Gulf Journal of Mathematics
Vol 23, Issue 2 (2026) 1-14

ISSN: 2309-4966
https://doi.org/10.56947/w0sqw731

SIMPLIFIED LYAPUNOV STABILITY FOR CHAOTIC SYSTEMS

SAMIR AZIOUNE[1,2,*], WIDAD LAOUIRA[1,2], MESSAOUD BERKAL[1,2,*], E. M. ELSAYED[3] AND M. B. ALMATRAFI[4]

Abstract. In this paper, based on Lyapunov stability theory, sufficient conditions are established for the local stabilization of the equilibrium point of a class of chaotic nonlinear systems. The proposed conditions are expressed as explicit algebraic inequalities, making them easy to verify without computing the eigenvalues of the Jacobian matrix. The results support the design of control strategies for suppressing chaos and stabilizing system trajectories. Their effectiveness is demonstrated through several three-, four-, and five-dimensional chaotic systems, with numerical simulations confirming the theoretical results.

Keywords. Nonlinear dynamical systems, chaos control, Lyapunov stability, stability criteria, chaotic and hyperchaotic dynamics
2020 Mathematics Subject Classification. Primary 34H10; Secondary 37N35, 93C10, 93C15, 93C95.

1. Introduction

Non-linear dynamical systems that exhibit chaotic and hyperchaotic behavior have attracted sustained interest over the last decades, driven by both their fundamental complexity and their growing relevance in engineering and applied sciences. Chaotic dynamics are characterized by strong sensitivity to initial conditions and parameter variations, which may generate irregular trajectories, broadened spectra, and limited predictability. While such features can be beneficial in some contexts (e.g., secure communications and encryption), they often degrade performance and reliability in control, signal processing, and decision-making systems, thereby motivating the development of effective stabilization and chaos-suppression strategies.

Discrete dynamical systems, which offer an effective mathematical structure for reproducing and interpreting nonlinear behaviors in many different kinds of scientific and practical applications, are crucial to the examination of complex phenomena which vary in discrete time stages [14, 15, 16]

Date: Received: Jun 9, 2026; Revised: Jul 7, 2026; Accepted: Jul 14, 2026.
* Corresponding author
© The Author(s) 2026. This article is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License. To view a copy of the licence, visit https://creativecommons.org/licenses/by-nc-nd/4.0/.

Beyond classical engineering applications, chaotic behavior has also emerged naturally in socio-economic and game-theoretic models when firms adopt adaptive or boundedly rational adjustment mechanisms. Recent contributions highlight how stability loss of equilibrium points may trigger complex oscillations, period-doubling cascades, and chaos, together with the necessity of control to restore economically meaningful equilibria. For instance, chaos and mixed-mode oscillations have been reported in a Bertrand duopoly game under bounded rationality and quadratic costs, where a state-feedback scheme successfully stabilizes the Bertrand–Nash equilibrium and yields explicit stability boundaries in parameter space [1]. Similarly, a Cournot triopoly game with bounded rationality can experience destabilization through flip and Neimark–Sacker bifurcations, and chaos control techniques (e.g., OGY-type methods) can be employed to recover stable Nash outcomes [2]. These studies illustrate a broader message: in high-dimensional non-linear models, stabilization hinges on rigorous yet practically verifiable stability conditions.

In non-linear control theory, feedback stabilization of chaotic dynamics is typically established through analytical criteria such as the Routh–Hurwitz conditions or Lyapunov's direct method. In particular, Lyapunov theory provides a systematic framework to certify stability by constructing an appropriate Lyapunov function and enforcing negativity of its derivative along system trajectories. However, for higher-order chaotic and hyperchaotic systems, standard Lyapunov-based derivations can become algebraically involved, conservative, or difficult to verify, which limits their usability in controller synthesis and real-time analysis.

Along another active line of research, several feedback and synchronization results have been obtained using Jacobian-based conditions and related eigenvalue constraints. For example, Zambrano-Serrano et al. [13] investigated a four-dimensional jerk-based chaotic oscillator and assessed local stability via Routh–Hurwitz arguments. Qi et al. [7] derived sufficient synchronization conditions in terms of LMIs, which implicitly incorporate Jacobian spectral properties. Lyapunov and LMI-based designs have also been widely used in Takagi–Sugeno fuzzy control of complex systems [9]. Moreover, Jacobian conditions appear naturally in biological and neuromuscular modeling [12] and in discrete/fractional strategies for chaotic satellite systems [8]. In [4], additional Jacobian-related requirements combined with Routh–Hurwitz analysis were proposed to stabilize several benchmark chaotic systems via feedback. Despite their effectiveness, Jacobian/eigenvalue-based approaches may still lead to non-trivial computations for high-order dynamics and may not always yield compact, directly checkable inequalities. Motivated by these challenges, the present work develops a simplified Lyapunov-based analytical approach for the stability assessment and stabilization of a class of nth-order chaotic and hyperchaotic non-linear systems. The main objective is to reduce the computational and algebraic burden typically encountered in Lyapunov-based analysis while preserving theoretical rigor.

More precisely, we establish explicit sufficient stability conditions expressed in terms of algebraic inequalities, which can be verified directly without the need to compute the eigenvalues of the Jacobian matrix. These conditions provide a

simple and practical alternative to classical eigenvalue-based methods, especially for high-dimensional systems.

Furthermore, the proposed results can be employed in the design of state-feedback control laws to suppress chaotic dynamics and stabilize the system around its equilibrium point. The effectiveness of the approach is illustrated through several representative chaotic systems of different dimensions.

It is important to emphasize that while the fundamental stability conditions utilized in this work are rooted in classical matrix theory, the primary contribution of this 'simplified approach' lies in its practical engineering utility. Rather than claiming high mathematical complexity, this framework is specifically tailored for the ease of controller synthesis and real-time verification by practitioners. By translating established stability theorems into straightforward algebraic inequalities, this approach provides a highly accessible, computation-light tool that bypasses the cumbersome eigenvalue extractions typically required when stabilizing complex, high-dimensional systems.

2. Preliminaries

In this section, we recall fundamental concepts from Lyapunov stability theory that will be used to establish the stability results presented later in this paper.

2.1. **Lyapunov Stability.**

Consider the autonomous nonlinear system

$$\dot{x} = f(x), \qquad f(0) = 0, \tag{2.1}$$

with $x \in \mathbb{R}^n$.

A continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ is called a Lyapunov function for the equilibrium $x = 0$ if

$$V(0) = 0, \qquad V(x) > 0 \quad \text{for all } x \neq 0, \tag{2.2}$$

and

$$\dot{V}(x) = \nabla V(x)\, f(x) \leq 0. \tag{2.3}$$

Theorem 2.1 ([5]). *If there exists a Lyapunov function V such that $\dot{V}(x) \leq 0$ in a neighborhood of the origin, then the equilibrium $x = 0$ of system (2.1) is stable. If, in addition, $\dot{V}(x) < 0$ for all $x \neq 0$, then the equilibrium is asymptotically stable.*

2.2. **Quadratic Lyapunov Functions for Linear Systems.**

Consider the linear system

$$\dot{x} = Ax. \tag{2.4}$$

A classical Lyapunov candidate is the quadratic function

$$V(x) = x^\top P x, \qquad P = P^\top > 0. \tag{2.5}$$

The derivative of V along the trajectories of (2.4) is

$$\dot{V}(x) = x^\top (A^\top P + PA)x. \tag{2.6}$$

Theorem 2.2 ([3]). *The equilibrium $x = 0$ of system (2.4) is asymptotically stable if there exists a symmetric positive definite matrix P such that*

$$A^\top P + PA < 0. \tag{2.7}$$

These results form the theoretical foundation for the stability analysis carried out in the next section, where they are applied to a nonlinear system written as the sum of a linear term and higher-order nonlinearities.

3. Main Results

Proposition 3.1. *Let the nonlinear system be written as*

$$\dot{x} = Ax + h(x), \tag{3.1}$$

where $A \in \mathbb{R}^{n\times n}$ and $h : \mathbb{R}^n \to \mathbb{R}^n$ is continuously differentiable in a neighborhood of the origin. Assume that

$$h(0) = 0, \quad Dh(0) = 0.$$

Then there exist constants $c > 0$ and $r > 0$ such that

$$\|h(x)\| \le c\|x\|^2, \quad \forall \|x\| < r. \tag{3.2}$$

Proof. Since h is continuously differentiable in a neighborhood of the origin, with

$$h(0) = 0, \quad Dh(0) = 0,$$

the first-order Taylor expansion of h at $x = 0$ implies that the linear part vanishes at the origin. Hence the nonlinear term is of order at least two with respect to x. Therefore, there exist constants $c > 0$ and $r > 0$ such that

$$\|h(x)\| \le c\|x\|^2, \quad \forall \|x\| < r.$$

This completes the proof. □

Theorem 3.2. *Consider the nonlinear system (3.1), where $h(0) = 0$, $Dh(0) = 0$, and h is continuously differentiable in a neighborhood of the origin. Suppose that for every $k = 1, \dots, n$, the following condition holds:*

$$2a_{kk} + \sum_{\substack{i=1 \\ i\neq k}}^{n} |a_{ki} + a_{ik}| < 0. \tag{3.3}$$

Then the equilibrium point $x = 0$ of system (3.1) is locally asymptotically stable.

Proof. Consider the quadratic Lyapunov function

$$V(x) = \frac{1}{2}x^\top x. \tag{3.4}$$

Clearly, $V(x)$ is positive definite. Its derivative along the trajectories of system (3.1) is

$$\dot{V}(x) = x^\top(Ax + h(x)) = x^\top Ax + x^\top h(x). \tag{3.5}$$

Note that

$$x^\top Ax = x^\top \left(\frac{A + A^\top}{2}\right) x. \tag{3.6}$$

Let $x = (x_1, \dots, x_n)^\top$. Then

$$x^\top Ax = \sum_{k=1}^{n} a_{kk}x_k^2 + \sum_{1\le i<j\le n} (a_{ij} + a_{ji})x_i x_j. \tag{3.7}$$

Using the inequality

$$2|x_i x_j| \le x_i^2 + x_j^2, \tag{3.8}$$

we obtain

$$(a_{ij} + a_{ji})x_i x_j \le \frac{1}{2}|a_{ij} + a_{ji}|(x_i^2 + x_j^2). \tag{3.9}$$

Thus, from (3.7) and (3.9),

$$x^\top A x \le \frac{1}{2}\sum_{k=1}^{n}\left(2a_{kk} + \sum_{\substack{i=1\\ i\neq k}}^{n}|a_{ki} + a_{ik}|\right)x_k^2. \tag{3.10}$$

Define

$$\eta_k = -\left(2a_{kk} + \sum_{\substack{i=1\\ i\neq k}}^{n}|a_{ki} + a_{ik}|\right) > 0, \qquad k = 1, \dots, n. \tag{3.11}$$

Let

$$\eta = \frac{1}{2}\min_{1\le k\le n}\eta_k > 0. \tag{3.12}$$

Then (3.10) yields

$$x^\top A x \le -\eta\|x\|^2. \tag{3.13}$$

On the other hand, by Proposition 3.1, there exist constants $c > 0$ and $r > 0$ such that

$$\|h(x)\| \le c\|x\|^2, \quad \forall \|x\| < r. \tag{3.14}$$

Therefore,

$$|x^\top h(x)| \le \|x\|\,\|h(x)\| \le c\|x\|^3. \tag{3.15}$$

Combining (3.5), (3.13), and (3.15), we obtain

$$\dot{V}(x) \le -\eta\|x\|^2 + c\|x\|^3 = -\|x\|^2(\eta - c\|x\|). \tag{3.16}$$

Choose

$$\rho = \min\left\{r, \frac{\eta}{2c}\right\}. \tag{3.17}$$

Then for every $x \neq 0$ such that $\|x\| < \rho$, we have

$$\eta - c\|x\| \ge \frac{\eta}{2} > 0.$$

Hence,

$$\dot{V}(x) \le -\frac{\eta}{2}\|x\|^2 < 0. \tag{3.18}$$

Therefore, $V(x)$ is positive definite and $\dot{V}(x)$ is negative definite in a neighborhood of the origin. By Lyapunov's direct method, the equilibrium point $x = 0$ is locally asymptotically stable. □

Remark 3.3. Without loss of generality, the stability analysis is performed around the origin. If the system admits an equilibrium point $x^* \neq 0$, the change of variables $y = x - x^*$ shifts the equilibrium to the origin, allowing the proposed results to be directly applied.

Remark 3.4. It is worth noting that the stability condition established in Theorem 3.2 is mathematically equivalent to requiring that the symmetric part of the Jacobian matrix, $\frac{A+A^\top}{2}$, is strictly diagonally dominant with negative diagonal entries. By the well-known Gershgorin circle theorem, this strict diagonal dominance guarantees that the symmetric part of the matrix is negative definite, which inherently ensures that the Lyapunov derivative satisfies $\dot{V}(x) < 0$. While this foundation is rooted in classical linear algebra, explicitly formulating the condition as the straightforward algebraic inequality presented in our theorem serves a distinct practical purpose. For applied control engineering—particularly when stabilizing high-dimensional chaotic and hyperchaotic systems—this explicit formulation translates a theoretical matrix property into a highly accessible, computation-light tool. It allows practitioners to analytically determine the required stabilizing feedback gains without the need for cumbersome numerical eigenvalue extractions or complex matrix transformations.

4. Application to a Three-Dimensional Controlled Lorenz System

In this section, we apply the proposed control strategy to a three-dimensional controlled Lorenz system in order to demonstrate the effectiveness of the developed theorem in stabilizing chaotic dynamics. The Lorenz system is a classical nonlinear dynamical system that exhibits chaotic behavior and has been widely used as a benchmark in control theory. Due to its strong nonlinear coupling and sensitivity to initial conditions, it provides an appropriate framework for testing the applicability of the proposed stabilization method.

4.1. **System Definition.**

The Lorenz system [6] is defined by:

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = \rho x - y - xz, \\ \dot{z} = xy - \beta z, \end{cases} \tag{4.1}$$

where $\sigma = 10$, $\rho = 28$, and $\beta = \dfrac{8}{3}$.

The system admits the equilibrium point

$$E_0 = (0, 0, 0).$$

4.2. **Control at E_0.**

The controlled Lorenz system is given by:

$$\begin{cases} \dot{x} = \sigma(y - x) - u_1, \\ \dot{y} = \rho x - y - xz - u_2, \\ \dot{z} = xy - \beta z - u_3, \end{cases} \tag{4.2}$$

where the feedback controllers are defined as:

$$u_1 = k_1 x, \quad u_2 = k_2 y, \quad u_3 = k_3 z,$$

with $k_1, k_2, k_3 \in \mathbb{R}$.

Proof. The Jacobian matrix of the controlled system (4.2) at the equilibrium point $(0,0,0)$ is:

$$A = \begin{pmatrix} -(\sigma + k_1) & \sigma & 0 \\ \rho & -(1+k_2) & 0 \\ 0 & 0 & -(\beta + k_3) \end{pmatrix}. \tag{4.3}$$

We apply the sufficient condition for local asymptotic stability:

$$2a_{kk} + \sum_{\substack{i=1 \\ i \neq k}}^{3} |a_{ki} + a_{ik}| < 0, \quad k = 1,2,3. \tag{4.4}$$

For $k = 1$:

$$2a_{11} + |a_{12} + a_{21}| + |a_{13} + a_{31}| = -2(\sigma + k_1) + |\sigma + \rho|.$$

Since $\sigma > 0$ and $\rho > 0$, we obtain

$$-2(\sigma + k_1) + \sigma + \rho < 0 \iff k_1 > \frac{\rho - \sigma}{2}.$$

For $k = 2$:

$$2a_{22} + |a_{21} + a_{12}| + |a_{23} + a_{32}| = -2(1 + k_2) + |\rho + \sigma|.$$

Thus,

$$-2(1 + k_2) + \rho + \sigma < 0 \iff k_2 > \frac{\sigma + \rho - 2}{2}.$$

For $k = 3$:

$$2a_{33} + |a_{31} + a_{13}| + |a_{32} + a_{23}| = -2(\beta + k_3) < 0 \iff k_3 > -\beta.$$

Therefore, if

$$k_1 > \frac{\rho - \sigma}{2}, \quad k_2 > \frac{\sigma + \rho - 2}{2}, \quad k_3 > -\beta,$$

then all the conditions of the theorem are satisfied, and the equilibrium point $E_0 = (0,0,0)$ is locally asymptotically stable.

□

4.2.1. *Numerical Validation.* For the classical Lorenz parameters $\sigma = 10$, $\rho = 28$, and $\beta = \dfrac{8}{3}$, the stability conditions become

$$k_1 > 9, \quad k_2 > 18, \quad k_3 > -\frac{8}{3}.$$

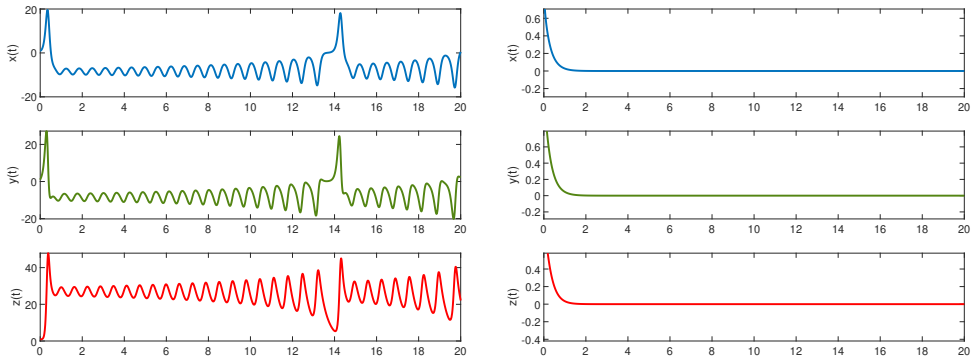
We choose the feedback gains

$$k_1 = 10, \quad k_2 = 19, \quad k_3 = 1.$$

Since the theoretical result ensures only local asymptotic stability, the initial condition is selected in a neighborhood of the equilibrium point:

$$(x_0, y_0, z_0) = (1,1,1).$$

The simulation results show that all state variables converge asymptotically to the equilibrium point $E_0 = (0,0,0)$, confirming the theoretical prediction. The time evolution of the system states is illustrated in Figure 1.



(A) (B)

FIGURE 1. Time series of the Lorenz system: (a) chaotic dynamics of the uncontrolled system, (b) stabilization of the system under the proposed control law with gains $k_1 = 10$, $k_2 = 19$, and $k_3 = 1$, showing convergence to the equilibrium point $E_0 = (0, 0, 0)$ from the initial condition $(1, 1, 1)$.

5. Application to a Four-Dimensional Biological System

In this section, we consider a four-dimensional autonomous nonlinear system inspired by the work in [11]. By introducing the parameters a, b, c, μ, α, and β, we formulate the following four-dimensional snap system:

$$\begin{cases} \dot{x} = y, \\ \dot{y} = az, \\ \dot{z} = w, \\ \dot{w} = -x - bz + cy - w + \mu y\left(1 - x^2 + \alpha x^4 - \beta x^6\right), \end{cases} \tag{5.1}$$

where (x, y, z, w) denote the state variables and $(a, b, c, \mu, \alpha, \beta)$ are real-valued system parameters. Due to its strong nonlinearity and high dimensionality, the system is capable of exhibiting a rich variety of dynamical behaviors, including periodic, quasi-periodic, and chaotic motions, depending on the choice of parameters.

5.1. **Uncontrolled Case.** For instance, when

$$(a, b, c, \alpha, \beta, \mu) = (5,\ 24,\ -0.05,\ 2.55,\ 1.7,\ 2.001),$$

the trajectories of system (5.1) display chaotic behavior (see Figure 2a).

5.2. **Controlled Case via Feedback.** To stabilize the system, we introduce a linear feedback control law:

$$\begin{cases} \dot{x} = y - u_1, \\ \dot{y} = az - u_2, \\ \dot{z} = w - u_3, \\ \dot{w} = -x - bz + cy - w + \mu y\,(1 - x^2 + \alpha x^4 - \beta x^6) - u_4, \end{cases} \tag{5.2}$$

with

$$u_1 = kx, \quad u_2 = ky, \quad u_3 = kz, \quad u_4 = kw,$$

where $k > 0$ is the feedback gain. It follows that, for $k > 14$, system (5.2) asymptotically converges to the equilibrium point $(0, 0, 0, 0)$.

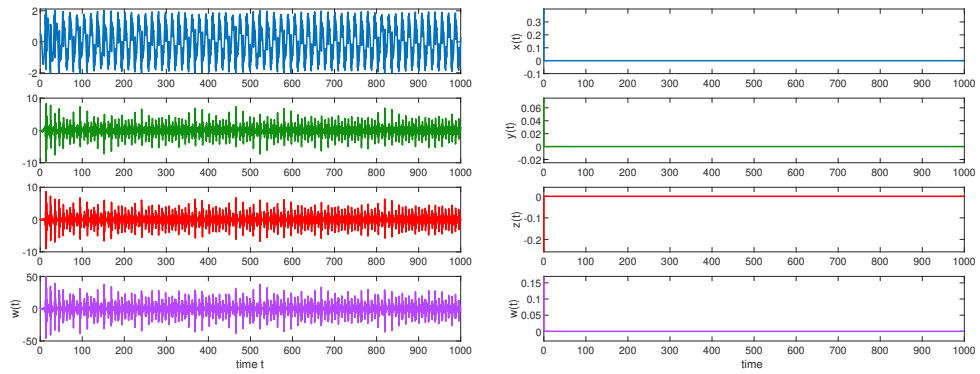
Proof. The Jacobian matrix of the controlled system at the equilibrium point $(0, 0, 0, 0)$ is:

$$A = \begin{pmatrix} -k & 1 & 0 & 0 \\ 0 & -k & a & 0 \\ 0 & 0 & -k & 1 \\ -1 & c & -b & -1-k \end{pmatrix}.$$

Applying the sufficient conditions of the theorem 3.2, we obtain:

$$\begin{aligned} 2a_{11} + |a_{12} + a_{21}| + |a_{13} + a_{31}| + |a_{14} + a_{41}| < 0 \quad &\Leftrightarrow \quad k > 1, \\ 2a_{22} + |a_{12} + a_{21}| + |a_{23} + a_{32}| + |a_{24} + a_{42}| < 0 \quad &\Leftrightarrow \quad k > 3.9755, \\ 2a_{33} + |a_{13} + a_{31}| + |a_{23} + a_{32}| + |a_{34} + a_{43}| < 0 \quad &\Leftrightarrow \quad k > 14, \\ 2a_{44} + |a_{14} + a_{41}| + |a_{24} + a_{42}| + |a_{34} + a_{43}| < 0 \quad &\Leftrightarrow \quad k > 11.9755. \end{aligned}$$

Thus, when $k = 15$, the system (5.2) gradually converges to the equilibrium point $(0, 0, 0, 0)$. This proves the effectiveness of the feedback control approach (see Figure 2b). □



(A) (B)

FIGURE 2. Time series before control (a) and after control (b) in a neighborhood of the origin, for $x(0) = (0.5, 0.1, -0.3, 0.2)$ with feedback coefficient $k = 15$.

6. Application to a Five-Dimensional hyperchaotic System

In this section, we extend our investigation to a higher-dimensional chaotic model. We consider a novel five-dimensional hyperchaotic system [10] which generates a sequence of five state variables (x, y, z, u, v) evolving dynamically over time.

The complexity of the 5D phase space ensures a stronger nonlinearity and unpredictability, which are highly desirable properties for secure image encryption. The additional degrees of freedom intensify the diffusion and confusion effects that are fundamental to cryptographic security.

The system is governed by the following set of differential equations:

$$\begin{cases} \dfrac{dx}{dt} = \gamma(y - x) + \kappa y + x, \\ \dfrac{dy}{dt} = \gamma x + \delta y - xz^2 + yz, \\ \dfrac{dz}{dt} = -\beta z + x^2 + xy + \kappa z, \\ \dfrac{du}{dt} = \varepsilon y + \vartheta u\alpha, \\ \dfrac{dv}{dt} = \rho x + \kappa v + z, \end{cases} \tag{6.1}$$

where the control parameters are chosen as

$$\alpha = 40, \quad \beta = 8, \quad \gamma = 40, \quad \delta = 1, \quad \varepsilon = -0.5, \quad \vartheta = -0.5, \quad \rho = 25.5, \quad \kappa = 0.05.$$

6.1. **Feedback Control Design.** To stabilise the chaotic trajectories around the equilibrium point $(0, 0, 0, 0, 0)$, we introduce a linear feedback control of the form:

$$u_1 = kx, \quad u_2 = ky, \quad u_3 = kz, \quad u_4 = ku, \quad u_5 = kv.$$

The controlled system then becomes:

$$\begin{cases} \dfrac{dx}{dt} = \gamma(y-x) + \kappa y + x - u_1, \\ \dfrac{dy}{dt} = \gamma x + \delta y - xz^2 + yz - u_2, \\ \dfrac{dz}{dt} = -\beta z + x^2 + xy + \kappa z - u_3, \\ \dfrac{du}{dt} = \varepsilon y + \vartheta u\alpha - u_4, \\ \dfrac{dv}{dt} = \rho x + \kappa v + z - u_5. \end{cases} \tag{6.2}$$

When $k > 41.275$, the system (6.2) will gradually converge to the equilibrium point $(0, 0, 0, 0)$.

Proof. The Jacobian matrix of system (6.2) at the equilibrium point $(0, 0, 0, 0, 0)$ is given by:

$$A = \begin{pmatrix} -k-39 & 40.05 & 0 & 0 & 0 \\ 40 & -k+1 & 0 & 0 & 0 \\ 0 & 0 & -k+8.05 & 0 & 0 \\ 0 & -0.5 & 0 & -k-20 & 0 \\ 25.5 & 0 & 1 & 0 & -k+0.05 \end{pmatrix}.$$

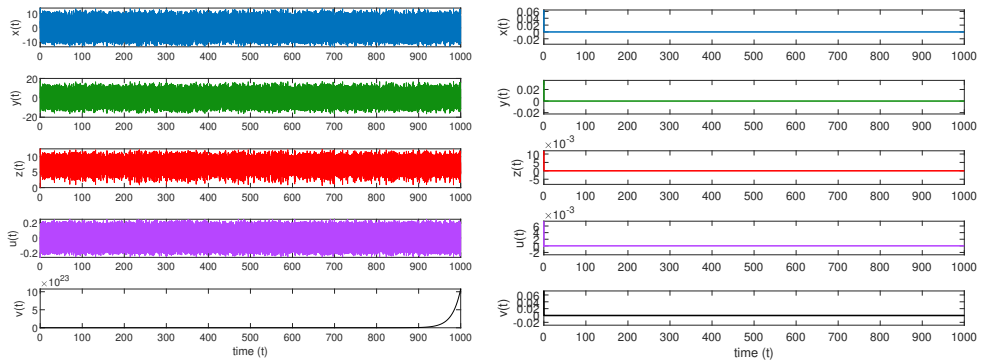
By applying the sufficient stability conditions (Theorem 3.2), we obtain the following inequalities:

$$\begin{cases} -2k + 27.55 < 0 & \Leftrightarrow \quad k > 13.775, \\ -2k + 82.55 < 0 & \Leftrightarrow \quad k > 41.275, \\ -2k + 17.1 < 0 & \Leftrightarrow \quad k > 8.55, \\ -2k - 39.5 < 0 & \Leftrightarrow \quad k > -19.75, \\ -2k + 0.1 < 0 & \Leftrightarrow \quad k > 0.05. \end{cases}$$

The most restrictive condition corresponds to

$$k > 41.275.$$

Therefore, according to Theorem 3.2, the feedback-controlled system (6.2) gradually converges to the equilibrium point $(0, 0, 0, 0, 0)$ when $k > 41.275$. This confirms the effectiveness of the control law in suppressing the hyperchaotic oscillations and ensuring system stabilisation (see Figure 3b). □



(A) (B)

FIGURE 3. Time series before control (a) and after control (b) in a neighborhood of the origin, for $x(0) = (0.1, 0.05, 0.02, 0.01, 0.01)$ with feedback coefficient $k = 42$.

7. Conclusion

Local stability criteria for the equilibrium point of a class of nonlinear dynamical systems have been established using a Lyapunov-based framework. The resulting sufficient conditions are both straightforward and practical, as they provide explicit algebraic inequalities for assessing stability without requiring the computation of the Jacobian eigenvalues.

The main contribution of this work lies in reformulating the stability problem into a set of easily verifiable algebraic inequalities, leading to a simple and computationally efficient stability criterion. This formulation facilitates the synthesis and validation of stabilizing feedback gains, particularly for high-dimensional chaotic and hyperchaotic systems, thereby providing a practical link between theoretical stability analysis and control implementation.

The proposed methodology is applicable to a broad range of nonlinear dynamical systems arising in applied mathematics and engineering, where understanding qualitative system behavior is of fundamental importance. Owing to its simplicity and low computational cost, the proposed criterion can serve as an effective tool for both stability analysis and feedback controller design.

Finally, several representative examples are presented to illustrate the applicability of the proposed results. The numerical simulations confirm the theoretical analysis and demonstrate that suitably designed feedback controllers drive the trajectories of the considered systems asymptotically toward the desired equilibrium point.

References

1. Azioune, M., & Abdelouahab, M. S. (2025). Controlling chaos and mixed mode oscillations in a Bertrand duopoly game with homogeneous expectations and quadratic cost functions. Mathematics and Computers in Simulation, 233, 553-566. https://doi.org/10.1016/j.matcom.2025.02.015

2. Azioune, M., Abdelouahab, M. S., & Lozi, R. (2025). Bifurcation analysis of a cournot triopoly game with bounded rationality and chaos control via the ogy method. International Journal of Bifurcation and Chaos, 35(08), 2530019. https://doi.org/10.1142/S0218127425300198
3. Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). Linear matrix inequalities in system and control theory. Society for industrial and applied mathematics.
4. Laouira, W., & Hamri, N. (2018). Feedback control of chaotic systems by using jacobian matrix conditions. Nonlinear Dynamics and Systems Theory, 18(3), 285-296.
5. LaSalle, J. (1960). Some extensions of Liapunov's second method. IRE Transactions on circuit theory, 7(4), 520-527. https://doi.org/10.1109/TCT.1960.1086720
6. Lorenz, E. N. (1963). Deterministic Nonperiodic Flow. Journal of the Atmospheric Sciences, 20(2), 130–141. https://doi.org/10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2
7. Qi, F., Qu, J., Chai, Y., Chen, L., & Lopes, A. M. (2022). Synchronization of incommensurate fractional-order chaotic systems based on linear feedback control. Fractal and Fractional, 6(4), 221. https://doi.org/10.3390/fractalfract6040221
8. Rashid, S., Hamidi, S. Z., Akram, S., Alosaimi, M., & Chu, Y. M. (2024). Enhancing the trustworthiness of chaos and synchronization of chaotic satellite model: a practice of discrete fractional-order approaches. Scientific Reports, 14(1), 10674. https://doi.org/10.1038/s41598-024-60268-3
9. Rasooli Berardehi, Z., Zhang, C., Taheri, M., Roohi, M., & Khooban, M. H. (2023). Implementation of TS fuzzy approach for the synchronization and stabilization of non-integer-order complex systems with input saturation at a guaranteed cost. Transactions of the Institute of Measurement and Control, 45(13), 2536-2553. https://doi.org/10.1177/01423312231155273
10. Subathra, S., & Thanikaiselvan, V. (2025). Enhanced security for medical images using a new 5D hyperchaotic map and deep learning based segmentation. Scientific Reports, 15(1), 22628. https://doi.org/10.1038/s41598-025-04906-4
11. Vaidyanathan, S., Feki, M., Sambas, A., & Lien, C. H. (2018). A new biological snap oscillator: its modelling, analysis, simulations and circuit design. International Journal of Simulation and Process Modelling, 13(5), 419-432. https://doi.org/10.1504/IJSPM.2018.094734
12. Vignesh, D., Fataf, N. A. A., & Abdul Rahim, M. F. (2023). Synchronization of discrete time fractional order neuromuscular models in the presence and absence of stimulus. Physica Scripta, 98(10), 105242. https://doi.org/10.1088/1402-4896/acf969
13. Zambrano-Serrano, E., & Anzo-Hernández, A. (2021). A novel antimonotic hyperjerk system: Analysis, synchronization and circuit design. Physica D: Nonlinear Phenomena, 424, 132927. https://doi.org/10.1016/j.physd.2021.132927
14. Berkal, M., & Almatrafi, M. B. (2023). Bifurcation and stability of two-dimensional activator–inhibitor model with fractional-order derivative. Fractal and Fractional, 7(5), 344. https://doi.org/10.3390/fractalfract7050344
15. Almatrafi, M. B., Berkal, M., & Hamada, M. Y. (2025). Qualitative Behavior of a Discrete-Time Predator–Prey Model With Holling–Type III Functional Response and Gompertz Growth of Prey. Mathematical Methods in the Applied Sciences, 48(13), 13100-13112. https://doi.org/10.1002/mma.11087
16. Bellout, A., Bououden, R., Houmor, T., & Berkal, M. (2025). Nonlinear dynamics and chaos in fractional-order cardiac action potential duration mapping model with fixed memory length. Gulf Journal of Mathematics, 19(2), 369-383. https://doi.org/10.56947/gjom.v19i2.2765
17. Boudjedour, A., Batiha, I., Boucetta, S., Dalah, M., Zennir, K., & Ouannas, A. (2025). Afinite difference method on uniform meshes for solving the time-space fractional advection-diffusion equation. Gulf Journal of Mathematics, 19(1), 156-168. https://doi.org/10.56947/gjom.v19i1.2524

[1] Laboratory of Mathematics and their Interactions, Department of Mathematics, Abdelhafid Boussouf University, Algeria.
Email address: samir.azioune@centre-univ-mila.dz

[2] Department of Mathematics, Abdelhafid Boussouf University, Mila, R.P 26, Mila, 43000, Algeria.
Email address: w.laouira@centre-univ-mila.dz, m.berkal@centre-univ-mila.dz

[3] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia.
Email address: emmelsayed@yahoo.com

[4] Department of Mathematics, College of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia.
Email address: mmutrafi@taibahu.edu.sa