Gulf Journal of Mathematics
Vol 24, Issue 2 (2026) 1-14

ISSN: 2309-4966
https://doi.org/10.56947/x7drbg27

COEFFICIENT ESTIMATES FOR A $\mathfrak{q}$-BI–BAZILEVIČ-TYPE CLASS OF COMPLEX ORDER ASSOCIATED WITH $\mathfrak{q}$-BERNOULLI POLYNOMIALS

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Abstract. This paper introduces a new $\mathfrak{q}$-analogue of a bi–Bazilevič-type class of complex order generated by the $\mathfrak{q}$-Bernoulli polynomials. The class is defined through suitable subordination conditions involving the Jackson $\mathfrak{q}$-derivative of both the function and its inverse. Using standard coefficient techniques for functions with positive real part, we derive bounds for the initial coefficients $|\delta_2|$ and $|\delta_3|$, together with an estimate for the corresponding Fekete–Szegö functional. Several special cases are also obtained by appropriate choices of the parameters, showing how the proposed class connects with related subclasses of analytic and bi-univalent functions. The results provide a flexible $\mathfrak{q}$-calculus framework for further coefficient and extremal problems in geometric function theory.

Keywords. q-Bernoulli polynomial; q-calculus; Bi-univalent functions; Coefficient estimates.
2020 Mathematics Subject Classification. Primary 30C45 ; Secondary 33D15.

1. Introduction

Geometric function theory studies analytic functions through the geometry of the domains they produce. Questions related to univalence, starlikeness, convexity, subordination, and coefficient estimates are among the main themes of this area and have led to the development of many interesting subclasses of analytic functions; see [8].

Subordination is one of the simplest and most useful tools for describing such classes. Instead of imposing a geometric condition directly on a function, one compares it with a suitable dominant function. This idea gives a flexible way to control the image of an analytic function and has been used successfully in many problems of geometric function theory. In particular, the Ma–Minda approach [12] provides a unified setting for several well-known families of starlike and convex functions.

Date: Received: Aug 21, 2026; Revised: Sep 19, 2026; Accepted: Sep 25, 2026.
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The problem becomes more delicate when the inverse function is also involved. A function is called bi-univalent if both the function and its inverse are univalent in the open unit disk. Although the definition is simple, the corresponding coefficient theory is much more difficult than in the classical univalent case. For this reason, estimates for the initial coefficients and related extremal problems continue to attract considerable attention.

Another useful direction comes from quantum calculus, or $\mathfrak{q}$-calculus. The Jackson $\mathfrak{q}$-derivative [10, 11] gives a parameter-dependent version of the ordinary derivative. This additional parameter makes it possible to construct broader families of analytic functions while keeping a direct connection with the classical setting, since the usual derivative is recovered as $\mathfrak{q} \to 1^-$. This idea has been used extensively in the study of starlike, convex, and bi-univalent functions [6, 7, 5, 13, 14, 19, 17, 16].

Special polynomial families provide another natural source for constructing new subclasses. Their generating functions can be used as dominant functions in subordination conditions and often lead to useful coefficient relations. In the present work, we focus on the $\mathfrak{q}$-Bernoulli polynomials, whose structure combines the classical Bernoulli family with the flexibility of $\mathfrak{q}$-calculus; see [1, 15, 18, 2].

The aim of this paper is to use these ideas in the construction and study of a new family of bi-univalent functions associated with the $\mathfrak{q}$-Bernoulli polynomial generating function. Our approach combines analytic subordination, the Jackson $\mathfrak{q}$-derivative, and a generalized difference-type expression. The resulting framework allows several related situations to be recovered by suitable choices of the parameters.

The main interest is in the initial Taylor–Maclaurin coefficients. In particular, we derive bounds for δ_2 and δ_3 , and investigate the corresponding Fekete–Szegö-type problem. We also discuss several special cases and show how the classical setting appears naturally when $\mathfrak{q} \to 1^-$.

2. Preliminaries

We begin with the notation and basic facts needed throughout the paper.

Let

$$\mathscr{O} = \{\zeta \in \mathbb{C} : |\zeta| < 1\}$$

be the open unit disk. Denote by $\mathscr{A}$ the class of functions analytic in $\mathscr{O}$ and normalized by

$$F(0) = 0, \qquad F'(0) = 1.$$

Hence every function $F \in \mathscr{A}$ has a Taylor–Maclaurin expansion of the form

$$F(\zeta) = \zeta + \sum_{\kappa=2}^{\infty} \delta_\kappa \zeta^\kappa, \qquad \zeta \in \mathscr{O}, \tag{2.1}$$

where $\delta_\kappa \in \mathbb{C}$.

The subclass of $\mathscr{A}$ consisting of univalent functions will be denoted by S.

For analytic functions F_1 and F_2 in $\mathscr{O}$, we write

$$F_1 \prec F_2$$

whenever there exists a Schwarz function ω, analytic in $\mathscr{O}$, such that

$$\omega(0) = 0, \qquad |\omega(\zeta)| < 1,$$

and

$$\mathcal{F}_1(\zeta) = \mathcal{F}_2\big(\omega(\zeta)\big).$$

Thus, subordination means that $\mathcal{F}_1$ is obtained from $\mathcal{F}_2$ after the variable ζ is first moved inside the unit disk by the Schwarz function ω. Equivalently,

$$\mathcal{F}_1 = \mathcal{F}_2 \circ \omega.$$

When $\mathcal{F}_2$ is univalent, this relation has a simple geometric meaning:

$$\mathcal{F}_1 \prec \mathcal{F}_2 \quad \Longleftrightarrow \quad \mathcal{F}_1(0) = \mathcal{F}_2(0) \quad \text{and} \quad \mathcal{F}_1(\mathscr{O}) \subset \mathcal{F}_2(\mathscr{O}).$$

In other words, the image of $\mathscr{O}$ under $\mathcal{F}_1$ lies inside the image produced by $\mathcal{F}_2$.

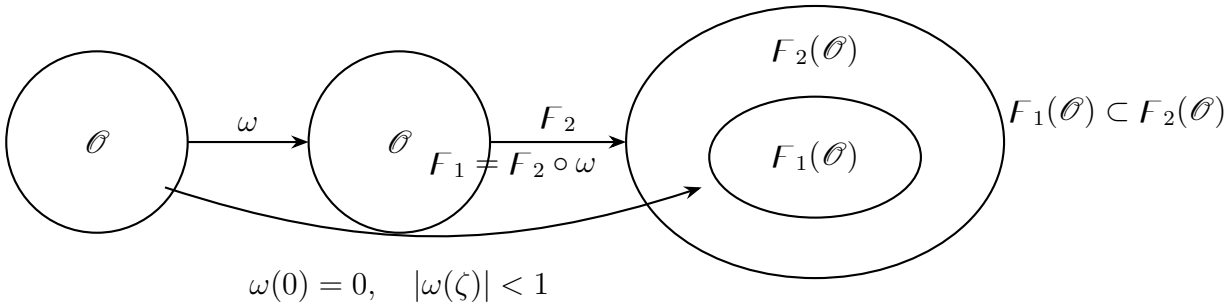


FIGURE 1. Geometric interpretation of the subordination relation $\mathcal{F}_1 \prec \mathcal{F}_2$. The Schwarz function ω maps the unit disk into itself, and the factorization $\mathcal{F}_1 = \mathcal{F}_2 \circ \omega$ implies that, whenever $\mathcal{F}_2$ is univalent, the image $\mathcal{F}_1(\mathscr{O})$ is contained in $\mathcal{F}_2(\mathscr{O})$.

We shall also use the Carathéodory class $\mathscr{P}$, consisting of functions

$$\mathscr{P}(\zeta) = 1 + \mathscr{P}_1\zeta + \mathscr{P}_2\zeta^2 + \mathscr{P}_3\zeta^3 + \cdots \tag{2.2}$$

that satisfy

$$\Re\{\mathscr{P}(\zeta)\} > 0, \qquad \zeta \in \mathscr{O}.$$

Equivalently,

$$\mathscr{P}(\zeta) \prec \frac{1+\zeta}{1-\zeta}.$$

For $\mathcal{F} \in \mathsf{S}$, the inverse function is analytic in a neighborhood of the origin. If $\mathcal{F}$ is given by (2.1), then

$$\begin{aligned} \mathcal{F}^{-1}(\xi) = \xi - \delta_2\xi^2 + \left(2\delta_2^2 - \delta_3\right)\xi^3 \\ - \left(5\delta_2^3 - 5\delta_2\delta_3 + \delta_4\right)\xi^4 + \cdots . \end{aligned} \tag{2.3}$$

A function $\mathcal{F} \in \mathscr{A}$ is called *bi-univalent* if both $\mathcal{F}$ and $\mathcal{F}^{-1}$ are univalent in $\mathscr{O}$. The family of all such functions is denoted by Σ.

We now recall the basic notation from $\mathfrak{q}$-calculus. For $0 < \mathfrak{q} < 1$, the $\mathfrak{q}$-number of a parameter λ is defined by

$$\lceil \lambda \rfloor_{\mathfrak{q}} = \frac{1 - \mathfrak{q}^{\lambda}}{1 - \mathfrak{q}}. \tag{2.4}$$

For $\kappa \in \mathbb{N}$,

$$\lceil \kappa \rfloor_{\mathfrak{q}} = 1 + \mathfrak{q} + \mathfrak{q}^2 + \cdots + \mathfrak{q}^{\kappa-1},$$

and

$$\lim_{\mathfrak{q} \to 1^-} \lceil \lambda \rfloor_{\mathfrak{q}} = \lambda.$$

The corresponding $\mathfrak{q}$-factorial is

$$\lceil \kappa \rfloor_{\mathfrak{q}}! = \lceil \kappa \rfloor_{\mathfrak{q}} \lceil \kappa - 1 \rfloor_{\mathfrak{q}} \cdots \lceil 2 \rfloor_{\mathfrak{q}} \lceil 1 \rfloor_{\mathfrak{q}}, \qquad \lceil 0 \rfloor_{\mathfrak{q}}! = 1.$$

The Jackson $\mathfrak{q}$-derivative of an analytic function $\mathcal{F}$ is defined by

$$\eth_{\mathfrak{q}} \langle \mathcal{F}(\zeta) \rangle = \begin{cases} \dfrac{\mathcal{F}(\zeta) - \mathcal{F}(\mathfrak{q}\zeta)}{(1 - \mathfrak{q})\zeta}, & \zeta \neq 0, \\ \mathcal{F}'(0), & \zeta = 0. \end{cases} \tag{2.5}$$

For a function of the form (2.1), this gives

$$\eth_{\mathfrak{q}} \langle \mathcal{F}(\zeta) \rangle = 1 + \sum_{\kappa=2}^{\infty} \lceil \kappa \rfloor_{\mathfrak{q}} \, \delta_{\kappa} \zeta^{\kappa-1}. \tag{2.6}$$

Applying the same operator to the inverse expansion, we obtain

$$\begin{aligned} \eth_{\mathfrak{q}} \langle \mathcal{F}^{-1}(\xi) \rangle = 1 &- \lceil 2 \rfloor_{\mathfrak{q}} \, \delta_2 \xi + \lceil 3 \rfloor_{\mathfrak{q}} \left(2\delta_2^2 - \delta_3 \right) \xi^2 \\ &- \lceil 4 \rfloor_{\mathfrak{q}} \left(5\delta_2^3 - 5\delta_2\delta_3 + \delta_4 \right) \xi^3 + \cdots . \end{aligned} \tag{2.7}$$

As $\mathfrak{q} \to 1^-$, the Jackson operator approaches the ordinary derivative:

$$\lim_{\mathfrak{q} \to 1^-} \eth_{\mathfrak{q}} \langle \mathcal{F}(\zeta) \rangle = \mathcal{F}'(\zeta).$$

The $\mathfrak{q}$-exponential function is defined by

$$\mathrm{e}_{\mathfrak{q}}(\zeta) = \sum_{\kappa=0}^{\infty} \frac{\zeta^{\kappa}}{\lceil \kappa \rfloor_{\mathfrak{q}}!}. \tag{2.8}$$

It satisfies

$$\eth_{\mathfrak{q}} \left\langle \mathrm{e}_{\mathfrak{q}}(\zeta) \right\rangle = \mathrm{e}_{\mathfrak{q}}(\zeta).$$

We finally recall the $\mathfrak{q}$-Bernoulli polynomials $\mathscr{B}_{\mathfrak{q},\kappa}(\vartheta)$, which will be used to define the dominant function in the next section. Their generating function is

$$\mathscr{B}_{\mathfrak{q}}(\vartheta, \mathsf{h}) = \frac{\mathsf{h}}{\mathrm{e}_{\mathfrak{q}}(\mathsf{h}) - 1} \, \mathrm{e}_{\mathfrak{q}}(\vartheta \mathsf{h}) = \sum_{\kappa=0}^{\infty} \mathscr{B}_{\mathfrak{q},\kappa}(\vartheta) \frac{\mathsf{h}^{\kappa}}{\lceil \kappa \rfloor_{\mathfrak{q}}!}. \tag{2.9}$$

For the coefficient calculations developed later, only the first few polynomials are needed:

$$\mathscr{B}_{\mathfrak{q},0}(\vartheta) = 1,$$

$$\mathscr{B}_{\mathfrak{q},1}(\vartheta) = \vartheta - \frac{\mathfrak{q}}{\lceil 2 \rfloor_{\mathfrak{q}}},$$

and

$$\mathscr{B}_{\mathfrak{q},2}(\vartheta) = \vartheta(\vartheta-1) + \frac{\mathfrak{q}}{\lceil 2\rfloor_{\mathfrak{q}}\lceil 3\rfloor_{\mathfrak{q}}}.$$

These terms will enter directly into the coefficient comparisons used in the proofs of our main results.

3. The $\mathfrak{q}$-Bi–Bazilevič Function Class

The theory of $\mathfrak{q}$-calculus provides an effective and flexible framework for constructing and investigating new subclasses of analytic and bi-univalent functions. In particular, the generating function of $\mathfrak{q}$-Bernoulli polynomials plays an important role in defining polynomial-generated subclasses with adjustable analytic and geometric properties.

Motivated by the structure of $\mathfrak{q}$-Bernoulli polynomials and by recent developments concerning Bazilevič-type bi-univalent functions, we introduce in this section a new subclass of the bi-univalent function class $\sum$. This class is of $\mathfrak{q}$-Bi–Bazilevič type and of complex order, and it is denoted by $\mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$ where the involved parameters control the Bazilevič-type operator, the complex order, and the $\mathfrak{q}$-Bernoulli polynomial structure. The class is defined by means of the Jackson $\mathfrak{q}$-derivative and suitable subordination conditions imposed on both the function and its inverse.

Definition 3.1. *Let $0 < \mathfrak{q} < 1$, $\gamma \in \mathbb{C}$, $M \in \mathbb{C}^*$, and $\vartheta \in [-\pi, \pi]$. A bi-univalent function $\digamma$, normalized by (2.1), is said to belong to the class $\mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$, if both $\digamma$ and its inverse function $\hbar = \digamma^{-1}$, given by (2.3), satisfy the following subordination conditions:*

$$1 + \frac{1}{M}\left[\frac{\zeta^{1-\gamma}\,\eth_{\mathfrak{q}}\langle \digamma(\zeta)\rangle}{(\digamma(\zeta))^{1-\gamma}} - 1\right] \prec \mathscr{B}_{\mathfrak{q}}(\vartheta, \zeta), \qquad (\zeta \in \mathscr{O}), \tag{3.1}$$

and

$$1 + \frac{1}{M}\left[\frac{\varpi^{1-\gamma}\,\eth_{\mathfrak{q}}\langle \hbar(\varpi)\rangle}{(\hbar(\varpi))^{1-\gamma}} - 1\right] \prec \mathscr{B}_{\mathfrak{q}}(\vartheta, \varpi), \qquad (\varpi \in \mathscr{O}). \tag{3.2}$$

The principal specializations of Definition 3.1 are summarized below. In each row the same subordination condition is imposed on both $\digamma$ and its inverse $\hbar = \digamma^{-1}$, with ζ replaced by ϖ for the latter. This compact presentation avoids repeating identical hypotheses.

Parameters	Differential expression	Dominant function
$M = 1$	$\dfrac{\zeta^{1-\gamma}\eth_{\mathfrak{q}}\langle \digamma(\zeta)\rangle}{(\digamma(\zeta))^{1-\gamma}}$	$\mathscr{B}_{\mathfrak{q}}(\vartheta, \zeta)$
$M = 1,\ \gamma = 0$	$\dfrac{\zeta\eth_{\mathfrak{q}}\langle \digamma(\zeta)\rangle}{\digamma(\zeta)}$	$\mathscr{B}_{\mathfrak{q}}(\vartheta, \zeta)$
$\mathfrak{q} \to 1^-$	$1 + \dfrac{1}{M}\left[\dfrac{\zeta^{1-\gamma}\digamma'(\zeta)}{(\digamma(\zeta))^{1-\gamma}} - 1\right]$	$\mathscr{B}(\vartheta, \zeta)$
$M = 1,\ \mathfrak{q} \to 1^-$	$\dfrac{\zeta^{1-\gamma}\digamma'(\zeta)}{(\digamma(\zeta))^{1-\gamma}}$	$\mathscr{B}(\vartheta, \zeta)$
$M = 1,\ \gamma = 0,\ \mathfrak{q} \to 1^-$	$\dfrac{\zeta\digamma'(\zeta)}{\digamma(\zeta)}$	$\mathscr{B}(\vartheta, \zeta)$

4. Main Results

Motivated by the generating function of $\mathfrak{q}$-Bernoulli polynomials and the framework of $\mathfrak{q}$-calculus, we now investigate the coefficient problem for functions belonging to the class $\mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$. In particular, we obtain estimates for the initial Taylor–Maclaurin coefficients $|\delta_2|$ and $|\delta_3|$, which serve as the basis for the Fekete–Szegö results established later.

Firstly, let $\mathscr{P}(\zeta) = 1 + \mathscr{P}_1\zeta + \mathscr{P}_2\zeta^2 + \mathscr{P}_3\zeta^3 + \ldots$, and $\mathscr{P}(\zeta) \prec \mathscr{B}_{\mathfrak{q}}(\vartheta, \zeta)$. Then there exist $\varphi \in \mathscr{P}$ such that $|\varphi(\zeta)| < 1$ in $\mathscr{O}$ and $\mathscr{P}(\zeta) = \mathscr{B}_{\mathfrak{q}}(\vartheta, \varphi(\zeta))$, we have

$$\hbar(\zeta) = (1 + \varphi(\zeta))(1 - \varphi(\zeta))^{-1} = 1 + \ell_1\zeta + \ell_2\zeta^2 + \cdots \in \mathscr{P} \qquad (\zeta \in \mathscr{O}). \tag{4.1}$$

It follows that

$$\varphi(\zeta) = \frac{\ell_1\zeta}{2} + \left(\ell_2 - \frac{\ell_1^2}{2}\right)\frac{\zeta^2}{2} + \left(\ell_3 - \ell_1\ell_2 - \frac{\ell_1^3}{4}\right)\frac{\zeta^3}{2} + \cdots, \tag{4.2}$$

and

$$\begin{aligned}\mathscr{B}_{\mathfrak{q}}(\vartheta, \varphi(\zeta)) &= 1 + \mathscr{B}_{\mathfrak{q},1}(\vartheta)\ \varphi(\zeta) + \frac{1}{\lceil 2 \rfloor_{\mathfrak{q}}}\mathscr{B}_{\mathfrak{q},2}(\vartheta)\ (\varphi(\zeta))^2 + \cdots \\ &= 1 + \mathscr{B}_{\mathfrak{q},1}(\vartheta)\ \ell_1\zeta + \left[\mathscr{B}_{\mathfrak{q},1}(\vartheta)\ \ell_2 + \frac{1}{\lceil 2 \rfloor_{\mathfrak{q}}}\mathscr{B}_{\mathfrak{q},2}(\vartheta)\ell_1^2\right]\zeta^2 + \cdots.\end{aligned} \tag{4.3}$$

It is a classical result that whenever $|\varphi(\zeta)| < 1$ holds for all $\zeta \in \mathscr{O}$, the coefficients ℓ_κ in its expansion satisfy the uniform bound

$$|\ell_\kappa| \leq 1, \qquad \forall\ \kappa \in \mathbb{N}. \tag{4.4}$$

Similarly, the second subordination condition involves a Schwarz function ν, analytic in $\mathscr{O}$, such that $\nu(0) = 0$ and $|\nu(\xi)| < 1$. Accordingly, the function

$$\kappa(\xi) = (1 + \nu(\xi))(1 - \nu(\xi))^{-1} = 1 + \tau_1\xi + \tau_2\xi^2 + \cdots \tag{4.5}$$

belongs to the class $\mathscr{P}$.

It follows that

$$\nu(\xi) = \frac{\tau_1\xi}{2} + \left(\tau_2 - \frac{\tau_1^2}{2}\right)\frac{\xi^2}{2} + \left(\tau_3 - \tau_1\tau_2 - \frac{\tau_1^3}{4}\right)\frac{\xi^3}{2} + \cdots, \tag{4.6}$$

and

$$\mathscr{B}_{\mathfrak{q}}(\vartheta, \nu(\xi)) = 1 + \mathscr{B}_{\mathfrak{q},1}(\vartheta)\ \tau_1\xi + \left[\mathscr{B}_{\mathfrak{q},1}(\vartheta)\ \tau_2 + \frac{1}{\lceil 2 \rfloor_{\mathfrak{q}}}\mathscr{B}_{\mathfrak{q},2}(\vartheta)\tau_1^2\right]\xi^2 + \cdots. \tag{4.7}$$

In the following theorem we determine the initial Taylor–Maclaurin coefficients $|\delta_2|$ and $|\delta_3|$ for the class $\mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$.

Theorem 4.1. *Let $0 < \mathfrak{q} < 1$, $\gamma \in \mathbb{C} \setminus \{0\}$, and $M \in \mathbb{C} \setminus \{0\}$. Suppose that*

$$F(\zeta) = \zeta + \delta_2\zeta^2 + \delta_3\zeta^3 + \cdots$$

belongs to the class $\mathcal{B}_{\Sigma_\mathfrak{q}}(\gamma, M; \vartheta)$. *Then*

$$|\delta_2| \leq \min\left\{ \frac{|M| \left|\lceil 2\rfloor_\mathfrak{q}\,\vartheta - \mathfrak{q}\right|}{\lceil 2\rfloor_\mathfrak{q}\,|\gamma+\mathfrak{q}|}, \sqrt{\frac{2\,\lceil 3\rfloor_\mathfrak{q}\,|M|^2\left|\lceil 2\rfloor_\mathfrak{q}\,\vartheta-\mathfrak{q}\right|^3}{|\mathscr{D}_\mathfrak{q}|}} \right\},$$

and

$$|\delta_3| \leq \frac{|M|\left|\lceil 2\rfloor_\mathfrak{q}\,\vartheta-\mathfrak{q}\right|}{\lceil 2\rfloor_\mathfrak{q}\left|\gamma+\mathfrak{q}\,\lceil 2\rfloor_\mathfrak{q}\right|} + \min\left\{ \frac{|M|^2\left|\lceil 2\rfloor_\mathfrak{q}\,\vartheta-\mathfrak{q}\right|^2}{\lceil 2\rfloor_\mathfrak{q}^2\,|\gamma+\mathfrak{q}|^2}, \frac{2\,\lceil 3\rfloor_\mathfrak{q}\,|M|^2\left|\lceil 2\rfloor_\mathfrak{q}\,\vartheta-\mathfrak{q}\right|^3}{|\mathscr{D}_\mathfrak{q}|} \right\}.$$

where

$$\begin{aligned}\mathscr{D}_\mathfrak{q} = {}& \lceil 2\rfloor_\mathfrak{q}\,\lceil 3\rfloor_\mathfrak{q}\,M(\lceil 2\rfloor_\mathfrak{q}\,\vartheta-\mathfrak{q})^2\mathscr{K}_1(\gamma;\mathfrak{q}) \\ & - 2\,\lceil 2\rfloor_\mathfrak{q}\,\mathscr{Y}(\vartheta;\mathfrak{q})(\gamma+\mathfrak{q})^2,\end{aligned}$$

and

$$\mathscr{K}_1(\gamma;\mathfrak{q}) = 2\left(\gamma+\mathfrak{q}\,\lceil 2\rfloor_\mathfrak{q}\right) + (\gamma-1)(\gamma+2\mathfrak{q}),$$

and

$$\mathscr{Y}(\vartheta;\mathfrak{q}) = \lceil 2\rfloor_\mathfrak{q}\,\lceil 3\rfloor_\mathfrak{q}\,\vartheta(\vartheta-1)+\mathfrak{q}.$$

Proof. Let $F \in \mathcal{B}_{\Sigma_\mathfrak{q}}(\gamma, M; \vartheta)$, and let $\hbar = F^{-1}$ be given by (2.3). By the subordination conditions (3.1) and (3.2), there exist two Schwarz functions φ and ν, analytic in $\mathscr{O}$, such that

$$\varphi(0) = \nu(0) = 0, \qquad |\varphi(\zeta)| < 1, \qquad |\nu(\xi)| < 1.$$

Let

$$\varphi(\zeta) = \ell_1\zeta + \ell_2\zeta^2 + \cdots \quad \text{and} \quad \nu(\xi) = \tau_1\xi + \tau_2\xi^2 + \cdots.$$

Then

$$1 + \frac{1}{M}\left[\frac{\zeta^{1-\gamma}\,\eth_\mathfrak{q}\langle F(\zeta)\rangle}{(F(\zeta))^{1-\gamma}} - 1\right] = \mathscr{B}_\mathfrak{q}(\vartheta, \varphi(\zeta)), \tag{4.8}$$

and

$$1 + \frac{1}{M}\left[\frac{\xi^{1-\gamma}\,\eth_\mathfrak{q}\langle \hbar(\xi)\rangle}{(\hbar(\xi))^{1-\gamma}} - 1\right] = \mathscr{B}_\mathfrak{q}(\vartheta, \nu(\xi)). \tag{4.9}$$

Using the series expansions of F and $\hbar$, we obtain

$$\begin{aligned}1 + \frac{1}{M}\left[\frac{\zeta^{1-\gamma}\eth_\mathfrak{q}\langle F(\zeta)\rangle}{(F(\zeta))^{1-\gamma}} - 1\right] = {}& 1 \\ & + \frac{\gamma+\mathfrak{q}}{M}\delta_2\zeta \\ & + \frac{1}{M}\left[(\gamma+\mathfrak{q}\,\lceil 2\rfloor_\mathfrak{q})\delta_3 + \frac{(\gamma-1)(\gamma+2\mathfrak{q})}{2}\delta_2^2\right]\zeta^2 + \cdots.\end{aligned} \tag{4.10}$$

and, similarly,

$$1+\frac{1}{M}\left[\frac{\xi^{1-\gamma}\eth_{\mathfrak{q}}\langle\hbar(\xi)\rangle}{(\hbar(\xi))^{1-\gamma}}-1\right]=1 - \frac{\gamma+\mathfrak{q}}{M}\delta_2\xi + \frac{1}{M}\left[-(\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}})\delta_3 + \left\{2(\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}})+\frac{(\gamma-1)(\gamma+2\mathfrak{q})}{2}\right\}\delta_2^2\xi^2+\cdots. \tag{4.11}$$

Comparing the corresponding coefficients in (4.8)–(4.11), we obtain

$$\frac{\gamma+\mathfrak{q}}{M}\,\delta_2=\mathscr{B}_{\mathfrak{q},1}(\vartheta)\ell_1, \tag{4.12}$$

$$-\frac{\gamma+\mathfrak{q}}{M}\,\delta_2=\mathscr{B}_{\mathfrak{q},1}(\vartheta)\tau_1. \tag{4.13}$$

$$\frac{1}{M}\left[(\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}})\delta_3+\frac{(\gamma-1)(\gamma+2\mathfrak{q})}{2}\delta_2^2\right] = \mathscr{B}_{\mathfrak{q},1}(\vartheta)\ell_2+\frac{\mathscr{B}_{\mathfrak{q},2}(\vartheta)}{\lceil 2\rfloor_{\mathfrak{q}}}\ell_1^2. \tag{4.14}$$

$$\frac{1}{M}\left[-(\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}})\delta_3+\left\{2(\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}) + \frac{(\gamma-1)(\gamma+2\mathfrak{q})}{2}\right\}\delta_2^2\right] = \mathscr{B}_{\mathfrak{q},1}(\vartheta)\tau_2+\frac{\mathscr{B}_{\mathfrak{q},2}(\vartheta)}{\lceil 2\rfloor_{\mathfrak{q}}}\tau_1^2. \tag{4.15}$$

From (4.12) and (4.13), we have

$$\ell_1=-\tau_1 \qquad\Longrightarrow\qquad \ell_1^2=\tau_1^2, \tag{4.16}$$

and

$$\ell_1^2+\tau_1^2=\frac{2(\gamma+\mathfrak{q})^2}{M^2\mathscr{B}_{\mathfrak{q},1}^2(\vartheta)}\,\delta_2^2. \tag{4.17}$$

First, from (4.12), we obtain

$$\delta_2=\frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)\ell_1}{\gamma+\mathfrak{q}}.$$

Since $|\ell_1|\leq 1$, it follows that

$$|\delta_2|\leq\frac{|M|\,|\mathscr{B}_{\mathfrak{q},1}(\vartheta)|}{|\gamma+\mathfrak{q}|}.$$

Using

$$\mathscr{B}_{\mathfrak{q},1}(\vartheta)=\frac{\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}}{\lceil 2\rfloor_{\mathfrak{q}}},$$

we get

$$|\delta_2| \leq \frac{|M| \left| \lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q} \right|}{\lceil 2 \rfloor_{\mathfrak{q}} |\gamma + \mathfrak{q}|}. \tag{4.18}$$

Next, adding (4.14) and (4.15), and using (4.17), we obtain

$$\delta_2^2 = \frac{\lceil 3 \rfloor_{\mathfrak{q}} M^2 \left(\lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q} \right)^3 (\ell_2 + \tau_2)}{\mathscr{D}_{\mathfrak{q}}}, \tag{4.19}$$

where

$$\mathscr{K}_1(\gamma; \mathfrak{q}) = 2 \left(\gamma + \mathfrak{q} \lceil 2 \rfloor_{\mathfrak{q}} \right) + (\gamma - 1)(\gamma + 2\mathfrak{q}),$$

and

$$\mathscr{Y}(\vartheta; \mathfrak{q}) = \lceil 2 \rfloor_{\mathfrak{q}} \lceil 3 \rfloor_{\mathfrak{q}} \vartheta(\vartheta - 1) + \mathfrak{q}.$$

Since $|\ell_2 + \tau_2| \leq 2$, we obtain

$$|\delta_2| \leq \sqrt{\frac{2 \lceil 3 \rfloor_{\mathfrak{q}} |M|^2 \left| \lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q} \right|^3}{|\mathscr{D}_{\mathfrak{q}}|}}. \tag{4.20}$$

Combining (4.18) and (4.20), we arrive at

$$|\delta_2| \leq \min \left\{ \frac{|M| \left| \lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q} \right|}{\lceil 2 \rfloor_{\mathfrak{q}} |\gamma + \mathfrak{q}|}, \sqrt{\frac{2 \lceil 3 \rfloor_{\mathfrak{q}} |M|^2 \left| \lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q} \right|^3}{|\mathscr{D}_{\mathfrak{q}}|}} \right\}. \tag{4.21}$$

Now, in order to estimate $|\delta_3|$, subtracting (4.15) from (4.14) and using (4.16), we obtain

$$\frac{2 \left(\gamma + \mathfrak{q} \lceil 2 \rfloor_{\mathfrak{q}} \right)}{M} \left(\delta_3 - \delta_2^2 \right) = \mathscr{B}_{\mathfrak{q},1}(\vartheta)(\ell_2 - \tau_2). \tag{4.22}$$

Hence,

$$\delta_3 = \delta_2^2 + \frac{M \mathscr{B}_{\mathfrak{q},1}(\vartheta)}{2 \left(\gamma + \mathfrak{q} \lceil 2 \rfloor_{\mathfrak{q}} \right)} (\ell_2 - \tau_2). \tag{4.23}$$

Taking absolute values in (4.23), using

$$\mathscr{B}_{\mathfrak{q},1}(\vartheta) = \frac{\lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q}}{\lceil 2 \rfloor_{\mathfrak{q}}},$$

and the estimate $|\ell_2 - \tau_2| \leq 2$, we obtain

$$|\delta_3| \leq |\delta_2|^2 + \frac{|M| \left| \lceil 2 \rfloor_{\mathfrak{q}} \vartheta - \mathfrak{q} \right|}{\lceil 2 \rfloor_{\mathfrak{q}} \left| \gamma + \mathfrak{q} \lceil 2 \rfloor_{\mathfrak{q}} \right|}. \tag{4.24}$$

Applying the estimate (4.21), we obtain

$$|\delta_3| \leq \frac{|M| \left|\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right|}{\lceil 2\rfloor_{\mathfrak{q}}\left|\gamma+\mathfrak{q}\,\lceil 2\rfloor_{\mathfrak{q}}\right|}+\min\left\{\frac{|M|^2\left|\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right|^2}{\lceil 2\rfloor_{\mathfrak{q}}^2\,|\gamma+\mathfrak{q}|^2},\ \frac{2\,\lceil 3\rfloor_{\mathfrak{q}}\,|M|^2\left|\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right|^3}{|\mathscr{D}_{\mathfrak{q}}|}\right\}. \tag{4.25}$$

This completes the proof. □

By employing the coefficient estimates for δ_2 and δ_3, and inspired by the recent result of Zaprawa [20], recalled in Lemma 4.1, we now derive the Fekete–Szegö inequality for the class $\mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$.

Lemma 4.1. [20] *Let $\beta_1, \beta_2 \in \mathbb{R}$ and $\varrho_1, \varrho_2 \in \mathbb{C}$. If*

$$|\varrho_1| < L \quad \text{and} \quad |\varrho_2| < L,$$

then

$$\left|(\beta_1+\beta_2)\varrho_1+(\beta_1-\beta_2)\varrho_2\right| \leq \begin{cases} 2|\beta_1|L, & |\beta_1| \geq |\beta_2|, \\ 2|\beta_2|L, & |\beta_1| \leq |\beta_2|. \end{cases}$$

Theorem 4.2. *Let*

$$\mathcal{F}(\zeta) = \zeta + \delta_2\zeta^2 + \delta_3\zeta^3 + \cdots$$

belong to the class $\mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$. Then, for any $\varepsilon_1 \in \mathbb{R}$,

$$\left|\delta_3 - \varepsilon_1\delta_2^2\right| \leq \begin{cases} 2|\Psi_1(\varepsilon_1)|, & |\Psi_1(\varepsilon_1)| \geq \dfrac{|M|\left|\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right|}{2\,\lceil 2\rfloor_{\mathfrak{q}}\left|\gamma+\mathfrak{q}\,\lceil 2\rfloor_{\mathfrak{q}}\right|}, \\ \dfrac{|M|\left|\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right|}{\lceil 2\rfloor_{\mathfrak{q}}\left|\gamma+\mathfrak{q}\,\lceil 2\rfloor_{\mathfrak{q}}\right|}, & |\Psi_1(\varepsilon_1)| \leq \dfrac{|M|\left|\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right|}{2\,\lceil 2\rfloor_{\mathfrak{q}}\left|\gamma+\mathfrak{q}\,\lceil 2\rfloor_{\mathfrak{q}}\right|}, \end{cases} \tag{4.26}$$

where

$$\Psi_1(\varepsilon_1) = (1-\varepsilon_1)\frac{\lceil 3\rfloor_{\mathfrak{q}}\,M^2\left(\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right)^3}{\mathscr{D}_{\mathfrak{q}}}, \tag{4.27}$$

with

$$\mathscr{K}_1(\gamma;\mathfrak{q}) = 2\left(\gamma+\mathfrak{q}\,\lceil 2\rfloor_{\mathfrak{q}}\right)+(\gamma-1)(\gamma+2\mathfrak{q}),$$

and

$$\mathscr{Y}(\vartheta;\mathfrak{q}) = \lceil 2\rfloor_{\mathfrak{q}}\,\lceil 3\rfloor_{\mathfrak{q}}\,\vartheta(\vartheta-1)+\mathfrak{q}.$$

Proof. Let $\mathcal{F} \in \mathcal{B}_{\Sigma_{\mathfrak{q}}}(\gamma, M; \vartheta)$. From (4.19) and (4.23), we have

$$\delta_2^2 = \frac{\lceil 3\rfloor_{\mathfrak{q}}\,M^2\left(\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}\right)^3(\ell_2+\tau_2)}{\mathscr{D}_{\mathfrak{q}}},$$

and

$$\delta_3 = \delta_2^2 + \frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)}{2\left(\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right)}(\ell_2 - \tau_2).$$

Therefore,

$$\begin{aligned}\delta_3 - \varepsilon_1\delta_2^2 &= \frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)}{2\left(\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right)}(\ell_2 - \tau_2) + (1-\varepsilon_1)\delta_2^2 \\ &= \frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)(\ell_2 - \tau_2)}{2\left(\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right)} + \frac{(1-\varepsilon_1)\lceil 3\rfloor_{\mathfrak{q}} M^2\left(\lceil 2\rfloor_{\mathfrak{q}}\vartheta - \mathfrak{q}\right)^3(\ell_2 + \tau_2)}{\mathscr{D}_{\mathfrak{q}}}.\end{aligned} \tag{4.28}$$

Hence, by (4.27), equation (4.28) becomes

$$\delta_3 - \varepsilon_1\delta_2^2 = \left(\Psi_1(\varepsilon_1) + \frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)}{2\left(\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right)}\right)\ell_2 + \left(\Psi_1(\varepsilon_1) - \frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)}{2\left(\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right)}\right)\tau_2. \tag{4.29}$$

Writing

$$\beta_1 = \Psi_1(\varepsilon_1), \qquad \beta_2 = \frac{M\mathscr{B}_{\mathfrak{q},1}(\vartheta)}{2\left(\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right)},$$

we have

$$\delta_3 - \varepsilon_1\delta_2^2 = (\beta_1 + \beta_2)\ell_2 + (\beta_1 - \beta_2)\tau_2.$$

Since ℓ_2 and τ_2 are coefficients of Schwarz functions,

$$|\ell_2| \le 1, \qquad |\tau_2| \le 1.$$

Thus, by Lemma 4.1, we obtain

$$\left|\delta_3 - \varepsilon_1\delta_2^2\right| \le \begin{cases} 2|\beta_1|, & |\beta_1| \ge |\beta_2|, \\ 2|\beta_2|, & |\beta_1| \le |\beta_2|. \end{cases}$$

Finally, since

$$\mathscr{B}_{\mathfrak{q},1}(\vartheta) = \frac{\lceil 2\rfloor_{\mathfrak{q}}\vartheta - \mathfrak{q}}{\lceil 2\rfloor_{\mathfrak{q}}},$$

we have

$$|\beta_2| = \frac{|M|\left|\lceil 2\rfloor_{\mathfrak{q}}\vartheta - \mathfrak{q}\right|}{2\lceil 2\rfloor_{\mathfrak{q}}\left|\gamma + \mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}\right|}.$$

This gives the desired estimate (4.26). □

The coefficient bounds and Fekete–Szegö estimates corresponding to the five specializations above are summarized in Table 1.

TABLE 1. Coefficient and Fekete–Szegö bounds for the five specializations.

Parameters	$\lvert\delta_2\rvert$	$\lvert\delta_3\rvert$	$\lvert\delta_3-\varepsilon_1\delta_2^2\rvert$
$M=1$	$\min\{A_1,\sqrt{B_1}\}$	$C_1+\min\{A_1^2,B_1\}$	$\max\{2\lvert\Psi_1\rvert,C_1\}$
$M=1,\ \gamma=0$	$\min\{A_2,\sqrt{B_2}\}$	$C_2+\min\{A_2^2,B_2\}$	$\max\{2\lvert\Psi_3\rvert,C_2\}$
$\mathfrak{q}\to1^-$	$\min\{A_3,\sqrt{B_3}\}$	$C_3+\min\{A_3^2,B_3\}$	$\max\{2\lvert\Psi_4\rvert,C_3\}$
$M=1,\ \mathfrak{q}\to1^-$	$\min\{A_4,\sqrt{B_4}\}$	$C_4+\min\{A_4^2,B_4\}$	$\max\{2\lvert\Psi_5\rvert,C_4\}$
$M=1,\ \gamma=0,\ \mathfrak{q}\to1^-$	$\min\{A_5,\sqrt{B_5}\}$	$C_5+\min\{A_5^2,B_5\}$	$\max\{2\lvert\Psi_6\rvert,C_5\}$

In this table $\Psi_k=\Psi_k(\varepsilon_1)$, where $\varepsilon_1\in\mathbb{R}$. Put $T=\lceil 2\rfloor_{\mathfrak{q}}\,\vartheta-\mathfrak{q}$, $S=2\vartheta-1$, and

$$D_1=\lceil 2\rfloor_{\mathfrak{q}}\lceil 3\rfloor_{\mathfrak{q}}T^2\mathscr{K}_1(\gamma;\mathfrak{q})-2\lceil 2\rfloor_{\mathfrak{q}}\mathscr{Y}(\vartheta;\mathfrak{q})(\gamma+\mathfrak{q})^2,\quad D_2=\lceil 2\rfloor_{\mathfrak{q}}\lceil 3\rfloor_{\mathfrak{q}}T^2\mathscr{K}_2(\mathfrak{q})-2\lceil 2\rfloor_{\mathfrak{q}}\mathscr{Y}(\vartheta;\mathfrak{q})\mathfrak{q}^2,$$
$$D_3=6MS^2\mathscr{K}_3(\gamma)-4\mathscr{Y}_1(\vartheta)(\gamma+1)^2,\qquad D_4=6S^2\mathscr{K}_3(\gamma)-4\mathscr{Y}_1(\vartheta)(\gamma+1)^2,$$
$$D_5=12S^2-4\mathscr{Y}_1(\vartheta).$$

$$(A_1,B_1,C_1)=\left(\frac{|T|}{\lceil 2\rfloor_{\mathfrak{q}}|\gamma+\mathfrak{q}|},\frac{2\lceil 3\rfloor_{\mathfrak{q}}|T|^3}{|D_1|},\frac{|T|}{\lceil 2\rfloor_{\mathfrak{q}}|\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}|}\right),$$
$$(A_2,B_2,C_2)=\left(\frac{|T|}{\lceil 2\rfloor_{\mathfrak{q}}|\mathfrak{q}|},\frac{2\lceil 3\rfloor_{\mathfrak{q}}|T|^3}{|D_2|},\frac{|T|}{\lceil 2\rfloor_{\mathfrak{q}}|\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}|}\right),$$
$$(A_3,B_3,C_3)=\left(\frac{|M||S|}{2|\gamma+1|},\frac{6|M|^2|S|^3}{|D_3|},\frac{|M||S|}{2|\gamma+2|}\right),$$
$$(A_4,B_4,C_4)=\left(\frac{|S|}{2|\gamma+1|},\frac{6|S|^3}{|D_4|},\frac{|S|}{2|\gamma+2|}\right),$$
$$(A_5,B_5,C_5)=\left(\frac{|S|}{2},\frac{6|S|^3}{|D_5|},\frac{|S|}{4}\right).$$
$$\Psi_1=\frac{(1-\varepsilon_1)\lceil 3\rfloor_{\mathfrak{q}}T^3}{D_1},\quad \Psi_3=\frac{(1-\varepsilon_1)\lceil 3\rfloor_{\mathfrak{q}}T^3}{D_2},\quad \Psi_4=\frac{3(1-\varepsilon_1)M^2S^3}{D_3},$$
$$\Psi_5=\frac{3(1-\varepsilon_1)S^3}{D_4},\qquad \Psi_6=\frac{3(1-\varepsilon_1)S^3}{D_5}.$$
$$\mathscr{K}_1(\gamma;\mathfrak{q})=2(\gamma+\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}})+(\gamma-1)(\gamma+2\mathfrak{q}),\qquad \mathscr{K}_2(\mathfrak{q})=2\mathfrak{q}\lceil 2\rfloor_{\mathfrak{q}}-2\mathfrak{q},$$
$$\mathscr{K}_3(\gamma)=(\gamma+1)(\gamma+2),\qquad \mathscr{Y}(\vartheta;\mathfrak{q})=\lceil 2\rfloor_{\mathfrak{q}}\lceil 3\rfloor_{\mathfrak{q}}\vartheta(\vartheta-1)+\mathfrak{q},$$
$$\mathscr{Y}_1(\vartheta)=6\vartheta(\vartheta-1)+1.$$

5. Conclusion

In this paper, we have introduced a new subclass of bi-univalent functions of $\mathfrak{q}$-Bi–Bazilevič type of complex order associated with $\mathfrak{q}$-Bernoulli polynomials. The proposed class was defined in the open unit disk by means of the Jackson $\mathfrak{q}$-derivative and subordination principles applied to both the analytic function and its inverse. This construction provides a flexible framework that unifies the influence of $\mathfrak{q}$-calculus, complex-order operators, Bazilevič-type conditions, and polynomial-generated analytic structures.

For functions belonging to the newly defined class, we obtained upper bounds for the initial Taylor coefficients $|\delta_2|$ and $|\delta_3|$. Moreover, an estimate for the Fekete–Szegö functional $|a_3-\mu a_2^2|$ was derived for a complex parameter μ. These coefficient estimates contribute to the ongoing study of analytic and bi-univalent functions associated with special polynomial families and quantum calculus operators.

Several special cases and consequences were also discussed by assigning particular values to the involved parameters. These reductions show that the results obtained in this work not only generalize a number of known results in the literature but also generate new subclasses of bi-univalent functions. Hence, the present investigation enriches the theory of bi-univalent functions and opens possible directions for further studies involving Hankel determinants, higher-order coefficient bounds, and other families of $\mathfrak{q}$-orthogonal polynomials.

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