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THREE-DIMENSIONAL PARA-KENMOTSU MANIFOLDS ADMITTING η-RICCI SOLITONS

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Abstract. In the present paper we study η-Ricci solitons on three-dimensional para-Kenmotsu manifolds with the curvature condition $R.Q = 0$. Also we study conformally flat, projectively flat and concircularly flat η-Ricci soliton on three-dimensional para-Kenmotsu manifolds. Finally, we have cited an example of a three-dimensional para-Kenmotsu manifold which admits η-Ricci solitons.

1. Introduction

In 1982, Hamilton [13] introduced the notion of the Ricci flow to find a canonical metric on a smooth manifold. The Ricci flow is an evolution equation for metrics on a Riemannian manifold

$$\frac{\partial}{\partial t} g_{ij}(t) = -2R_{ij}.$$

A Ricci soliton is a natural generalization of Einstein metric and defined on a Riemannian manifold (M, g). A Ricci soliton is a triple (g, V, λ) with g a Riemannian metric, V a vector field and λ a real scalar such that

$$\pounds_V g + 2S + 2\lambda g = 0,$$

where S is a Ricci tensor of M and $\pounds_V$ denotes the Lie derivative operator along the vector field V. The Ricci soliton is said to be shrinking, steady and expanding according as $\lambda < 0, \lambda = 0$, or $\lambda > 0$, respectively [8]. Ricci solitons have been studied by many authors, such as [1, 6, 9, 10, 14, 16] and several authors.

As a generalization of Ricci solitons, the notion of an η-Ricci solitons was introduced by Cho and Kimura [7]. This notion has been studied in [5], for Hopf hypersurfaces in complex space form. An Ricci soliton is a tuple (g, V, λ, μ), where V is a vector field on M, λ and μ are real constants, and g is a Riemannian (or pseudo-Riemannian) metric satisfying the equation

$$\pounds_V g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0. \tag{1.1}$$

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An η-Ricci solitons on para-Kenmotsu manifolds were studied by A. M. Blaga [3] and η-Ricci solitons on Lorentzian Para-Sasakian Manifolds were also studied by A. M. Blaga [4]. In particular, if $\mu = 0$, then the notion of η-Ricci solitons (g, V, λ, μ) reduces to the notion of Ricci solitons (g, V, λ). If $\mu \neq 0$, then the η-Ricci solitons are called proper η-Ricci solitons. M. Atçeken and et. al. [2] studied anti-inveriant submanifolds of a normal paracontact metric manifold. Also, Y. Gunduzalp [12] studied slant submersions from Lorentzian almost paracontact manifolds.

Gray [11] introduced the notion of Codazzi type of the Ricci tensor. A pseudo-Riemannian manifold is said to satisfy Codazzi type of the Ricci tensor if its Ricci tensor S of type (0,2) is non-zero and satisfies the condition

$$(\nabla_X S)(Y, Z) = (\nabla_Y S)(X, Z),$$

which implies that div R=0, where div denotes divergence and R is the Riemannian curvature tensor of type (1,3). If the Weyl tensor is harmonic, then we get

$$(\nabla_X S)(Y, Z) - (\nabla_Y S)(X, Z) = \frac{1}{2(n-1)}[g(Y, Z)dr(X) - g(X, Z)dr(Y)], \quad (1.2)$$

where r is the scalar curvature. The projective curvature tensor P [19] in an n-dimensional Riemannian manifold M is defined by

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1}[g(Y, Z)QX - g(X, Z)QY]. \quad (1.3)$$

The concircular curvature tensor C [19] in an n-dimensional Riemannian manifold M is defined by

$$C(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y], \quad (1.4)$$

where Q is the Ricci operator defined by $S(X, Y) = g(QX, Y)$ and $X, Y, Z \in \chi(M)$, $\chi(M)$ being the Lie algebra of vector fields of M.

In this paper, first we give some basic formulas in para-Kenmotsu manifolds. In the next section, we study η-Ricci soliton in a three-dimensional para-Kenmotsu manifold satisfying the curvature property $R.Q = 0$. Also we study η-Ricci soliton on conformally flat, projectively flat and concircularly flat para-Kenmotsu of dimension three. In the last Section, we have cited [18] an example to prove the existence of proper η-Ricci solitons on three-dimensional para-Kenmotsu manifolds and verify some results.

2. Para-Kenmotsu Manifolds

Let $(M, \varphi, \eta, \xi, g)$ be an n-dimensional smooth manifold, where φ is an $(1, 1)$ tensor field, ξ is a vector field and η is an 1-form such that

$$\varphi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \quad \eta\varphi = 0. \quad (2.1)$$

Then (φ, η, ξ, g) is called an almost paracontact metric structure and M an almost paracontact manifold. Moreover, if M admits a semi-Riemannian metric g such that

$$g(X,\xi)=\eta(X), \quad g(\varphi X,\varphi Y)=-g(X,Y)+\eta(X)\eta(Y). \tag{2.2}$$

Then (φ,ξ,η,g) is called an almost paracontact metric structure and M an almost paracontact metric manifold [17]. The fundamental 2-form Φ of an almost paracontact metric structure (M,φ,ξ,η,g) is defined by $\Phi(X,Y)=g(X,\varphi Y)$. If $\Phi=d\eta$, then the manifold is called a paracontact metric manifold and g is an associated metric. An almost paracontact metric manifold is normal if $[\varphi,\varphi](X,Y)+2d\eta(X,Y)\xi=0$, where $[\varphi,\varphi](X,Y)=\varphi^2[X,Y]+[\varphi X,\varphi Y]-\varphi[\varphi X,\varphi Y]-\varphi[X,\varphi Y]$. In a para-Kenmotsu manifold, we have the following formulas [3, 20]

$$(\nabla_X\varphi)Y=g(\varphi X,Y)\xi-\eta(Y)\varphi X, \tag{2.3}$$

$$\nabla_X\xi=X-\eta(X)\xi, \tag{2.4}$$

$$(\nabla_X\eta)Y=g(X,Y)-\eta(X)\eta(Y), \tag{2.5}$$

$$\eta(R(X,Y)Z)=-\eta(X)g(Y,Z)+\eta(Y)g(X,Z), \tag{2.6}$$

$$R(X,Y)\xi=\eta(X)Y-\eta(Y)X, \tag{2.7}$$

$$R(\xi,X)Y=\eta(Y)X-g(X,Y)\xi, \tag{2.8}$$

$$R(\xi,X)\xi=X-\eta(X)\xi, \tag{2.9}$$

$$S(X,\xi)=(1-n)\eta(X), \tag{2.10}$$

$$Q\xi=(1-n)\xi, \tag{2.11}$$

$$(\pounds_\xi g)(X,Y)=-2\{g(X,Y)-\eta(X)\eta(Y)\}, \tag{2.12}$$

where S is the Ricci tensor, R is the Riemannian curvature tensor field and ∇ is the Levi-Civita connection associated to g. The curvature tensor of a three-dimensional Riemannian manifold is given by

$$\begin{aligned} R(X,Y)Z=&[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]\\ &-\frac{r}{2}[g(Z,Y)X-g(X,Z)Y], \end{aligned} \tag{2.13}$$

where S and r are the Ricci tensor and scalar curvature respectively and Q is the Ricci operator defined by $g(QX,Y)=S(X,Y)$. It is known that the Ricci tensor of a three-dimensional para-Kenmotsu manifold is given by [15]

$$S(X,Y)=\frac{1}{2}[(r+2)g(X,Y)-(r+6)\eta(X)\eta(Y)], \tag{2.14}$$

where r is the scalar curvature which needs not be constant, in general. So, g Einstein (hence has constant curvature -1) if and only if $r = -6$. From above equation we have

$$S(X, \xi) = -2\eta(X). \tag{2.15}$$

Proposition 2.1. [3] *For an η-Ricci soliton on a para-Kenmotsu manifold the Ricci tensor S is of the form*

$$S(X, Y) = -(1 + \lambda)g(X, Y) - (\mu - 1)\eta(X)\eta(Y), \tag{2.16}$$

$$QX = -(1 + \lambda)X - (\mu - 1)\eta(X)\xi. \tag{2.17}$$

Proposition 2.2. *For an η-Ricci soliton on a three-dimensional para-Kenmotsu manifold we have $\lambda + \mu = 2$.*

Proof. The Ricci tensor of a three-dimensional para-Kenmotsu manifold is given by

$$S(X, Y) = \frac{1}{2}[(r + 2)g(X, Y) - (r + 6)\eta(X)\eta(Y)], \tag{2.18}$$

where r is the scalar curvature. Comparing the above equation with $S(X, Y) = -(1 + \lambda)g(X, Y) - (\mu - 1)\eta(X)\eta(Y)$, we get $\lambda = -\frac{1}{2}(r + 4)$ and $\mu = \frac{1}{2}(r + 8)$. From which it follows that

$$\lambda + \mu = 2. \tag{2.19}$$

This completes the proof.

□

3. η-Ricci Solitons of para-Kenmotsu manifolds satisfying some curvature restrictions

In this section we study η-Ricci solitons in a three-dimensional para-Kenmotsu manifold satisfying $R.Q = 0$. Also, we obtain the conditions for η-Ricci solitons on conformally flat, projectively flat and concircular flat para-Kenmotsu manifolds.

Theorem 3.1. *An η-Ricci soliton on a three-dimensional para-Kenmotsu manifold satisfying $R.Q = 0$ is expanding.*

Proof. Let us consider para-Kenmotsu manifols satisfying $R.Q = 0$. Then we have

$$(R(X, Y).Q)Z = 0, \tag{3.1}$$

that is

$$R(X, Y)QZ - Q(R(X, Y)Z) = 0, \tag{3.2}$$

for all smooth vector fields X, Y, Z in M, where Q stands for the Ricci operator defined by $S(X, Y) = g(QX, Y)$ and R is the Riemannian curvature tensor.
Using (2.7) and (2.17) in (3.2) we get

$$(\mu - 1)\eta(R(X, Y)Z)\xi - (\mu - 1)[(\eta(X)Y - \eta(Y)X)]\eta(Z) = 0. \tag{3.3}$$

Using (2.16) and (2.17) in (2.13) we obtain

$$\begin{aligned} R(X,Y)Z = &-(\frac{r}{2}+2\lambda+2)[g(Y,Z)X-g(X,Z)Y] \\ &-(\mu-1)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\xi \\ &-(\mu-1)[\eta(Y)X-\eta(X)Y]\eta(Z). \end{aligned} \tag{3.4}$$

From the last equation we have

$$\eta(R(X,Y)Z) = -(\frac{r}{2}+2\lambda+\mu+1)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]. \tag{3.5}$$

In the view of (3.3) and (3.5) we get

$$\begin{aligned} &-(\mu-1)(\frac{r}{2}+2\lambda+\mu+1)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\xi \\ &-(\mu-1)[(\eta(X)Y-\eta(Y)X)]\eta(Z)=0. \end{aligned} \tag{3.6}$$

Replacing Z by $\varphi^2 Z$ in (3.6) and using (2.2) we have

$$(\mu-1)(\frac{r}{2}+2\lambda+\mu+1)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\xi=0. \tag{3.7}$$

Putting $Y=\xi$ in (3.7) and using (2.2) we obtain

$$(\mu-1)(\frac{r}{2}+2\lambda+\mu+1)[\eta(X)\eta(Z)-g(X,Z)]\xi=0. \tag{3.8}$$

Contracting X and Z in (3.8) we find

$$(\mu-1)(\frac{r}{2}+2\lambda+\mu+1)=0, \tag{3.9}$$

from the above equation we get either $\mu=1$ or $\frac{r}{2}+2\lambda+\mu+1=0$.
Contracting X,Y in (2.16) we get

$$r=-3\lambda-\mu-2. \tag{3.10}$$

Using (3.10) in $\frac{r}{2}+2\lambda+\mu+1=0$, we get

$$\lambda+\mu=0, \tag{3.11}$$

which contradicts the Proposition 2.2. Hence $\mu=1$ and from (2.19) we obtain $\lambda=1$. Thus the η-Ricci soliton is expanding.
This completes the proof. □

Theorem 3.2. *A proper η-Ricci soliton on a conformally flat three dimensional para-Kenmotsu manifold is of the type $(g,\xi,1,1)$.*

Proof. Taking the covariant differentiation of (2.16) with respect to Z we obtain

$$\begin{aligned} (\nabla_Z S)(X,Y) &= -(\mu-1)[(\nabla_Z\eta)(X)\eta(Y)+\eta(X)(\nabla_Z\eta)(Y)] \\ &= -(\mu-1)[g(X,Z)\eta(Y)+g(Y,Z)\eta(X)-2\eta(X)\eta(Y)\eta(Z)]. \end{aligned} \tag{3.12}$$

Using (3.12) in (1.2), we have

$$-(\mu-1)[g(X,Z)\eta(Y)-g(Y,Z)\eta(X)] = \frac{1}{4}[g(Y,Z)dr(X)-g(X,Z)dr(Y)]. \quad (3.13)$$

Putting $X = \xi$ in (3.13) and using (2.2), we get

$$4(\mu - 1)g(\varphi Y, \varphi Z)] = \eta(Z)dr(Y). \quad (3.14)$$

Replacing Z by φZ in (3.14), we have

$$(\mu - 1)g(\varphi Y, Z) = 0. \quad (3.15)$$

It follows that

$$\mu = 1. \quad (3.16)$$

From the relation (2.19), we obtain $\lambda = 1$. Therefore, a conformally flat three-dimensional para-Kenmotsu manifold admits an η-Ricci soliton.
Hence the theorem is proved. □

Theorem 3.3. *An η-Ricci soliton on a projectively flat three-dimensional para-Kenmotsu manifold is of the type $(g, \xi, -\frac{r}{3} - 1, \frac{r}{3} + 3)$.*

Proof. Using (2.16) and (3.4) in (1.3), we get

$$\begin{aligned} P(X,Y)Z = &-\frac{(r+3\lambda+3)}{2}[g(Y,Z)X - g(X,Z)Y] \\ &- (\mu-1)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi \\ &- \frac{1}{2}(\mu-1)[\eta(Y)X - \eta(X)Y]\eta(Z). \end{aligned} \quad (3.17)$$

Putting $X = \varphi X$ and $Y = \varphi Y$ in (3.17), we get

$$\frac{(r+3\lambda+3)}{2}[g(\varphi Y, Z)\varphi X - g(\varphi X, Z)\varphi Y] = 0. \quad (3.18)$$

Taking the inner product of (3.18) with respect to arbitary vector field W, we obtain

$$\frac{(r+3\lambda+3)}{2}[g(\varphi Y, Z)g(\varphi X, W) - g(\varphi X, Z)g(\varphi Y, W)] = 0. \quad (3.19)$$

Contracting X and Z in (3.19) and using $Tr\varphi = 0$ and (2.2), we get

$$(r + 3\lambda + 3)[g(Y, W) - \eta(Y)\eta(W)] = 0. \quad (3.20)$$

Contracting Y and W in the above equation we get

$$\lambda = -\frac{r}{3} - 1. \quad (3.21)$$

By virtue of (2.19) and (3.21), we obtain $\mu = \frac{r}{3} + 3$. Since λ is constant, then from (3.21) it follows that r is constant.
Hence the theorem is proved. □

Theorem 3.4. *An η-Ricci soliton on a concircular flat three-dimensional para-Kenmotsu manifold is of the type $(g, \xi, -\frac{r}{3} - 1, \frac{r}{3} + 3)$.*

Proof. Using (2.16) and (3.4) in (1.4), we get

$$\begin{aligned} C(X,Y)Z = &-2(\frac{r}{3}+\lambda+1)[g(Y,Z)X-g(X,Z)Y] \\ &-(\mu-1)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\xi \\ &-(\mu-1)[\eta(Y)X-\eta(X)Y]\eta(Z). \end{aligned} \tag{3.22}$$

Putting $X=\varphi X$ and $Y=\varphi Y$ in (3.22), we get

$$(\frac{r}{3}+\lambda+1)[g(\varphi Y,Z)\varphi X-g(\varphi X,Z)\varphi Y]=0. \tag{3.23}$$

Taking the inner product of (3.23) with respect to arbitary vector field W, we obtain

$$(\frac{r}{3}+\lambda+1)[g(\varphi Y,Z)g(\varphi X,W)-g(\varphi X,Z)g(\varphi Y,W)]=0. \tag{3.24}$$

Contracting X and Z in (3.24) and using $Tr\varphi=0$ and (2.2), we get

$$(\frac{r}{3}+\lambda+1)[g(Y,W)-\eta(Y)\eta(W)]=0. \tag{3.25}$$

Contracting Y and W in the above equation we get

$$\lambda=-\frac{r}{3}-1. \tag{3.26}$$

By virtue of (2.19) and (3.26), we obtain $\mu=\frac{r}{3}+3$. Since λ is constant, then from (3.26) it follows that r is constant.
Hence the theorem is proved.

□

4. Example

We consider the three-dimensional manifold $M=\{(x,y,z)\in R^3, z\neq 0\}$, where (x,y,z) are the standard coordinates in R^3. The vector fields

$$e_1=\frac{\partial}{\partial x},\qquad e_2=\frac{\partial}{\partial y},\qquad e_3=x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+\frac{\partial}{\partial z},$$

are linearly independent at each point of M. Furthermore, by direct calculatins, we have

$$[e_1,e_2]=0,[e_1,e_3]=e_1,[e_2,e_3]=e_2.$$

Let η be the 1-form defined by $\eta(Z)=g(Z,e_3)$ for any $Z\in\chi(M)$, where $\chi(M)$ is the set of alldifferentiable vector fields on M.
Let φ be the (1,1) tensor field defined by $\varphi(e_1)=e_2$, $\varphi(e_2)=e_1$, $\varphi(e_3)=0$. Then using the linearity of φ and g we have $\eta(e_3)=1$, $\varphi^2(Z)=Z-\eta(Z)e_3$, $g(\varphi Z,\varphi W)=-g(Z,W)+\eta(Z)\eta(W)$, for any $Z,W\in\chi(M)$. Thus for $e_3=\xi$, (φ,ξ,η,g) defines an almost contact metric structure on M.
Let the Levi-Civita connection with respect to g be ∇, then using Koszul's formula we get the following:

$$\begin{array}{lll} \nabla_{e_1}e_3 = e_1, & \nabla_{e_1}e_2 = 0, & \nabla_{e_1}e_1 = -e_3, \\ \nabla_{e_2}e_3 = e_2, & \nabla_{e_2}e_2 = e_3, & \nabla_{e_2}e_1 = 0, \\ \nabla_{e_3}e_3 = 0, & \nabla_{e_3}e_2 = 0, & \nabla_{e_3}e_1 = 0. \end{array}$$

From above relations we see that the manifold satisfies (2.6) for $e_3 = \xi$. Therefore the structure $M^3(\varphi, \xi, \eta, g)$ is a three-dimensional para-Kenmotsu manifold. With the help of the above relations we find the curvature tensor with respect to Levi-Civita connectiom as follows:

$$\begin{array}{lll} R(e_1, e_2)e_3 = 0, & R(e_2, e_3)e_3 = -e_2, & R(e_1, e_3)e_3 = -e_1, \\ R(e_1, e_2)e_2 = e_1, & R(e_2, e_3)e_2 = -e_3, & R(e_1, e_3)e_2 = 0, \\ R(e_1, e_2)e_1 = e_2, & R(e_2, e_3)e_1 = 0, & R(e_1, e_3)e_1 = e_3. \end{array}$$

Using the expressions of the curvature tensor we find the values of the Ricci tensor as follows:

$$S(e_1, e_1) = -2, \quad S(e_2, e_2) = 2, \quad S(e_3, e_3) = -2,$$
$$S(e_1, e_2) = 0, \quad S(e_1, e_3) = 0, \quad S(e_2, e_3) = 0.$$

From (2.16) we obtain $S(e_1, e_1) = -(\lambda + 1)$ and $S(e_3, e_3) = -(\lambda + \mu)$, therefore $\lambda = 1$ and $\mu = 1$. Also we have constant scalar curvature as follows:

$$r = S(e_1, e_1) - S(e_2, e_2) + S(e_3, e_3) = -6.$$

Thus the manifold (g, ξ, λ, μ) admits an proper η-Ricci soliton. Hence the Theorem 3.1, Theorem 3.2, Theorem 3.3 and Theorem 3.4 are verified.

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