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ADDITIVE MAPPINGS COVERING GENERALIZED (α_1, α_2)-DERIVATIONS IN SEMIPRIME RINGS

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Abstract. In this paper we study additive maps satisfying some identities and investigate the following: Let R be a semiprime ring, α_1, α_2 be automorphisms on R and Δ, δ be additive maps from R to R. If Δ and δ satisfying any one of the following identities

(i) $\Delta(x^n y^n) = \Delta(x^n)\alpha_1(y^n) + \alpha_2(x^n)\delta(y^n)$

(ii) $\Delta(x^n) = \Delta(x^{n-m})\alpha_1(x^m) + \alpha_2(x^{n-m})\delta(x^m)$, for all $x, y \in R$,

then Δ will be a generalized (α_1, α_2)-derivation with associated (α_1, α_2)-derivation δ on R.

1. Introduction and preliminaries

We consider R to be an associative ring with identity e throughout. Recall that a ring R is semiprime if for any $x \in R$, $xRx = 0$, implies $x = 0$. A ring R is said to be n-torsion free, if whenever $nx = 0$ for $x \in R$, implies $x = 0$, where $n > 1$ is any integer. A map $\delta : R \to R$ will be called a derivation if it is additive and fulfils the condition $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in R$. In case δ satisfying for all $x \in R$, $\delta(x^2) = \delta(x)x + x\delta(x)$, we call such δ a Jordan derivation on R. An additive map $\Delta : R \to R$ such that $\Delta(xy) = \Delta(x)y + x\delta(y)$ for all $x, y \in R$ termed as generalized derivation if there exists a derivation $\delta : R \to R$. Following Zalar [10], an additive map $H : R \to R$ termed as left centralizer and Jordan left centralizer of R if $H(xy) = H(x)y$ and $H(x^2) = H(x)x$ respectively holds for all $x, y \in R$. One can think about the structure of right centralizer and Jordan right centralizer by same approach. It is obvious fact that every left centralizer and right centralizer is Jordan left centralizer and Jordan right centralizer respectively on R. The converse statement to this fact is not guaranteed. A brief description of the allied matter can be found in [10]. For given two automorphisms α_1 and α_2 on R, an additive map $\delta : R \to R$ is recognized as (α_1, α_2)-derivation and Jordan (α_1, α_2)-derivation if $\delta(xy) = \delta(x)\alpha_1(y) + \alpha_2(x)\delta(y)$ and $\delta(x^2) = \delta(x)\alpha_1(x) + \alpha_2(x)\delta(x)$ respectively holds for all $x, y \in R$. An additive mapping $\delta : R \to R$ is called an α_1-derivation (or a skew derivation) on R if $\delta(xy) = \delta(x)y + \alpha_1(x)\delta(y)$ for all pairs $x, y \in R$. An additive mapping $\Delta : R \to R$ is called a generalized α_1-derivation

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(or a generalized skew derivation) on R if $\Delta(xy) = \Delta(x)y + \alpha_1(x)\delta(y)$ associated with an α_1-derivation (or a skew derivation) δ on R for all pairs $x, y \in R$. An additive map $\Delta : R \to R$ is called a generalized (α_1, α_2)-derivation, if there exists an (α_1, α_2)-derivation $\delta : R \to R$ such that $\Delta(xy) = \Delta(x)\alpha_1(x) + \alpha_2(x)\delta(y)$ holds for all $x, y \in R$.

An additive map $\Delta : R \to R$ is called Jordan generalized (α_1, α_2)-derivation if there exists a Jordan (α_1, α_2)-deviation such that $\Delta(x^2) = \Delta(x)\alpha_1(x) + \alpha_2(x)\delta(x)$, for all $x \in R$. An additive mapping $H : R \to R$ is called left α_1-centralizer and Jordan left α_1-centralizer of R if $H(xy) = H(x)\alpha_1(y)$ and $H(x^2) = H(x)\alpha_1(x)$ respectively holds for all $x, y \in R$. One can define the right α_1-centralizer and Jordan right α_1-centralizer of R by familiar approach. Obviously, every left (resp. right) α_1-centralizer is a Jordan left (resp. right) α_1-centralizer on R. It is facile to see that $\Delta : R \to R$ is a generalized (α_1, α_2)-derivation if and only if Δ is of the form $\Delta = \delta + H$, where δ is an (α_1, α_2)-derivation and H is left α_1-centralizer of R. The notion of generalized (α_1, α_2)-derivations cover the concept of derivation and the concept of left α_1-centralizer. Some results on generalized (α_1, α_2)-derivation α_1-centralizer has done by Muthana and Rehman in [7]. Let $\Delta, \delta : R \to R$ be two additive mappings which satisfy the following identities

(i) $\Delta(x^n y^n) = \Delta(x^n)\alpha_1(y^n) + \alpha_2(x^n)\delta(y^n)$
(ii) $\Delta(x^n) = \Delta(x^{n-m})\alpha_1(x^m) + \alpha_2(x^{n-m})\delta(x^m)$ for all $x, y \in R$.

Now, a natural question arises that the additive mappings $\Delta, \delta : R \to R$ satisfying (i) and (ii) having some restrictions on m, n will be a generalized (α_1, α_2)-derivation and (α_1, α_2)-derivation respectively on R. We have the affirmative answer of this question. Infact, we prove that $\Delta, \delta : R \to R$ are generalized (α_1, α_2)-derivation and (α_1, α_2)-derivation respectively on semiprime ring R if satisfying (i) and (ii). For more literature review one can look in to [2, 3, 6, 8].

Some pertinent lemmas are listed below for the development of the proofs of the main theorems:

Lemma 1.1 ([1, Theorem 3.1]). *Let R be a 2-torsion free semiprime ring and δ a Jordan (α_1, α_2)-derivation on R, where α_1 and α_2 are automorphisms on R, and Δ is a generalized Jordan (α_1, α_2)- derivation on R, then Δ is a generalized (α_1, α_2)-derivation on R.*

Lemma 1.2 ([4, Lemma 1]). *Let R be a $m!$-torsion free semiprime ring. Suppose that $y_1, y_2, ..., y_m \in R$ satisfy $\sum_{i=1}^{m} \alpha^i y_i = 0$, for $\alpha = 1, 2, ..., m$. Then $y_i = 0$ for all i.*

Lemma 1.3 ([5, Theorem 2]). *Let R be a 2-torsion free semiprime ring. Suppose that α_1, α_2 are automorphisms on R. If δ is a Jordan (α_1, α_2)-derivation on R, then δ is an (α_1, α_2)-derivation on R.*

2. Main results

We begin our investigation with following theorems.

Theorem 2.1. *Let n be any integer greater than 1, R be a $n!$-torsion free semiprime ring and α_1, α_2 be automorphisms on R. If two additive mappings $\Delta, \delta : R \to R$ are such that $\Delta(x^n y^n) = \Delta(x^n)\alpha_1(y^n) + \alpha_2(x^n)\delta(y^n)$ for all $x, y \in R$, then Δ will be a generalized (α_1, α_2)-derivation associated with (α_1, α_2)-derivation δ on R.*

Proof. We have

$$\Delta(x^n y^n) = \Delta(x^n)\alpha_1(y^n) + \alpha_2(x^n)\delta(y^n) \text{ for all } x, y \in R. \tag{2.1}$$

Replacing x and y by e in the above equation, we get $\delta(e) = 0$. Again, replacing x by $x + e$ in (2.1), we get

$$\begin{aligned} &\binom{n}{1}[\Delta(x^{n-1}y^n) - \Delta(x^{n-1})\alpha_1(y^n) - \alpha_2(x^{n-1})\delta(y^n)] \\ &+\binom{n}{2}[\Delta(x^{n-2}y^n) - \Delta(x^{n-2})\alpha_1(y^n) - \alpha_2(x^{n-2})\delta(y^n)] + ... \\ &+\binom{n}{n-1}[\Delta(xy^n) - \Delta(x)\alpha_1(y^n) - \alpha_2(x)\delta(y^n)] \\ &+\binom{n}{n}[\Delta(y^n) - \Delta(e)\alpha_1(y^n) - \delta(y^n)] = 0. \end{aligned} \tag{2.2}$$

Particularly take $x = e$ in (2.1), we find

$$\Delta(y^n) = \Delta(e)\alpha_1(y^n) + \delta(y^n) \text{ for all } x, y \in R. \tag{2.3}$$

In view of (2.3), (2.2) reduces to the form

$$\begin{aligned} &\binom{n}{1}[\Delta(x^{n-1})y^n - \Delta(x^{n-1})\alpha_1(y^n) - \alpha_2(x^{n-1})\delta(y^n)] \\ &+\binom{n}{2}[\Delta(x^{n-2}y^n) - \Delta(x^{n-2})\alpha_1(y^n) - \alpha_2(x^{n-2})\delta(y^n)] \\ &+... + \binom{n}{n-1}[\Delta(xy^n) - \Delta(x)\alpha_1(y^n) - \alpha_2(x)\delta(y^n)] = 0. \end{aligned} \tag{2.4}$$

Substitute kx for x in (2.4) to get

$$\begin{aligned} \binom{n}{1}k^{n-1}[\Delta(x^{n-1}y^n) \;&-\; \Delta(x^{n-1})\alpha_1(y^n) - \alpha_2(x^{n-1})\delta(y^n)] \\ &+ \;\binom{n}{2}k^{n-2}[\Delta(x^{n-2}y^n) - \Delta(x^{n-2})\alpha_1(y^n) - \alpha_2(x^{n-2})\delta(y^n)] + ... \\ &+ \;\binom{n}{n-1}k[\Delta(xy^n) - \Delta(x)\alpha_1(y^n) - \alpha_2(x)\delta(y^n)] = 0. \end{aligned}$$

Which implies that

$$\sum_{r=1}^{n-1}\binom{n}{r}k^{n-r}[\Delta(x^{n-r}y^n) - \Delta(x^{n-r})\alpha_1(y^n) - \alpha_2(x^{n-r})\delta(y^n)] = 0 \text{ for all } x, y \in R.$$

By application of Lemma 1.2, we arrive at

$$\binom{n}{r}[\Delta(x^{n-r}y^n - \Delta(x^{n-r})\alpha_1(y^n) - \alpha_2(x^{n-r})\delta(y^n)] = 0 \text{ for all } x, y \in R,$$

$r = 1, 2, \cdots, n-1$. Next putting $r = n-1$, we get

$$n[\Delta(xy^n) - \Delta(x)\alpha_1(y^n) - \alpha_2(x)\delta(y^n)] = 0 \text{ for all } x, y \in R.$$

Since R is n-torsion free, we find that

$$\Delta(xy^n) = \Delta(x)\alpha_1(y^n) + \alpha_2(x)\delta(y^n) \text{ for all } x, y \in R. \tag{2.5}$$

In place of y, put $y + e$ in the above equation to get

$$\begin{aligned}&\binom{n}{1}[\Delta(xy^{n-1}) - \Delta(x)\alpha_1(y^{n-1}) - \alpha_2(x)\delta(y^{n-1})]\\ &+\binom{n}{2}[\Delta(xy^{n-2}) - \Delta(x)\alpha_1(y^{n-2}) - \alpha_2(x)\delta(y^{n-2})]\\ &+ ... + \binom{n}{n-1}[\Delta(xy) - \Delta(x)\alpha_1(y) - \alpha_2(x)\delta(y)]] = 0.\end{aligned} \tag{2.6}$$

Substitute ky in place of y in (2.6) to get

$$\sum_{r=1}^{n-1}\binom{n}{r}k^{n-r}[\Delta(xy^{n-r}) - \Delta(x)\alpha_1(y^{n-r}) - \alpha_2(x)\delta(y^{n-r})] = 0 \text{ for all } x, y \in R.$$

Application of Lemma 1.2 yields that

$$\binom{n}{r}[\Delta(xy^{n-r}) - \Delta(x)\alpha_1(y^{n-r}) - \alpha_2(x)\delta(y^{n-r})] = 0 \text{ for all } x, y \in R,$$

$r = 1, 2, \cdots, n-1$. Particularly, take $r = n-1$, we obtain

$$n[\Delta(xy) - \Delta(x)\alpha_1(y) - \alpha_2(x)\delta(y)] = 0 \text{ for all } x, y \in R.$$

Using torsion restriction on R, we find that

$$\Delta(xy) = \Delta(x)\alpha_1(y) + \alpha_2(x)\delta(y) \text{ for all } x, y \in R. \tag{2.7}$$

If putting yz in place of y in the above relation and manipulating in two ways, then observe the resulting two relation as follows:

$$\begin{aligned}\Delta(xyz) &= \Delta(x(yz))\\ &= \Delta(x)\alpha_1(y)\alpha_1(z) + \alpha_2(x)\delta(yz) \text{ for all } x, y, z \in R.\end{aligned} \tag{2.8}$$

Again,

$$\begin{aligned}\Delta(xyz) &= \Delta((xy)z)\\ &= \Delta(x)\alpha_1(y)\alpha_1(z) + \alpha_2(x)\delta(y)\alpha_1(z) + \alpha_2(x)\alpha_2(y)\delta(z).\end{aligned} \tag{2.9}$$

Equating (2.8) and (2.9), we obtain

$$\alpha_2(x)[\delta(yz) - \delta(y)\alpha_1(z) - \alpha_2(y)\delta(z)] = 0 \text{ for all } x, y, z \in R.$$

Replace x by $\alpha_2^{-1}(x)$ to get $x[\delta(yz) - \delta(y)\alpha_1(z) - \alpha_2(y)\delta(z)] = 0$ for all $x, y, z \in R$. Multiplying left side by $\delta(yz) - \delta(y)\alpha_1(z) - \alpha_2(y)\delta(z)$ and using semiprimeness of R, we obtained that $\delta(yz) = \delta(y)\alpha_1(z) + \alpha_2(y)\delta(z)$ for all $y, z \in R$. Hence δ is acting as an (α_1, α_2)-derivation. Therefore Δ is a generalized (α_1, α_2)-derivation on R as desired.

□

The following results are some immediate consequences of the above theorem:

Corollary 2.2 ([2, Theorem 1.3])**.** *Let $n > 1$ be any fixed integer and R be any $n!$-torsion free semiprime ring. If $\Delta, \delta : R \to R$ are additive mappings satisfying $\Delta(x^n y^n) = \Delta(x^n)y^n + x^n\delta(y^n)$ for all $x, y \in R$. Then, Δ is generalized derivation with associated derivation δ on R.*

Proof. We will get the required result by taking $\alpha_1 = \alpha_2 = I$ (identity mapping) in Theorem 2.1. □

Corollary 2.3. *Let $n > 1$ be any fixed integer and R be any $n!$-torsion free semiprime ring. If $\Delta : R \to R$ is an additive mapping satisfying $\Delta(x^n y^n) = \Delta(x^n)\alpha_1(y^n)$ for all $x, y \in R$, where α_1 is an automorphism on R. Then, Δ is a left α_1-centralizer on R.*

Proof. Taking $d = 0$ in Theorem 2.1, we get the required result. □

Corollary 2.4. *Let $n > 1$ be any fixed integer and R be any $n!$-torsion free semiprime ring. If $\Delta, \delta : R \to R$ are additive mappings satisfying $\Delta(x^n y^n) = \Delta(x^n)y^n + \alpha_2(x^n)\delta(y^n)$ for all $x, y \in R$, where α_2 is an automorphism on R. Then, Δ is generalized skew derivation with associated skew derivation δ on R.*

Proof. We will get the required result by taking $\alpha_1 = I$ (identity mapping) in Theorem 2.1. □

Corollary 2.5. *Let $n > 1$ be any fixed integer and R be any $n!$-torsion free semiprime ring. If $d : R \to R$ is additive mapping satisfying $\delta(x^n y^n) = \delta(x^n)y^n + \alpha_2(x^n)\delta(y^n)$ for all $x, y \in R$, where α_2 is an automorphism on R. Then, δ is skew derivation on R.*

Proof. Consequence of the above corollary.

□

The next step is to figure out and prove when the additive mapping Δ in (ii) is a generalized (α_1, α_2)-derivation on R.

Theorem 2.6. *Let $n > 1$ be any fixed integer and R be any $n!$-torsion free semiprime ring with $n - 1 \geq m \geq 1$. If $\Delta, \delta : R \to R$ are additive mappings satisfying $\Delta(x^n) = \Delta(x^{n-m})\alpha_1(x^m) + \alpha_2(x^{n-m})\delta(x^m)$ for all $x \in R$, where α_1 and α_2 are automorphisms on R. Then Δ is acting as a generalized (α_1, α_2)-derivation with associated (α_1, α_2)-derivation δ on R.*

Proof. Given that

$$\Delta(x^n) = \Delta(x^{n-m})\alpha_1(x^m) + \alpha_2(x^{n-m})\delta(x^m) \text{ for all } x \in R. \tag{2.10}$$

Replace x by e to get $\delta(e) = 0$. Again, replacing x by $x + ky$ in (2.10), we obtain

$$\Delta((x+ky)^n) = \Delta((x+ky)^{n-m})\alpha_1((x+ky)^m)+\alpha_2((x+ky)^{n-m})\delta((x+ky)^m) \tag{2.11}$$

for all $x, y \in R$. Which can be written as $kP_1(x,y)+k^2P_2(x,y)+...+k^nP_n(x,y) = 0$ for all $x, y \in R$, where $k = 1, 2, 3, ..., n$ and $P_i(x, y)$ denotes the sum of the terms involving i factors of $\alpha_1(y)$ and $\alpha_2(y)$ in (2.11), then from Lemma 1.2, we have $P_i(x, y) = 0$ for all $i = 1, 2, 3, ..., n$. In particular

$$\begin{aligned} P_1(x,y) &= \Delta(x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}) \\ &\quad -\Delta(x^{n-m-1}y + x^{n-m-2}yx + ... + yx^{n-m-1})\alpha_1(x^m) \\ &\quad -\Delta(x^{n-m})\alpha_1(x^{m-1}y + x^{m-2}yx + ... + yx^{m-1}) \\ &\quad -\alpha_2(x^{n-m-1}y + x^{n-m-2}yx + ... + yx^{n-m-1})\delta(x^m) \\ &\quad -\alpha_2(x^{n-m})\delta(x^{m-1}y + x^{m-2}yx + ... + yx^{m-1}) \\ &= 0. \end{aligned} \tag{2.12}$$

Then, putting $x = e$ in the above equation, we find that $m\Delta(y) - m\Delta(e)\alpha_1(y) - m\delta(y) = 0$. Since R is $n!$ torsion free, therefore

$$\Delta(y) = \Delta(e)\alpha_1(y) + \delta(y). \tag{2.13}$$

Replacing y by x^n, we get

$$\Delta(x^n) = \Delta(e)\alpha_1(x^n) + \delta(x^n). \tag{2.14}$$

Equating (2.10) and (2.14), we find that

$$\delta(x^n) = \Delta(x^{n-m})\alpha_1(x^m) - \Delta(e)\alpha_1(x^n) + \alpha_2(x^{n-m})\delta(x^m). \tag{2.15}$$

Substitute x^{n-m} for y in (2.13) to find

$$\Delta(x^{n-m}) = \Delta(e)\alpha_1(x^{n-m}) + \delta(x^{n-m}). \tag{2.16}$$

Multiplying right side by $\alpha_1(x^m)$, we get

$$\delta(x^{n-m})\alpha_1(x^m) = \Delta(x^{n-m})\alpha_1(x^m) - \Delta(e)\alpha_1(x^n). \tag{2.17}$$

Use (2.17) in (2.15) to get

$$\delta(x^n) = \delta(x^{n-m})\alpha_1(x^m) + \alpha_2(x^{n-m})\delta(x^m) \text{ for all } x \in R. \tag{2.18}$$

Again, replacing x by $x + ky$ in (2.18), we have

$$\delta((x+ky)^n) = \delta((x+ky)^{n-m})\alpha_1((x+ky)^m)+\alpha_2((x+ky)^{n-m})\delta((x+ky)^m) \tag{2.19}$$

for all $x, y \in R$. Expanding the above, we have $kP_1(x, y) + k^2P_2(x, y) + ... + k^nP_n(x, y) = 0$ for all $x, y \in R$, where $k = 1, 2, 3, ..., n$ and $P_i(x, y)$ denotes the sum of the terms involving i factors of $\alpha_1(y)$ and $\alpha_2(y)$ in (2.19), then from Lemma 1.2, we have $P_i(x, y) = 0$ for all $i = 1, 2, 3, ..., n$. In particular,

$$\begin{aligned} P_2(e,y) &= \frac{n(n-1)}{2}\delta(y^2) - \frac{(n-m)(n-m-1)}{2}\delta(y^2) \\ &\quad -m(n-m)\alpha_2(y)\delta(y) - \frac{m(m-1)}{2}\delta(y^2) \\ &\quad -m(n-m)\delta(y)\alpha_1(y) \\ &= 0. \end{aligned} \tag{2.20}$$

This reduces to $m(n-m)\delta(y^2) = m(n-m)\delta(y)\alpha_1(y) - m(n-m)\alpha_2(y)\delta(y)$. Since R is $n!$ torsion free, we get

$$\delta(y^2) = \delta(y)\alpha_1(y) + \alpha_2(y)\delta(y) \text{ for all } y \in R. \tag{2.21}$$

Hence δ is a Jordan (α_1,α_2)-derivation. Replacing y by y^2 in (2.13), we obtain

$$\begin{aligned}\Delta(y^2) &= \Delta(e)\alpha_1(y^2)+\delta(y^2)\\ &= \Delta(e)\alpha_1(y^2)+\delta(y)\alpha_1(y)+\alpha_2(y)\delta(y)\\ &= [\Delta(e)\alpha_1(y)+\delta(y)]\alpha_1(y)+\alpha_2(y)\delta(y)\\ &= \Delta(y)\alpha_1(y)+\alpha_2(y)\delta(y).\end{aligned} \tag{2.22}$$

Since R is semiprime ring, therefore by Lemma 1.1, Δ is a generalized (α_1,α_2)-derivation on R.

□

Some direct consequences driven by the above theorem are listed below:

Corollary 2.7 ([9, Theorem 1.3])**.** *Let $n>1$ be any fixed integer and R be any $n!$-torsion free semiprime ring with $n-1\geq m\geq 1$. If $\Delta,\delta:R\to R$ are additive mappings satisfying $\Delta(x^n)=\Delta(x^{n-m})x^m+x^{n-m}\delta(x^m)$ for all $x\in R$. Then, Δ is generalized derivation with associated derivation δ on R.*

Proof. In particular, put $\alpha_1=\alpha_2=I$ in Theorem 2.6, we obtain the result. □

Corollary 2.8. *Let $n>1$ be any fixed integer and R be any $n!$-torsion free semiprime ring with $n-1\geq m\geq 1$. If $\Delta:R\to R$ is an additive mapping satisfying $\Delta(x^n)=\Delta(x^{n-m})\alpha_1(x^m)$ for all $x\in R$, where α_1 is an automorphism on R. Then, Δ is left α_1-centralizer on R.*

Proof. Particularly, take $d=0$ in Theorem 2.6, we arrive at the conclusion. □

Corollary 2.9. *Let $n>1$ be any fixed integer and R be any $n!$-torsion free semiprime ring with $n-1\geq m\geq 1$. If $d:R\to R$ is an additive mapping satisfying $\delta(x^n)=\delta(x^{n-m})\alpha_1(x^m)+\alpha_2(x^{n-m})\delta(x^m)$ for all $x\in R$, where α_1 and α_2 are automorphisms on R. Then, δ is an (α_1,α_2)-derivation on R.*

Proof. Continue the proof of the Theorem 2.6 from (2.18), we find $\delta(y^2)=\delta(y)\alpha_1(y)+\alpha_2(y)\delta(y)$ for all $y\in R$. Use Lemma 1.3 to get the required result. □

Corollary 2.10 ([9, Corollary 1.4])**.** *Let $n>1$ be any fixed integer and R be any $n!$-torsion free semiprime ring with $n-1\geq m\geq 1$. If $d:R\to R$ is an additive mapping satisfying $\delta(x^n)=\delta(x^{n-m})(x^m)+(x^{n-m})\delta(x^m)$ for all $x\in R$. Then, δ is derivation on R.*

Proof. Putting $\alpha_1=\alpha_2=I$ in the above corollary, we find the result. □

3. Examples

The above theorems are supported by the following examples.

Example 3.1. $R=\left\{\begin{pmatrix} a & 0\\ 0 & b\end{pmatrix}\mid a,b\in 2\mathbb{Z}_8\right\}$ is a ring under matrix addition and multiplication, where $\mathbb{Z}_8$ denotes the ring of integers modulo 8. Define mappings $\Delta,\delta,\alpha_1,\alpha_2:R\to R$ by $\Delta\begin{pmatrix} a & 0\\ 0 & b\end{pmatrix}=\begin{pmatrix} 0 & 0\\ 0 & b\end{pmatrix}$, $d\begin{pmatrix} a & 0\\ 0 & b\end{pmatrix}=\begin{pmatrix} a & 0\\ 0 & 0\end{pmatrix}$,

$\alpha_1\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b^* \end{pmatrix}$ and $\alpha_2\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a^* & 0 \\ 0 & b \end{pmatrix}$, where a^* and b^* are the additive inverse of a and b respectively in $2\mathbb{Z}_8$. It is clear that R is neither a semiprime ring nor 2-torsion free and $\Delta, \delta, \alpha_1, \alpha_2$ satisfy the identity $\Delta(x^2y^2) = \Delta(x^2)\alpha_1(y^2) + \alpha_2(x^2)\delta(y^2)$ for all $x, y \in R$ but Δ is not a generalized (α_1, α_2)-derivation on R, so semiprimeness and torsion restriction are essential conditions for Theorem 2.1.

Example 3.2. Let $R, \Delta, \delta, \alpha_1, \alpha_2$ be the same as in above Example. In particular, if we consider $n = 4$ and $m = 2$, then it is clear that R is not 4!-torsion free and of course not semiprime and $\Delta, \delta, \alpha_1, \alpha_2$ satisfy the identity $\Delta(x^4) = \Delta(x^{4-2})\alpha_1(x^2) + \alpha_2(x^2)\delta(x^{4-2})$ for all $x \in R$ but Δ is not a generalized (α_1, α_2)-derivation on R. Hence semiprimeness and torsion restriction are not superfluous in Theorem 2.6.

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