice of LKF, A set of sufficient LMI conditions is developed to guarantee the stability of the NCS closed-loop dynamics together with a prescribed disturbance attenuation level H_∞. Simulation results demonstrate that the proposed approach effectively reduces network utilization and enlarges the admissible delay bounds compared with the existing event-triggered mechanism while maintaining strong robustness against disturbances and measurement noise. These findings confirm the effectiveness of the proposed framework in achieving a desirable trade-off between communication efficiency and control performance. Further research will be devoted to extending the present framework to a fault-tolerant control mechanism able to handle actuator and sensor failures, as well as resilient strategies against cyber-attacks such as denial-of-service and false-data injection.

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