

2-POINT SET DOMINATION NUMBER OF A CACTUS

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ABSTRACT. A set $D \subseteq V(G)$ is a *2-point set dominating set* (2-psd set) of a graph G if for any subset $S \subseteq V - D$, there exists a non-empty subset $T \subseteq D$ containing at most two vertices such that the subgraph $\langle S \cup T \rangle$ induced by $S \cup T$ is connected. The *2-point set domination number* of G , denoted by $\gamma_{2ps}(G)$, is the minimum cardinality of a 2-psd set of G . In this paper we give the 2-psd number $\gamma_{2ps}(G)$ for cactus. Also we give an alternate proof for $\gamma_{2ps}(G)$ of unicyclic graphs which also reflects the structure of minimum 2-psd sets of unicyclic graphs.

1. INTRODUCTION AND PRELIMINARIES

Unless defined or mentioned otherwise, we refer the reader to West [7] for standard terminology and notation in the theory of graphs. Also, in this paper by a graph we mean a finite, simple, undirected and connected graph.

The vertex set of a graph G is denoted by $V(G)$ and $n = |V(G)|$. For any vertex $v \in V(G)$, the *open neighborhood* $N(v)$ of v in G is the set $\{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* $N[v]$ of v in G is the set $N(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the *open neighborhood* $N(S)$ of S in G is $\cup_{v \in S} N(v)$ and the *closed neighborhood* $N[S]$ of S in G is $N(S) \cup S$.

In a connected graph G , the *distance* between two vertices u and v is the length of the shortest path joining u and v , and is denoted by $d(u, v)$. A subgraph H of a graph G is said to be *induced* if, for any pair of vertices x and y of H , xy is an edge of H if and only if xy is an edge of G . If the vertex set of H is the subset S of $V(G)$, then H can be written as $\langle S \rangle$ and is said to be *induced by S* .

Definition 1.1. A set $D \subseteq V(G)$ in a graph G is said to be a *dominating set* of G if for every vertex v in $V - D$, there exists a vertex $u \in D$ such that v is adjacent to u . The *domination number* of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G (See [3, 4]).

Various types of dominations, such as set domination [6] and point-set domination [1, 2, 5] have been widely studied.

Date: Received: Mar 11, 2016; Accepted: Aug 10, 2016.

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2010 *Mathematics Subject Classification.* Primary 05C69.

Key words and phrases. Point-set domination, 2-Point set domination, Cactus, Unicyclic Graph.

Definition 1.2. A set $D \subseteq V(G)$ in a graph G is said to be a *set dominating set* of G if for every subset $S \subseteq V - D$, there exists a non-empty set $T \subseteq D$ such that $\langle S \cup T \rangle$ is connected. The *set domination number* of G , denoted by $\gamma_s(G)$, is the minimum cardinality of a set dominating set of G .

The concept of point-set domination, a special case of set-domination was introduced by Sampathkumar and Pushplatha in [5].

Definition 1.3. A set $D \subseteq V(G)$ in a graph G is said to be a *point set dominating set* (or in short, psd-set) of G if for every subset $S \subseteq V - D$ there exists a vertex $v \in D$ such that the induced subgraph $\langle S \cup \{v\} \rangle$ is connected. The *point-set domination number* of G , denoted by $\gamma_{ps}(G)$, is the minimum cardinality of a psd-set of G .

In this paper we consider another special case of set domination, namely 2-point-set domination of a graph defined as follows:

Definition 1.4. A set $D \subseteq V(G)$ in a graph G is said to be a *2-point set dominating set* (or, in short, 2-psd set) of G if for every set $S \subseteq V - D$ there exists a non-empty set $T \subseteq D$ containing at most two vertices such that the subgraph $\langle S \cup T \rangle$ is connected. The *2-point set domination number* of G , denoted by $\gamma_{2ps}(G)$, is the minimum cardinality of a 2-psd set of G .

Clearly, $V(G)$ is always a 2-psd set for the graph G . A 2-psd set D with $|D| = \gamma_{2ps}(G)$ is said to be a $\gamma_{2ps}(G)$ -set or γ_{2ps} -set of G . The set of all 2-psd sets of G will be denoted by $\mathcal{D}_{2ps}(G)$.

Clearly, if G has a vertex of full degree, then $\gamma_{2ps}(G) = 1$. Thus $\gamma_{2ps}(K_n) = 1$, $\gamma_{2ps}(K_{1,n}) = 1$. Also, $\gamma_{2ps}(K_{m,n}) = 2$, where $m \geq 2, n \geq 2$. For a path P_n , $\gamma_{2ps}(P_n) = n - 2, n > 2$. For a cycle C_n , $\gamma_{2ps}(C_3) = 1$, $\gamma_{2ps}(C_4) = 2$, $\gamma_{2ps}(C_n) = n - 3$, where $5 \leq n \leq 7$ and $\gamma_{2ps}(C_n) = n - 2, n \geq 8$.

In this paper we will give the 2-psd number of cactus and unicyclic graphs.

2. MAIN RESULTS

We begin with the following theorem which is immediate from the definition of 2-psd set of a graph.

Theorem 2.1. Let G be a graph and D be a 2-psd set of G . For $x, y \in V - D$, $d(x, y) \leq 3$.

For any graph G it is easy to observe that for any vertex $v \in V(G)$, the set $V(G) - N(v)$ is a 2-psd set of G . Also for any two adjacent vertices $u, v \in G$, the set $V(G) - [N(\{u, v\}) - \{u, v\}] \in \mathcal{D}_{2ps}(G)$. This leads us to define a new parameter which gives an upper bound on $\gamma_{2ps}(G)$.

Definition 2.2. For a graph G , we define

$$r = \max |N(S) - S|,$$

where maximum is taken over all subsets S of V such that $|S| \leq 2$ and the subgraph $\langle S \rangle$ is connected.

Next theorem is immediate from the definition of r .

Theorem 2.3. *For any graph G with n vertices, $\gamma_{2ps}(G) \leq n - r$.*

Proof. Suppose r is attained at a set $S_0 \subseteq V(G)$, where $|S_0| \leq 2$ and $\langle S_0 \rangle$ is connected. Then $V(G) - \{N(S_0) - S_0\}$ is a 2-psd set of G and therefore, $\gamma_{2ps}(G) \leq |V(G)| - |N(S_0) - S_0| = n - r$. Thus the result. $\square$

Remark 2.4. Let G be a graph and v be a vertex of maximum degree Δ in G . Then for the set $S = \{v\}$, $|N(S) - S| = \Delta$. Thus by the definition of r , $r \geq \Delta$.

Corollary 2.5. *For any graph G with n vertices, $\gamma_{2ps}(G) \leq n - \Delta$.*

Proof. Since $r \geq \Delta$, therefore by Theorem 2.3, we get $\gamma_{2ps}(G) \leq n - r \leq n - \Delta$. $\square$

It can be easily seen that the upper bound on $\gamma_{2ps}(G)$ given in Theorem 2.3, is attained by a double-star $(S_{m,n})$, a graph with two adjacent vertices of degree m and n respectively, and all other vertices are of degree one. For cycles C_5, C_6 and C_7 , $\gamma_{2ps} < n - r$. Consider the graph shown in Figure 1. For this graph $r = 4$, whereas $\gamma_{2ps}(G) = 2$. Thus, $\gamma_{2ps}(G) < n - r$ for the graph.

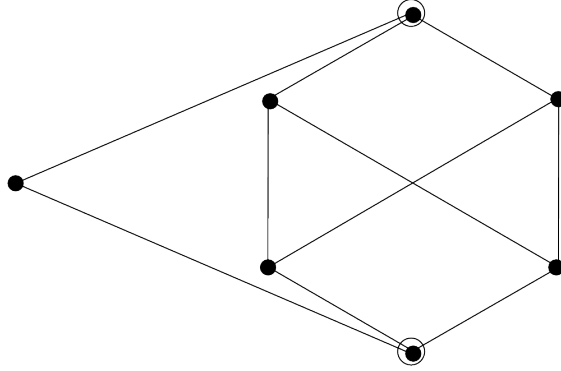


FIGURE 1. Set of circled vertices forms a $\gamma_{2ps}(G)$ -set for the graph .

Thus it is of interest to study classes of graphs which attain the upper bound on $\gamma_{2ps}(G)$ given by Theorem 2.3. In our forthcoming discussion we shall show that cactus is one such class for which $\gamma_{2ps}(G) = n - r$.

A *cactus* is a connected graph in which every block is an edge or a cycle. Since we know the $\gamma_{2ps}(G)$ number of paths and cycles, therefore, here we consider only separable cactus and show that 2-psd number of a separable cactus is $n - r$.

Theorem 2.6. *For a separable cactus G with n vertices, $\gamma_{2ps}(G) = n - r$.*

Proof. Let D be a $\gamma_{2ps}(G)$ -set of G . Since G is separable, $\Delta \geq 2$. If $\Delta = 2$ then G is a path of length at least two. In that case we know that $\gamma_{2ps}(G) = n - 2 = n - r$. Therefore, assume that $\Delta \geq 3$. By Corollary 2.5 we know that

$$\gamma_{2ps}(G) \leq n - \Delta \leq n - 3. \quad (2.1)$$

Therefore, $V - D$ contains at least three vertices. Now two cases arise:

Case(i): $V - D \subseteq V(B)$ for some block $B \in \mathcal{B}(G)$.

Since $V - D$ contains at least three vertices, therefore, B must be a cycle block.

Claim: $|V - D| \leq 3$.

Let $u \in V - D$ be arbitrary. If there exists $v \in (V - D) - \{u\}$ such that $v \notin N(u)$, then there exists a set $W \subseteq D \cap V(B)$ such that $|W| \leq 2$ and $\langle W \cup \{u, v\} \rangle$ is connected. This implies there exists a u - v path passing through vertices of W . Therefore, for each $v \in (V - D) - \{u\}$ such that $v \notin N(u)$ there exists $d_v \in D \cap V(B)$ such that $d_v \in N(u)$. Also note that d_v can be adjacent to at most two vertices of $V(B)$, therefore, for each $v \in (V - D) - \{u\}$ such that $v \notin N(u)$ we have a distinct $d_v \in D \cap V(B)$ such that $d_v \in N(u)$. Thus, $|N(u) \cap V(B)| \geq |V - D| - 1$. Since $|N(u) \cap V(B)| = 2$, therefore, $|V - D| \leq 3 \leq \Delta$. This proves the claim.

Therefore, we get $|V - D| = 3 = \Delta$. Hence, $\gamma_{2ps}(G) = n - \Delta$. We also know that $\gamma_{2ps}(G) \leq n - r \leq n - \Delta$, therefore, $n - r = n - \Delta$ in this case and $\gamma_{2ps}(G) = n - r$.

Case (ii): $V - D \not\subseteq V(B)$ for any block $B \in \mathcal{B}(G)$.

In this case $V - D$ contains vertices of more than one block of G . Again two cases arise:

Subcase(i): All vertices of $V - D$ are contained in blocks at a single cut-vertex.

In this case there exists a cut-vertex $w \in V(G)$, such that $V - D \subseteq \bigcup_{B \in \mathcal{B}_w(G)} V(B)$. Since $V - D$ contains vertices of different blocks at the cut-vertex w and the only path between these vertices passes through w , therefore, $w \in D$.

If $V - D \subseteq N(w)$ then $|V - D| \leq d(w) \leq \Delta$. So, suppose that $V - D \not\subseteq N(w)$.

Claim: There exists exactly one block $B \in \mathcal{B}_w(G)$ such that $(V - D) \cap V(B) \neq \phi$ and $(V - D) \cap V(B) \not\subseteq N(w)$.

Suppose not. This implies there exist at least two blocks $B_1, B_2 \in \mathcal{B}_w(G)$ such that $\phi \neq (V - D) \cap V(B_1) \not\subseteq N(w)$ and $\phi \neq (V - D) \cap V(B_2) \not\subseteq N(w)$. Let $x_1 \in (V - D) \cap V(B_1)$ and $x_2 \in (V - D) \cap V(B_2)$ such that $x_1, x_2 \notin N(w)$. Since x_1 and x_2 belong to two different blocks at the cut-vertex w , therefore, every x_1 - x_2 path must pass through w . Also $x_1, x_2 \notin N(w)$. This implies that $d(x_1, x_2) \geq 4$, which is a contradiction to Theorem 2.1. This proves the claim.

Let $B \in \mathcal{B}_w(G)$ be the block such that $(V - D) - V(B) \subseteq N(w)$, $(V - D) \cap V(B) \neq \phi$ and $(V - D) \cap V(B) \not\subseteq N(w)$. Then trivially B must be a cycle block at w . Let $v \in (V - D) \cap V(B)$ be such that $v \notin N(w)$. Let $x \in (V - D) - V(B)$ be arbitrary. Then $x \in N(w)$. Since D is a 2-psd set of G , there exists a set $W \subseteq D$ such that $|W| \leq 2$ and $\langle \{x, v\} \cup W \rangle$ is connected. Since x and v belong to two different blocks at w therefore, $w \in W$ and since $v \notin N(w)$, $|W| = 2$. This implies that for each $v \in (V - D) \cap V(B)$ such that $v \notin N(w)$, there

exists a vertex $d_v \in V(B) \cap D$ such that $d_v \in N(w)$. Also since d_v can be adjacent to at most two vertices in block B , therefore, such a vertex d_v is distinct for each $v \in (V - D) \cap V(B)$ which are not adjacent to w . This shows that $|(V - D) \cap V(B)| \leq |N(w) \cap V(B)| = 2$.

Therefore, $|V - D| \leq d(w) \leq \Delta$. This implies that

$$\gamma_{2ps}(G) \geq n - \Delta. \quad (2.2)$$

From inequalities (2.1) and (2.2) we get $\gamma_{2ps}(G) = n - \Delta$. Also we know that $\gamma_{2ps}(G) \leq n - r \leq n - \Delta$. Hence, $r = \Delta$ in this case and $\gamma_{2ps}(G) = n - r$.

Subcase(ii): vertices of $V - D$ are not contained in blocks at a single cut-vertex.

Since $V - D$ is not contained in blocks at a single cut-vertex, there exist two vertices $x_1, x_2 \in V - D$ such that for every block B_1 containing x_1 and for every block B_2 containing x_2 , $V(B_1) \cap V(B_2) = \emptyset$. Consider the set $\{x_1, x_2\} \subseteq V - D$. Since D is a 2-psd set of G , there exists a set $W \subseteq D$ such that $|W| \leq 2$ and $\langle \{x_1, x_2\} \cup W \rangle$ is connected. Also from our choice of vertices x_1 and x_2 , $|W| \neq 1$, otherwise there exist blocks B_1 and B_2 containing x_1 and x_2 respectively and having a common cut-vertex. Let $W = \{w_1, w_2\}$. Then (x_1, w_1, w_2, x_2) is a path in G . Since x_1 and x_2 belong to two different blocks with no common cut-vertex, therefore, $x_1 - x_2$ path must pass through cut vertices. Thus, w_1 and w_2 are cut vertices and are adjacent.

We shall show that all other vertices of $V - D$ are adjacent to w_1 or w_2 .

Claim: $V - D \subseteq N(w_1) \cup N(w_2)$.

Let $x_3 \in V - D$ and $x_3 \notin N(w_1) \cup N(w_2)$. Since D is a 2-psd set of G , for the set $\{x_1, x_2, x_3\}$, there exists a subset $W_1 \subseteq D$ such that $|W_1| \leq 2$ and $\langle W_1 \cup \{x_1, x_2, x_3\} \rangle$ is connected. Since every $x_1 - x_2$ path passes through w_1 and w_2 , therefore, $W_1 = \{w_1, w_2\}$. Since $x_3 \notin N(w_1) \cup N(w_2)$ and $\langle W \cup \{x_1, x_2, x_3\} \rangle$ is connected, therefore, x_3 is adjacent to either x_1 or x_2 . Without loss of generality we may assume that $x_3 \in N(x_1)$. Now for the set $\{x_3, x_2\}$, there exists a set $W_2 \subseteq D$ such that $|W_2| \leq 2$ and $\langle W_2 \cup \{x_3, x_2\} \rangle$ is connected. Here $W_2 \neq W$ as $x_3 \notin N(w_1) \cup N(w_2)$. This implies there exists a $x_1 - x_2$ path not containing w_1 , which is a contradiction. Thus $x_3 \in N(w_1) \cup N(w_2)$. Hence the claim.

Thus, $|V - D| \leq r$ which implies that $\gamma_{2ps}(G) = |D| \geq n - r$. Also by Theorem 2.3 we know that $\gamma_{2ps}(G) \leq n - r$. Therefore, $\gamma_{2ps}(G) = n - r$. $\square$

Now we discuss 2-psd number of unicyclic graphs. Though unicyclic graphs are cactus and in the previous theorem we have proved that the 2-psd number of a cactus with n vertices is $n - r$, therefore, 2-psd number of a unicyclic graph is also $n - r$. Here we give an independent and an alternate proof of the above theorem for separable unicyclic graphs. This proof also gives us an insight about the structure of the minimum 2-psd sets of a unicyclic graph.

Theorem 2.7. *Let m be an integer, ($m > 7$). Let G be a separable unicyclic graph with n vertices and the cycle of length m . Then $\gamma_{2ps}(G) = n - r$.*

Proof. Let G be a separable unicyclic graph. Then $\Delta \geq 3$. This implies $\gamma_{2ps}(G) \leq n - 3$ (Corollary 2.5). Let D be a $\gamma_{2ps}(G)$ -set. Then $V - D$ contains at least three vertices.

Claim: $V - D$ is independent.

Suppose $x, y \in V - D$ and $xy \in E(G)$. Let z be any other member of $V - D$. Now we have following three cases:

Case(i): z is adjacent to both x and y .

In this case we get a cycle (x, y, z) of length three, which is a contradiction to the assumption that the only cycle in G is of length greater than seven. So this case is not possible.

Case(ii): z is adjacent to one of x and y .

Without loss of generality we assume that z is adjacent to y and not to x . Now, x, z are non-adjacent vertices in $V - D$, therefore, there exists a set $W \subseteq D$ such that $|W| \leq 2$ and $\langle \{x, z\} \cup W \rangle$ is connected. Let P be the z - x path in $\langle \{x, z\} \cup W \rangle$. In this case (x, y, z, P, x) forms a cycle of length less than or equal to five, which is again a contradiction to our assumption that the only cycle in G is of length greater than seven. So this case also is not possible.

Case(iii): z is not adjacent to any of x and y .

For sets $\{z, x\}$ and $\{z, y\}$ of non-adjacent vertices there exist $W_1, W_2 \subseteq D$ such that $|W_1|, |W_2| \leq 2$ and $\langle \{z, x\} \cup W_1 \rangle, \langle \{z, y\} \cup W_2 \rangle$ are connected. Let P_1 and P_2 be the z - x and z - y paths in $\langle \{z, x\} \cup W_1 \rangle$ and $\langle \{z, y\} \cup W_2 \rangle$ respectively. Then $P_1 \cup P_2 \cup xy$ contains a cycle of length at most seven, which is again a contradiction to our assumption that the only cycle in G is of length greater than seven. So this case also is not possible.

From all the above cases we get that no two vertices of $V - D$ are adjacent and hence $V - D$ is independent. This proves the claim.

Now, for the set $V - D$, there exists a set $W \subseteq D$ such that $|W| \leq 2$ and $\langle (V - D) \cup W \rangle$ is connected. Then, there are three possibilities:

Case(i): $|W| = 1$ and $W = \{w\}$.

In this case $V - D \subseteq N(w)$. This implies that $|V - D| \leq d(w) \leq \Delta \leq r$. Therefore, $|D| = n - |V - D| \geq n - r$.

Case(ii): $|W| = 2$, $W = \{w_1, w_2\}$ and $w_1 w_2 \in E(G)$.

In this case $V - D \subseteq N(w_1) \cup N(w_2)$. Therefore, $|V - D| \leq |N(S) - S| \leq r$, where $S = \{w_1, w_2\}$. This implies that $|D| \geq n - r$.

Case(iii): $|W| = 2$, $W = \{w_1, w_2\}$ and $w_1 w_2 \notin E(G)$.

In this case there exists a vertex $v \in V - D$ such that $v \in N(w_1) \cap N(w_2)$ and $V - D \subseteq N(w_1) \cup N(w_2)$. Also, there exist vertices v_1 and v_2 in $V - D$ such that $v_1 \in N(w_1) - N(w_2)$ and $v_2 \in N(w_2) - N(w_1)$. Since D is a 2-psd set of G , therefore for the set $\{v_1, v_2\}$, there exists $W' \subseteq D$, $|W'| \leq 2$ such that $\langle W' \cup \{v_1, v_2\} \rangle$

is connected. Note that by our choice of vertices v_1 and v_2 , $W \neq W'$. Let P_1 and P_2 be the $v_1 - v_2$ paths in $\langle W \cup (V - D) \rangle$ and $\langle W' \cup \{v_1, v_2\} \rangle$ respectively. Then $P_1 \cup P_2$ contains a cycle of length less than or equal to seven, which is a contradiction to our assumption that the only cycle of G is of length greater than seven. Thus, Case(iii) is not possible.

Hence, by Case(i) and Case(ii), we get that $|D| \geq n - r$. Also, by Theorem 2.3 we know that $|D| \leq n - r$. Therefore, $|D| = n - r$. This proves the result. $\square$

Remark 2.8. We observed in Theorem 2.7 that for a unicyclic graph G ($m > 7$), where m is the length of the unique cycle, for a $\gamma_{2ps}(G)$ -set D , $V - D$ is independent. However, for $3 \leq m \leq 7$, $V - D$ need not be independent; for example, K_3^+ , obtained by adjunction of a pendant vertex at each vertex of the complete graph K_3 .

Now consider the unicyclic graphs G with unique cycle of length m ($3 \leq m \leq 7$). By Theorem 2.6, we know that $\gamma_{2ps}(G) = n - r$. Here we try to get the insight of the structure of $\gamma_{2ps}(G)$ -set for unicyclic graphs G having unique cycle of length m ($3 \leq m \leq 7$). By Remark 2.8, we know that for any $D \in \mathcal{D}_{2ps}(G)$, $V - D$ need not be independent. Note here that if $V - D$ does not contain any vertex of the unique cycle (C), then $V - D$ is independent. Now let D be any $\gamma_{2ps}(G)$ -set such that $V - D$ contains vertices of the unique cycle (C).

Note that if $|(V - D) \cap V(C)| = 1$, then also clearly $V - D$ is independent. Now assume that $|(V - D) \cap V(C)| \geq 2$. Let $u \in (V - D) \cap V(C)$ be arbitrary. Let $v \in \{(V - D) \cap V(C)\} - \{u\}$ be such that $v \notin N(u)$. Since, D is a 2-psd set of G , there exists a vertex $d_v \in D \cap V(C)$ such that $d_v \in N(u)$. Also since d_v can be adjacent to at most two vertices of C , therefore, for each $v \in \{(V - D) \cap V(C)\} - \{u\}$ such that $v \notin N(u)$ there exists a distinct vertex $d_v \in D \cap V(C)$. This implies that $|(V - D) \cap V(C)| \leq |N(u) \cap V(C)| + 1 = 3$ i.e., for any $D \in \mathcal{D}_{2ps}(G)$, $V - D$ can contain at most three vertices of the unique cycle C .

Now let $|(V - D) \cap V(C)| = 2$. Then $\langle (V - D) \cap V(C) \rangle$ may be connected or disconnected as shown in Figure 2.

Next let $|(V - D) \cap V(C)| = 3$.

Claim: $|V - D| = 3$.

Since $|(V - D) \cap V(C)| = 3$, therefore in order to show that $|V - D| = 3$ we will show that $V - D$ does not contain vertex of any other block. Suppose $z \in (V - D) - V(C)$. Then for $u \in (V - D) \cap V(C)$, there exists a set $W_u \subseteq D$ such that $|W_u| \leq 2$ and $\langle W_u \cup \{u, z\} \rangle$ is connected. Since u and z belong to different blocks of G , therefore, W_u must contain a cut-vertex (say w), belonging to $V(C) \cap D$. Note that $w \in \bigcap_{u \in (V - D) \cap V(C)} W_u$, otherwise there exists a path joining two vertices of $(V - D) \cap V(C)$ and contains z , which contradicts our assumption that $z \in (V - D) - V(C)$.

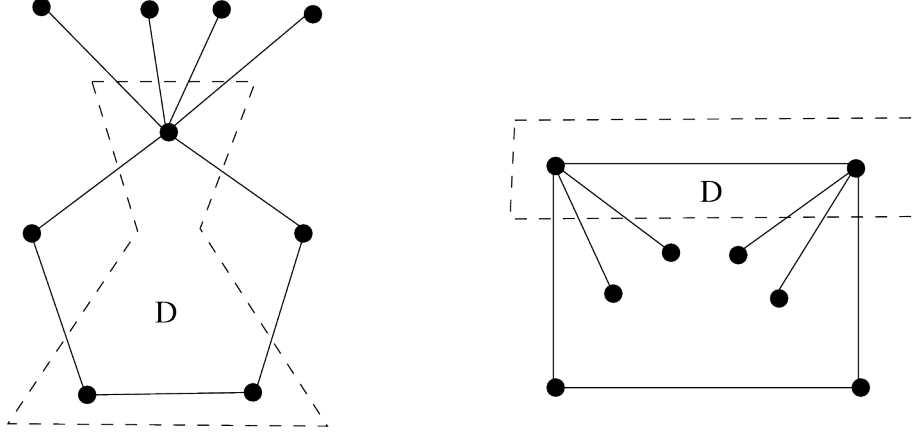


FIGURE 2.

Now since $|(V - D) \cap V(C)| = 3$, therefore, there exists at least one vertex $x \in (V - D) \cap V(C)$ such that $x \notin N(w)$. Since D is a 2-psd set of G and $z \in (V - D) - V(C)$, therefore, there exists a vertex $d_x \in V(C) \cap D \cap N(w) \cap N(x)$. Also since $|N(d_x) \cap V(C)| = 2$, therefore, such a vertex d_x is distinct for each $x \in (V - D) \cap V(C)$.

Thus, if $x \in (V - D) \cap V(C)$ then either $x \in N(w)$ or there exists a distinct $d_x \in V(C) \cap D \cap N(w)$. Therefore, $|N(w) \cap V(C)| \geq |(V - D) \cap V(C)| = 3$, which is a contradiction as C is a cycle and $w \in V(C)$. This proves our claim.

From the above discussion we conclude that in a unicyclic graph G with unique cycle C of length m ($3 \leq m \leq 7$) and for any $D \in \mathcal{D}_{2ps}(G)$, if the set $V - D$ contains three vertices of the cycle C then $|V - D| = 3$.

Therefore, if $|(V - D) \cap V(C)| = 3$ then $V - D \subseteq V(C)$. Further if $\langle V - D \rangle$ is complete, then the unique cycle of G is C_3 and $V - D = V(C)$ in this case. So assume that $|(V - D) \cap V(C)| = 3$ and $\langle V - D \rangle$ is not complete. If $\langle V - D \rangle$ is connected but not complete then the unique cycle is of length either 4 or 5. The two such possible graphs are shown in Figure 3.

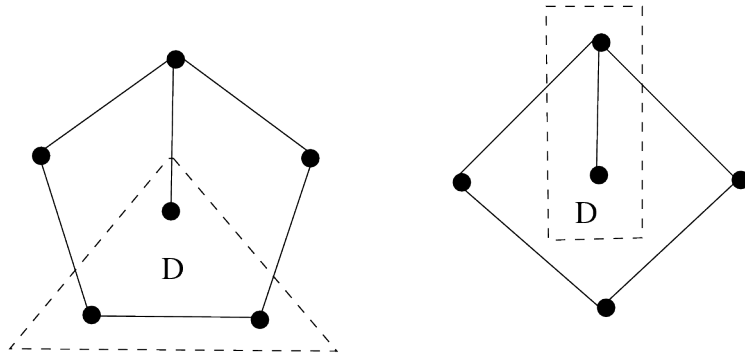


FIGURE 3.

Next if $|(V - D) \cap V(C)| = 3$ and $\langle (V - D) \cap V(C) \rangle$ is disconnected then the graph G must have one of the graphs shown in Figure 4 as a subgraph.

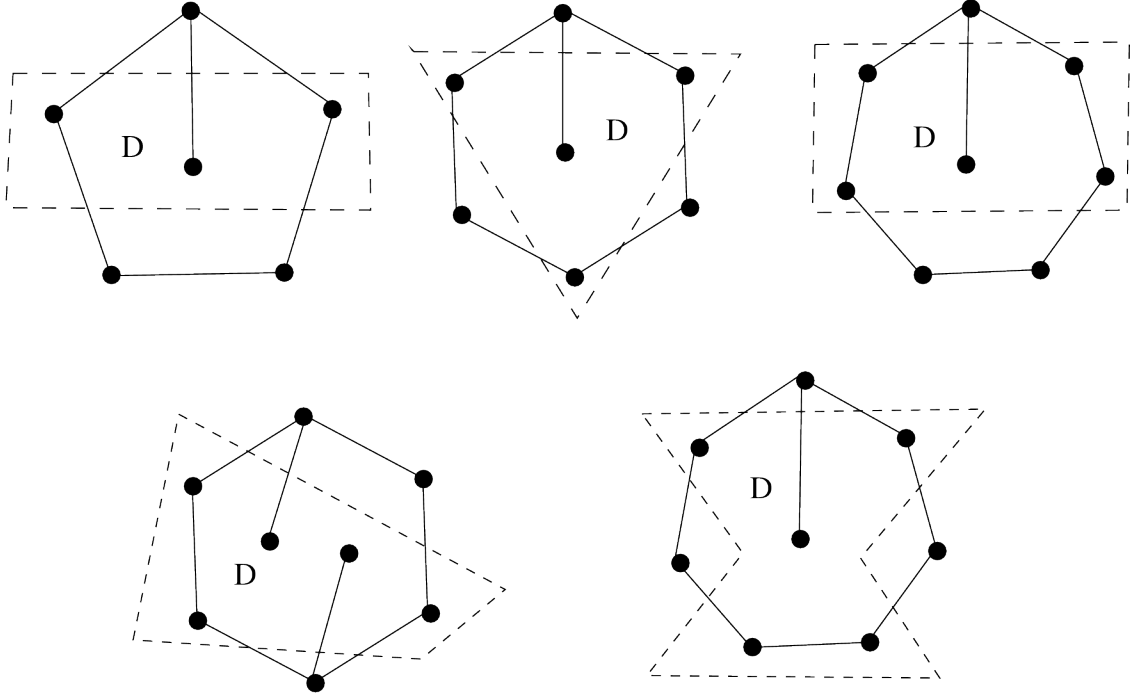


FIGURE 4.

3. CONCLUSION

In this paper we studied 2-point set domination of a graph and found an upper bound on $\gamma_{2ps}(G)$. Further we computed $\gamma_{2ps}(G)$ for cactus and showed that it attains the upper bound given in Theorem 2.3. We can look for some other extremal classes of graphs. In our subsequent papers we will continue our study of 2-point set domination on graphs.

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