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# FAULT-TOLERANT STRONG METRIC DIMENSION OF GENERALIZED NEIGHBORHOOD CORONA GRAPHS

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Abstract. In this work, we investigate the fault-tolerant strong metric dimension problem for generalized neighborhood corona graphs. We study the structural and distance-based properties of these graphs and establish the relationship between mutually maximally distant vertices in a graph and those in its generalized neighborhood corona.

*Keywords.* Metric dimension, strong metric dimension, fault-tolerant strong metric dimension, neighborhood corona graphs.
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## 1. Introduction

By a graph $G(V, E)$, we mean a finite, connected and undirected graph with neither loops nor multiple edges. The distance between vertices $x$ and $y$, denoted by $d_G(x, y)$, is defined as the length of a shortest path from $x$ to $y$ in $G$. Whenever there is no confusion, we simply write $d(x, y)$ instead of $d_G(x, y)$. The diameter of $G$ is the maximum distance between two vertices of $G$ and is denoted by $diam(G)$. For a vertex $v$ of $G$, $N(v)$ denotes the set of neighbors that $v$ has in $G$, i.e., $N(v) = \{u \in V : uv \in E\}$. The set $N(v)$ is called the open neighborhood and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood of $v$ in $G$. Two vertices $u, v$ are called true twins if $N[u] = N[v]$.

A vertex $u$ of $G$ is said to be maximally distant from $v$ if for every $w \in N(u)$, $d(u, v) \geq d(w, v)$. If $u$ is maximally distant from $v$ and $v$ is maximally distant from $u$, then $u$ and $v$ are said to be mutually maximally distant.

For $u, v \in V(G)$, let $R\{u, v\} = \{z \in V(G) : d(u, z) \neq d(v, z)\}$. A set $W \subseteq V(G)$ is called a resolving set of $G$ if $W \cap R\{u, v\} \neq \emptyset$ for any two distinct vertices $u, v \in V(G)$. The minimum cardinality of all resolving sets of $G$ is called the metric dimension of $G$ and it is denoted by $\dim(G)$.

The notion of metric dimension was first introduced by Slater [22], who defined resolving sets under the name locating sets. The same concept was independently

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formulated by Harary and Melter [4]. Since then, the concept of metric dimension has attracted considerable attention and has been extensively investigated by several authors; see for example [1, 2, 5, 7, 9, 10, 17] and the references therein. Moreover, Garey and Johnson [3] showed that the problem of computing the metric dimension of a graph is NP-hard.

A resolving set $W$ is said to be fault-tolerant if for every $u \in W$, $W \setminus \{u\}$ preserves the property of being a resolving set. Analogous to the metric dimension, the fault-tolerant metric dimension is the minimum number $\dim_f(G)$ such that there exists a fault-tolerant resolving set of order $\dim_f(G)$. For further study on fault-tolerant metric dimension, we refer the readers to [6, 8, 12, 19].

Another invariant, more restricted than the metric dimension was presented by Sebő and Tannier in [21], and studied further in several articles [11, 13, 15, 16, 20, 23]. Following [16], for two vertices $x$ and $y$ in a connected graph $G$, the interval $I[x, y]$ between $x$ and $y$ is said to be the collection of all vertices that belong to some shortest $x - y$ path. For $x, y \in V(G)$, let $P\{x, y\} = \{s \in V(G) : x \in I[y, s] \text{ or } y \in I[x, s]\}$. A set $S \subseteq V(G)$ is called a strong resolving set of $G$ if $S \cap P\{x, y\} \neq \emptyset$ for any two distinct vertices $x, y \in V(G)$. The minimum cardinality of all strong resolving sets of $G$ is called the strong metric dimension of $G$ and it is denoted by $\dim_s(G)$.

A strong resolving set $S$ for $G$ is fault-tolerant if $S \setminus \{s\}$ is also a strong resolving set for each $s$ in $S$. The minimum cardinality of all fault-tolerant strong resolving sets is called the fault-tolerant strong metric dimension of $G$ and is denoted by $\dim_{fs}(G)$. A fault-tolerant strong resolving set of cardinality $\dim_{fs}(G)$ is called fault-tolerant strong metric basis. The fault-tolerant strong metric dimension problem in a graph $G$ is to identify a fault-tolerant strong metric basis in $G$. The notion of fault-tolerant strong metric dimension was first proposed by Krishnan et al. [14] as an extension of the strong metric dimension that allows for vertex failure.

**Lemma 1.1.** [14] *Let $G$ be a simple connected graph and let $S$ be a fault-tolerant strong resolving set of $G$. Let $u, v \in V$ be mutually maximally distant in $G$. Then both $u$ and $v \in S$.*

For each $v_i \in V(G)$, let $H_i$ be a graph of order at least two. The generalized neighborhood corona of $G$ with the family $\{H_1, H_2 \dots H_n\}$, denoted by $G \star [H_i]_1^n$, is the graph obtained from $G$ and the disjoint union of the graphs $H_1, H_2 \dots H_n$ by joining every vertex of $H_i$ to all neighbors of $v_i$ in $G$, for each $i = 1, 2 \dots n$. Thus, the vertex set of $G \star [H_i]_1^n$ is

$$V(G \star [H_i]_1^n) = V(G) \ \cup \ \bigcup_{i=1}^{n} V(H_i),$$

and the edge set is

$$E(G \star [H_i]_1^n) = E(G) \ \cup \ \bigcup_{i=1}^{n} E(H_i) \ \cup \ \bigcup_{i=1}^{n} \{xy : x \in V(H_i), \ y \in N_G(v_i)\}.$$

When all graphs $H_i$ are isomorphic to a fixed graph $H$, the construction is called the neighborhood corona of $G$ and $H$, and is denoted simply by $G \star H$ [18].

## 2. Main results

Throughout this paper, $G$, $H_1, H_2 \dots H_n$ are graphs of orders at least two, unless stated otherwise.

**Lemma 2.1.** *If $u \in V(G)$, then*

$$N_{G\star[H_i]_1^n}(u) = N_G(u) \cup \bigcup_{v \in N_G(u)} V(H_v).$$

*Proof.* The proof follows directly from the definition of $G \star [H_i]_1^n$. □

**Lemma 2.2.** *For every $u, v \in V(G), d_{G\star[H_i]_1^n}(u, v) = d_G(u, v)$.*

*Proof.* Since every edge of $G$ remains an edge in $G \star [H_i]_1^n$, any $u - v$ path in $G$ is also a $u - v$ path in $G \star [H_i]_1^n$. Hence $d_{G\star[H_i]_1^n}(u, v) \leq d_G(u, v)$.

It remains to prove that $d_G(u, v) \leq d_{G\star[H_i]_1^n}(u, v)$. Define a map $\phi$ from $V(G \star [H_i]_1^n)$ to $V(G)$ by

$$\phi(z) = \begin{cases} z & \text{if } z \in V(G) \\ w & \text{if } z \in V(H_w) \end{cases}$$

Let $P : u = x_0\ x_1\ x_2 \dots\ x_l = v$ be a shortest path corresponding to $d_{G\star[H_i]_1^n}(u, v)$. Then clearly for some $i$, $0 < i < l$, $x_i \in V(H_{x_i})$ and $x_j \in V(G), j \neq i$. Hence under the mapping $\phi$, $\phi(P) = \phi(x_0)\ \phi(x_1) \dots\ \phi(x_i) \dots\ \phi(x_l) = x_0\ x_1 \dots\ x_i \dots\ x_l$. Since $d_{G\star[H_i]_1^n}(u, v) = l$, we obtain $d_G(u, v) \leq d_{G\star[H_i]_1^n}(u, v)$, as required. □

**Theorem 2.3.** *Let $u, v \in V(G)$ with $uv \in E(G)$ and $a \in V(H_u)$, $b \in V(H_v)$. Then*

*(i) $d_{G\star[H_i]_1^n}(a, b) = 2$ if $u$ and $v$ have a common neighbor in $G$.*

*(ii) $d_{G\star[H_i]_1^n}(a, b) = 3$ if $u$ and $v$ have no common neighbor in $G$.*

*(iii) $a$ and $b$ are mutually maximally distant in $G \star [H_i]_1^n$.*

*Proof.*
*(i)* $u$ and $v$ have a common neighbor in $G$.
Suppose $x \in N_G(u) \cap N_G(v)$. Then every vertex of $H_u$ is adjacent to $x$, and every vertex of $H_v$ is also adjacent to $x$. Hence $a - x - b$ is a path of length two. Since there is no edge between $a$ and $b$, $d_{G\star[H_i]_1^n}(a, b) = 2$.
*(ii)* $u$ and $v$ have no common neighbor in $G$.
Since $uv \in E(G)$, by Lemma 2.1, $a$ is adjacent to every vertex of $N_G(u)$, in particular to $v$; similarly $b$ is adjacent to every vertex of $N_G(v)$ including $u$. Therefore the path $a - v - u - b$ has length three and the distance between $a$ and $b$ is at most three.

Suppose a shortest path $a - x - b$ of length two exists. Then $x$ must be adjacent to both $a$ and $b$. Such a vertex $x$ cannot lie in any copy of $H_w$, because vertices belonging to different copies are never adjacent. Therefore $x \in V(G)$, and adjacent to both $a$ and $b$ means $x \in N_G(u) \cap N_G(v)$, contradicting the hypothesis. Hence $d_{G\star[H_i]_1^n}(a, b) = 3$.

*(iii)* By *(i)* and *(ii)*, $2 \leq d_{G\star[H_i]_1^n}(a,b) \leq 3$.
Let $a'$ be any neighbor of $a$. Then we have the following cases.
**Case 1:** $a'$ is a vertex of $H_u$.
In this case it is adjacent to every vertex of $N_G(u)$, in particular $v$. Hence $a' - v - u - b$ is a path of length three, $d_{G\star[H_i]_1^n}(a',b) \leq 3$.
**Case 2:** $a'$ is a vertex of $G$.
We consider the following two subcases.
**Case 2.1:** $a'$ is a common neighbor of $u$ and $v$.
Then $a' \in N_G(u) \cap N_G(v)$. Since every vertex of $H_v$ is adjacent to every vertex in $N_G(v)$, we have $a'b \in E(G \star [H_i]_1^n)$. Hence $d_{G\star[H_i]_1^n}(a',b) = 1$.
**Case 2.2:** $a'$ is not a common neighbor of $u$ and $v$.
Since $a' \in G$ is a neighbor of $a$, $a'$ is also a neighbor of $u$. Hence $a' - u - b$ is a path of length two.

Thus, in all possible cases, $d_{G\star[H_i]_1^n}(a',b) \leq d_{G\star[H_i]_1^n}(a,b)$. Hence, $a$ is maximally distant from $b$. Similarly, for every neighbor $b'$ of $b$, $d_{G\star[H_i]_1^n}(a,b') \leq d_{G\star[H_i]_1^n}(a,b)$, and therefore $b$ is maximally distant from $a$. Consequently, $a$ and $b$ are mutually maximally distant in $G \star [H_i]_1^n$. □

**Lemma 2.4.** *Let $u, v \in V(G)$ with $d_G(u,v) \geq 2$. Let $a \in V(H_u)$, $b \in V(H_v)$. Then $d_G(u,v) = d_{G\star[H_i]_1^n}(a,b)$.*

*Proof.* Let $P : u = x_0\ x_1\ x_2\ \ldots\ x_k = v$ be a shortest $u - v$ path in $G$, where $d_G(u,v) = k \geq 2$. In $G \star [H_i]_1^n$, each vertex $a \in V(H_u)$ is adjacent to every vertex of $N_G(u)$, and similarly each vertex $b \in V(H_v)$ is adjacent to all vertices of $N_G(v)$. By Lemma 2.2, $d_{G\star[H_i]_1^n}(u,v) = d_G(u,v) = k$. Consider a path from $a$ to $b$ in $G \star [H_i]_1^n$. Any such path must begin at $a$, move to some vertex in $N_G(u)$, continue through vertices of $G$ along $u - v$ path and finally to $b$ through a vertex in $N_G(v)$. The path $a\ x_1\ x_2\ \ldots\ x_{k-1}\ b$ exists in $G \star [H_i]_1^n$, since $a$ is adjacent to $x_1$, $b$ is adjacent to $x_{k-1}$, and each consecutive pair $x_i$ and $x_{i+1}$ are adjacent in $G$. Clearly this path has length $k$. Hence $d_G(u,v) = d_{G\star[H_i]_1^n}(a,b) = k$. □

**Theorem 2.5.** *Let $u, v \in V(G)$ such that $d_G(u,v) \geq 2$. If $u$ and $v$ are mutually maximally distant in $G$, then $u$ and $v$ are mutually maximally distant in $G\star[H_i]_1^n$.*

*Proof.* We first prove that $v$ is maximally distant from $u$. Let $z$ be an arbitrary neighbor of $v$ in $G \star [H_i]_1^n$. By Lemma 2.1, either $z \in N_G(v)$ or $z \in V(H_w)$ for some $w \in N_G(v)$. Let $d_G(u,v) = l$, $l \geq 2$.
**Case 1:** $z \in N_G(v)$.
Since $u$ and $v$ are mutually maximally distant in $G$, we have $d_G(u,z) \leq d_G(u,v) = l$. By Lemma 2.2, $d_{G\star[H_i]_1^n}(u,z) = d_G(u,z) \leq d_G(u,v) = l$. Hence $v$ is maximally distant from $u$ in $G \star [H_i]_1^n$.
**Case 2:** $z \in V(H_w)$ for some $w \in N_G(v)$.
In this case $z$ is adjacent to every vertex of $N_G(w)$. Let $d_G(u,w) = p$ and let $x \in N_G(w)$ be the neighbor of $w$ lying on a shortest $u - w$ path; then $d_G(u,x) = p-1$. Because any path from $u$ to $z$ must pass through some $x \in N_G(w)$, we have $d_{G\star[H_i]_1^n}(u,z) \leq 1 + \min\limits_{y \in N_G(w)} d_{G\star[H_i]_1^n}(u,y) = 1 + \min\limits_{y \in N_G(w)} d_G(u,y) \leq 1 + d_G(u,x) = 1 + p - 1 = p$. But $w \in N_G(v)$ and $u$ and $v$ are mutually maximally distant in

$G$, so $p = d_G(u,w) \leq d_G(u,v) = l$. Therefore $d_{G\star[H_i]_1^n}(u,z) \leq p \leq l$. Thus every neighbor $z$ of $v$ in $G \star [H_i]_1^n$ satisfies $d_{G\star[H_i]_1^n}(u,z) \leq l$.

This means that $v$ is maximally distant from $u$ in $G \star [H_i]_1^n$. Similarly it can be proved that $u$ is maximally distant from $v$ in $G \star [H_i]_1^n$. □

**Lemma 2.6.** *If $u$ and $v$ are not mutually maximally distant in $G$, then $u$ and $v$ are not mutually maximally distant in $G \star [H_i]_1^n$.*

*Proof.* By assumption $u$ and $v$ are not mutually maximally distant in $G$. This means that at least one of the following holds in $G$.
$(i)$ there exists a neighbor $x \in N_G(u)$ with $d_G(x,v) > d_G(u,v)$ or
$(ii)$ there exists a neighbor $y \in N_G(v)$ with $d_G(y,u) > d_G(u,v)$.

Without loss of generality assume that $(i)$ holds. The vertex $x$ remains a neighbor of $u$ in $G \star [H_i]_1^n$ since every edge of $G$ remains an edge of $G \star [H_i]_1^n$. Moreover by Lemma 2.2, $d_{G\star[H_i]_1^n}(w,t) = d_G(w,t)$ for all $w,t \in V(G)$. Therefore $d_{G\star[H_i]_1^n}(x,v) = d_G(x,v) > d_G(u,v) = d_{G\star[H_i]_1^n}(u,v)$. That is, $x$ is a neighbor of $u$ in $G \star [H_i]_1^n$ such that $d_{G\star[H_i]_1^n}(x,v) > d_{G\star[H_i]_1^n}(u,v)$. The symmetric argument covers the other case. Hence $u$ and $v$ are not mutually maximally distant in $G \star [H_i]_1^n$. □

**Theorem 2.7.** *Let $u,v \in V(G)$ such that $d_G(u,v) \geq 2$. If $u$ and $v$ are mutually maximally distant in $G$, then for every $a \in V(H_u)$ and every $b \in V(H_v)$, the vertices $a$ and $b$ are mutually maximally distant in $G \star [H_i]_1^n$.*

*Proof.* Let $d_G(u,v) = l$. Fix $a \in V(H_u)$ and $b \in V(H_v)$. By Lemma 2.4, $d_{G\star[H_i]_1^n}(a,b) = l$. We first prove that $b$ is maximally distant from $a$. Let $b'$ be any neighbor of $b$ in $G \star [H_i]_1^n$. Then either $b' \in N_G(v)$ or $b' \in V(H_v)$.
**Case 1:** $b' \in N_G(v)$.
Since $v$ is maximally distant from $u$, for all $b' \in N_G(v)$, $d_G(u,b') \leq d_G(u,v) = l$. As $d_{G\star[H_i]_1^n}(a,b) = l$, it means that for all $b' \in N_G(v)$, $d_{G\star[H_i]_1^n}(a,b') \leq d_{G\star[H_i]_1^n}(a,b) = l$. Hence $b$ is maximally distant from $a$.
**Case 2:** $b' \in V(H_v)$.
Since $b'$ is adjacent to every vertex of $N_G(v)$, consider a shortest path in $G$ from $a$ to $v$. Let $y$ be the vertex immediately before $v$ on this shortest path; then $y \in N_G(v)$ and $d_G(a,y) = d_G(a,v) - 1 = l - 1$. Since $b'$ is adjacent to every vertex of $N_G(v)$, we get $d_{G\star[H_i]_1^n}(a,y) + d_{G\star[H_i]_1^n}(y,b') = l - 1 + 1 = l$. It means that for all $b' \in N(H_v)$, $d_{G\star[H_i]_1^n}(a,b') = d_{G\star[H_i]_1^n}(a,b)$. Hence $b$ is maximally distant from $a$.

In both cases $b$ is maximally distant from $a$. Similar argument shows that $a$ is maximally distant from $b$. Hence $a$ and $b$ are mutually maximally distant in $G \star [H_i]_1^n$. □

**Theorem 2.8.** *Let $u,v \in V(G)$ with $d_G(u,v) \geq 2$. For vertices $a \in V(H_u)$ and $b \in V(H_v)$, if $u$ and $v$ are mutually maximally distant in $G$, then $u$ and $b$, as well as $v$ and $a$, are mutually maximally distant in $G \star [H_i]_1^n$.*

*Proof.* The proof is omitted as it is similar to that of Theorem 2.7. □

**Theorem 2.9.** $\dim_{fs}(G \star [H_i]_1^n) \geq \dim_{fs}(G) + \sum_{i=1}^{n} |V(H_i)|$.

*Proof.* Let $S$ be any fault-tolerant strong resolving set of $G \star [H_i]_1^n$. By Theorem 2.3 (iii) and Theorem 2.7, vertices belonging to different copies $H_i$ and $H_j$ are mutually maximally distant, and therefore by Lemma 1.1 every vertex in each $H_i$ and $H_j$ must be included in $S$. This contributes $\sum_{i=1}^{n} |V(H_i)|$ vertices to $S$.

By Theorem 2.5, every pair of mutually maximally distant vertices in $G$ remains mutually maximally distant in $G \star [H_i]_1^n$. Thus, by Lemma 1.1, $S$ must contain a fault-tolerant strong resolving set of $G$, contributing at least $\dim_{fs}(G)$ vertices. Consequently,

$$\dim_{fs}(G \star [H_i]_1^n) \geq \dim_{fs}(G) + \sum_{i=1}^{n} |V(H_i)|.$$

□

**Theorem 2.10.** $\dim_{fs}(G \star [H_i]_1^n) \leq \dim_{fs}(G) + \sum_{i=1}^{n} |V(H_i)|$.

*Proof.* Let $S$ be a fault-tolerant strong resolving set of $G$ with $|S| = \dim_{fs}(G)$. We construct the set

$$S^* = S \cup \bigcup_{i=1}^{n} V(H_i),$$

where all vertices of each $H_i$ are included.

To verify that $S^*$ is a fault-tolerant strong resolving set of $G \star [H_i]_1^n$, it is enough to consider vertices of $G$ which are not mutually maximally distant in $G$. Let $x, y \in V(G)$ be two such vertices. Since $S$ is a fault-tolerant strong resolving set for $G$, $S$ itself will strongly resolve $x$ and $y$. Thus, there is no need to include any additional vertices of $G$ beyond those in $S$. Therefore, $S^*$ is a fault-tolerant strong resolving set of $G \star [H_i]_1^n$. □

Theorems 2.9 and 2.10 imply the following result.

**Theorem 2.11.** $\dim_{fs}(G \star [H_i]_1^n) = \dim_{fs}(G) + \sum_{i=1}^{n} |V(H_i)|$.

## 3. Conclusion

In this work, we have investigated the fault-tolerant strong metric dimension problem of the generalized neighborhood corona graphs. Several structural and distance-based properties were analyzed to understand how the neighborhood corona operation influences fault-tolerant strong resolvability. We have shown that the concept of mutual maximal distance among vertices in the graph $G$ is preserved in its generalized neighborhood corona, establishing a clear connection between their corresponding invariants. This research extends the concept of fault-tolerant strong metric dimension to a broader class of graph operations and highlights its structural consistency and theoretical significance.

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