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EXACT SOLUTIONS OF NON-NEWTONIAN FLUID OF ROTATING MHD FLOWS THROUGH POROUS MEDIA WITH HALL EFFECT BY COMPLEX VARIABLE TECHNIQUE

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Abstract. In this paper steady, two dimensional, incompressible, electrically conducting, MHD fluid through porous media in a rotating reference frame with Hall effect under the presence of magnetic field is considered. The magnetic field is applied along the z-axis. Using complex variable technique the governing equations are recast in solvable form. Exact solutions are obtained for straight parallel flows.

1. Introduction

In this paper we have applied complex variable technique for the study of steady two dimensional incompressible, electrically conducting MHD fluid flows through porous media in a rotating frame of reference with Hall effect under the influence of magnetic field. We have found exact solutions for straight parallel flow in this work. The governing equations of the flow of a conducting fluid in the presence of a magnetic field are second and fourth order non-linear partial differential equations which are very difficult to solve for exact solutions. To convert these equations into solvable form various transformation techniques like hodograph/Magnetograph transformation, inverse, semi-inverse methods, applications of Martin's approach based on differential geometry etc, are applied. Also, another method involving complex variable is used for the analysis of the fluid flow problems and converting the flow equations into a suitable form for finding out the exact solutions. This complex variable technique has been used by some researchers for finding the exact solution. The first use of this method was done by Stallybrass [22], followed by Wan-Lee Yin [26], in 1983. Afterwards Nguyen and Chandna [13], Thakur and Kumar [23], Sil and Kumar [15], have applied this technique for studying different kinds of MHD fluid flows problems. The study of rotating fluid have gained a lot of importance over the years due to their various applications in different fields including oceanography, meteorology, atmospheric science and limnology etc. The study of earth's magnetic field done by Hide and Proberts [9], involved a frame rotating with earth. The theory of rotating fluid was also considered by Dieke [7], in the study of solar physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars. Various studies on rotating MHD/ non-MHD fluid or

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fluid in a rotating frame of reference have been carried out by many researchers [8, 25, 21, 12, 4, 18, 19, 20, 10, 17, 24, 14, 16, 5]. Many works are there in the literature where in MHD fluid analysis has been carried out with Hall effect. Exact solutions involving Haff effect MHD fluid flow have been found out by many authors [2, 6, 1, 27, 11]. Awanti, Jyoti and Katagi [3], also find the solution of MHD flow of viscous fluid.
In this paper an approach has been made where the complex variables are employed as new independent variables for the determination of the exact solutions for straight parallel flow problem of two dimensional motion of a steady MHD flows of an incompressible electrically conducting fluid through porous media having infinite electrical conductivity in a rotating reference frame with Hall effect in a magnetic field.

Nomenclature

Symbol	SI unit	Meaning
B	Wbm^{-1}	magnetic induction vector
$\mathbf{H_0}$	Am^{-1}	uniform applied magnetic field vector
E	Vm^{-1}	electric field vector
e	C	electric charge
J	Am^{-2}	current density
k	m^2	permeability of the medium
n_e	m^{-3}	number density of electron
p	Nm^{-2}	pressure
p'	Nm^{-2}	reduced pressure
p_e	Nm^{-2}	electron pressure
r	m	radius vector
u, v	ms^{-1}	components of velocity in cartesian coordinate
V	ms^{-1}	velocity vector
x,y	m	cartesian coordinates
μ	Hm^{-1}	magnetic permeability
η	$Kgm^{-1}s^{-1}$	coefficient of viscosity
ρ	Kgm^{-3}	density
σ	Sm^{-1}	electrical conductivity
τ	s	electron collision time
ω_e	s^{-1}	electron frequency
ϕ	...	Hall parameter
$\mathbf{\Omega}$	rad s^{-1}	angular velocity vector

2. Basic Equations

The basic equations governing the motion of a steady, homogeneous, infinite electrically conducting fluid flows through porous media in a rotating reference frame with Hall effect under the influence of magnetic field is given by

$$\vec{\nabla} \cdot \vec{V} = 0, \tag{2.1}$$

$$\rho[(\vec{V} \cdot \vec{\nabla})\vec{V} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})] = -\vec{\nabla}p + \eta\nabla^2\vec{V} + \vec{J} \times \vec{B} - \frac{\eta}{k}\vec{V}, \tag{2.2}$$

$$\vec{J}+\frac{w_e\tau_e}{H_0}(\vec{J}\times\vec{B})=\sigma[\vec{E}+\vec{v}\times\vec{B}+\frac{1}{en_e}\nabla p_e]. \tag{2.3}$$

Where $\vec{B}$ = magnetic induction vector, $\vec{E}$= electric field, $\vec{J}$=current density, ω_e= cyclotron frequency, H_0=strength of uniform applied magnetic field, σ=electrical conductivity, e=electron charge, n_e= number density of the electron, p_e=electron pressure $\vec{V}$ = velocity vector, p = fluid pressure , ρ = fluid density, $\vec{\Omega}$ = constant angular velocity vector, $\vec{r}$ = the radius vector, η = coefficient of viscosity, μ = magnetic permeability, k= permeability of the porous medium and τ_e= electron collision time.
We consider two dimensional flow $\vec{V}$=$\vec{V}(x,y)$ and $\vec{B}=\mu H_0\vec{K}$, and all variable are function of x and y. Also we introduce vorticity function and Bernoulli function

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}, \qquad (\text{ Vorticity function}) \tag{2.4}$$

$$h=\frac{1}{2}\rho V^2+p'+\frac{1}{2}\rho|\vec{\Omega}\times\vec{r}|^2, \qquad (\text{Bernoulli function}) \tag{2.5}$$

where $V^2=u^2+v^2$, p' is the reduced pressure given by $p'=p-\frac{1}{2}\rho|\vec{\Omega}\times\vec{r}|^2$. Now above system of equations are replaced by the following system of equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.6}$$

$$\eta\frac{\partial\omega}{\partial y}-\rho v\omega+Av+Fu=-\frac{\partial h}{\partial x}, \tag{2.7}$$

$$\eta\frac{\partial\omega}{\partial x}-\rho u\omega+Au-Fv=\frac{\partial h}{\partial y}, \tag{2.8}$$

where

$$A=2\rho\Omega-\frac{\sigma\mu^3H_0^2\phi}{1+\mu^2\phi^2} \qquad and \qquad F=\frac{\eta}{k}+\frac{\sigma\mu^2H_0^2}{1+\mu^2\phi^2}$$

are constants, where $\phi=\omega_e\tau$ is the Hall parameter.
We can write equation (2.7) and (2.8) in vector form

$$\vec{\nabla}h=\eta\nabla^2\vec{V}+(\rho\omega-A)\vec{V}\times\vec{k}-F\vec{V}, \tag{2.9}$$

where $\vec{k}$ is the unit vector along z-direction.

Taking curl of equation (2.9) and using integrability conditions $\frac{\partial^2 h}{\partial x\partial y}=\frac{\partial^2 h}{\partial y\partial x}$ we have

$$\vec{\nabla}\times\vec{\nabla}h=\eta\nabla^2(\vec{\nabla}\times\vec{V})+(\rho\omega-A)(\vec{\nabla}\times\vec{V}\times\vec{k})-F(\vec{\nabla}\times\vec{V}), \tag{2.10}$$

$$\eta\nabla^2\omega=F\omega, \tag{2.11}$$

here $\vec{\omega}=\vec{\nabla}\times\vec{V}$, using $u=\psi_y, \qquad v=-\psi_x$ and $\omega=-\nabla^2\psi\vec{k}$, equation (2.11) becomes

$$\eta\nabla^4\psi=-F\nabla^2\psi. \tag{2.12}$$

We introduce the complex variables

$$z=x+iy, \qquad \bar{z}=x-iy,$$

where $i=\sqrt{-1}$ and the following relation can be derived

$$m_x=m_z+m_{\bar{z}}, \qquad m_y=i(m_z-m_{\bar{z}}),$$

$$4m_z m_{\bar{z}} = m_x^2 + m_y^2, \qquad 4m_{z\bar{z}} = m_{xx} + m_{yy},$$

$$\frac{\partial(m,n)}{\partial(x,y)} = -4Im(m_{\bar{z}} m_z),$$

$$m_x n_x + m_y n_y = 4Re(m_z n_{\bar{z}}), \tag{2.13}$$

where m and n are scalar functions.

If $f(z) = \alpha(x,y) + i\beta(x,y)$ is a complex analytic function, then

$$f^{'}(z) = \alpha_x + i\beta_x = \alpha_x - i\alpha_y = 2\alpha_z = 2i\beta_z, \tag{2.14}$$

and

$$\overline{f'(z)} = 2\alpha_{\bar{z}} = 2i\beta_{\bar{z}}, \tag{2.15}$$

where prime denotes differentiation with respect to z.
using equation (2.13) in (2.12) we get,

$$\frac{\psi_{z\bar{z}z\bar{z}}}{\psi_{z\bar{z}}} = -D, \tag{2.16}$$

where $D = \frac{F}{\eta}$

3. Straight Parallel Flow

Let

$$\psi = \psi(\alpha), \qquad \psi^{'} \neq 0, \tag{3.1}$$

to be the stream function, where

$$\alpha(z,\bar{z}) = C_1(z+\bar{z}) + iC_2(\bar{z}-z), \tag{3.2}$$

where C_1 and C_2 are arbitrary constants.
from (3.1) and (3.2) we get

$$\psi_z = \bar{C}\psi^{'}, \psi_{\bar{z}} = C\psi^{'}, \psi_{z\bar{z}} = C\bar{C}\psi^{''},\ and\ \psi_{z\bar{z}z\bar{z}} = C^2\bar{C}^2\psi^{IV}, \tag{3.3}$$

where $C = (C_1 + iC_2)$ and prime denotes differntiation with respect to α.
Using equation (3.3) in (2.16) we get

$$\frac{\psi^{IV}}{\psi^{''}} = -R, \tag{3.4}$$

where $R = \frac{D}{C\bar{C}}$. Solving equation (3.4) we get

$$\psi = -\frac{1}{R}\Big[A_1 sin\sqrt{R}\alpha - Bcos\sqrt{R}\alpha\Big], \tag{3.5}$$

where A_1 and B are constant. Putting value of R in equation (3.5) we get

$$\psi = -\frac{1}{\frac{D}{C\bar{C}}}\Big[A_1 sin\sqrt{\frac{D}{C\bar{C}}}\alpha - Bcos\sqrt{\frac{D}{C\bar{C}}}\alpha\Big], \tag{3.6}$$

in terms of x and y

$$\psi = -\frac{1}{\frac{D}{C\bar{C}}}\Big[A_1 sin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) + Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big], \tag{3.7}$$

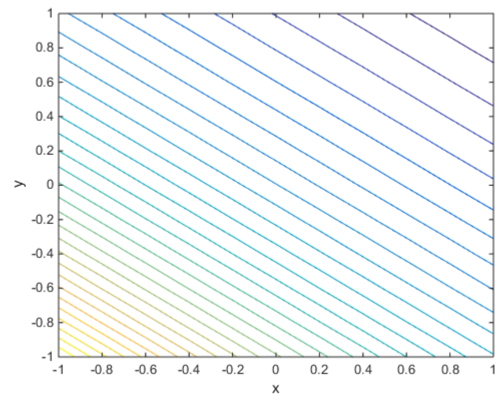

fig: 1 streamlines

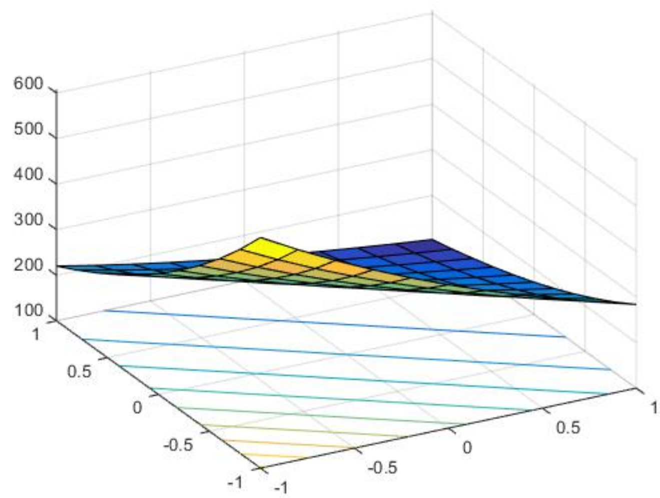

fig: 2 streamsurface

$$u = \frac{2C_2}{\sqrt{\frac{D}{C\bar{C}}}}\Big[Bsin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) - A_1cos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big], \quad (3.8)$$

$$v = \frac{2C_1}{\sqrt{\frac{D}{C\bar{C}}}}\Big[A_1cos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) - Bsin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big], \quad (3.9)$$

$$\omega = -4({C^2}_1 + {C^2}_2)\Big[A_1sin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) + Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big], \quad (3.10)$$

$$p = 4\eta({C^2}_1 + C_2^2)\Big[\frac{C_2}{C_1} - \frac{C_1}{C_2}\Big]\Big[A_1 sin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))$$
$$+Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big] - 8\rho(C_1 + C_2)^2(A_1^2 - B^2)sin^2\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2))$$
$$-\frac{4\rho({C^2}_1 + {C^2}_2)^2}{\sqrt{\frac{D}{C\bar{C}}}}A_1Bsin2\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))$$
$$-\frac{A}{2\sqrt{\frac{D}{C\bar{C}}}}\Big[\frac{1}{C_1} + \frac{1}{C_2}\Big]\Big[A_1 sin\sqrt{(}2(C_1x + C_y)) + Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big]$$
$$+\frac{F}{\frac{D}{C\bar{C}}}\Big[\frac{C_1}{C_2} - \frac{C_2}{C_1}\Big]\Big[A_1 sin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) + Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big]$$
$$-\frac{2\rho}{\frac{D}{C\bar{C}}}\Big[{C^2}_2\{Bsin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) - A_1cos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\}^2$$
$$+{C^2}_1\{A_1cos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) - Bsin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\}^2\Big] + p_0 \tag{3.11}$$

where p_0 is arbitrary constant. Putting the value of A and F in equation (3.11) we get

$$p = 4\eta({C^2}_1 + C_2^2)\Big[\frac{C_2}{C_1} - \frac{C_1}{C_2}\Big]\Big[A_1 sin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))$$
$$+Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big] - 8\rho(C_1^2 + C_2^2)(C_1 + C_2)^2(A_1^2 - B^2)$$
$$sin^2\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2)) - \frac{4\rho({C^2}_1 + {C^2}_2)^2}{\sqrt{\frac{D}{C\bar{C}}}}A_1Bsin2\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) -$$
$$\frac{2\rho\Omega - \frac{\sigma\mu^3H_0^2\phi}{1+\mu^2\phi^2}}{2\sqrt{\frac{D}{C\bar{C}}}}\Big[\frac{1}{C_1} + \frac{1}{C_2}\Big]\Big[A_1 sin\sqrt{(}2(C_1x + C_y)) + Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big]$$
$$+\frac{\frac{\eta}{k} + \frac{\sigma\mu^2H_0^2}{1+\mu^2\phi^2}}{\frac{D}{C\bar{C}}}\Big[\frac{C_1}{C_2} - \frac{C_2}{C_1}][A_1 sin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) + Bcos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\Big]$$
$$-\frac{2\rho}{\frac{D}{C\bar{C}}}\Big[{C^2}_2\{Bsin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) - A_1cos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\}^2$$
$$+{C^2}_1\{A_1cos\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y)) - Bsin\sqrt{\frac{D}{C\bar{C}}}(2(C_1x + C_2y))\}^2\Big] + p_0 \tag{3.12}$$

4. Conclusion

In this present work complex variable technique has been used for the determination of exact solution of steady, two dimensional, incompressible electrically conducting, MHD fluid in a rotating reference frame through porous media under the presence of magnetic field with Hall effect. The basic equations governing the motion have been given in vector form then we have considered the two dimensional flows so that $\vec{V}$ and $\vec{H}$ lies in (x, y) plane. We have taken the system of equations in the velocity field that the flow must satisfy and introduced the vorticity and Bernoulli functions and have written the flow equations in terms of these functions for our flows. Then we have employed the complex variables z=x+iy and $\bar{z}$=x-iy as our new independent variables to recast the equations in solvable form. Further we have determined exact solution for straight parallel flow. The expression for stream function, velocity field, vorticity and pressure function are found out. Also the streamlines and streamsurface are plotted.

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