Gulf Journal of Mathematics ISSN: 2309-4966
Vol 13, Issue 2 (2022) 106-112 https://doi.org/10.56947/gjom.v13i2.811

# DOMINATION AND DOMINATOR COLORING OF NEIGHBORHOOD CORONA OF CERTAIN GRAPHS

K. SATHISH KUMAR[1*], N. GNANAMALAR DAVID[2] AND K.G. SUBRAMANIAN[3]

Abstract. New classes of graphs are obtained by performing operations on known classes of graphs. The neighborhood corona is one such operation on graphs, having interesting applications as well. In this paper we perform this operation on certain graph classes and obtain the dominator chromatic number and the domination number of the resulting classes of graphs besides discussing the bounds in terms of domination number and dominator chromatic number.

## 1. Introduction

Let $G = (V, E)$ be a finite, undirected and simple graph, with $V = V(G)$, the vertex set and $E = E(G)$, the edge set. A subset $S \subseteq V$ is a dominating set of $G$ [9, 11] if every vertex in $V - S$ has at least one neighbor in $S$. A dominating set $S$ of $G$ is called a minimum dominating set, if $S$ consists of a minimum number of vertices among all dominating sets of $G$. The domination number $\gamma(G)$ is the number of vertices in a minimum dominating set of $G$. There has been a number of studies on domination and its variants (see [1, 4, 15] for some recent variants).

A proper coloring of a graph $G$ is an assignment of colors to the vertices of $G$, such that any two adjacent vertices have different colors, and the chromatic number $\chi(G)$ of $G$ is the minimum number of colors needed in a proper coloring of $G$ [3, 7, 11].

Based on the concept of domination in graphs, the notion of dominator coloring in a graph has been introduced and investigated. This concept of dominator coloring was introduced by Hedetniemi et al. [10] and later studied by many researchers [2, 5, 7]. There has been a number of studies on the dominator chromatic number for common graph classes [7] and also on the relationship between dominator chromatic number, domination number and chromatic number. A dominator coloring of $G$ [12, 13, 14] is a proper coloring of $G$ in which every vertex of $G$ dominates every vertex of at least one color class, with the convention that if $\{v\}$ is a color class, then $v$ dominates the color class $\{v\}$. In other words the vertex is either adjacent to all the vertices of one color class or is the only vertex in its color class, by which it will dominate its own color class.

*Date*: Received: Mar 2, 2022; Accepted: Apr 13, 2022.

* Corresponding author.

2010 *Mathematics Subject Classification.* Primary 05C15; Secondary 05C69.

*Key words and phrases.* Graphs, Coloring, Dominator coloring, Neighborhood Corona graph.

Given a graph $G$, while the chromatic number $\chi(G)$ represents the minimum number of colors required to color the vertices of $G$ so that no two adjacent vertices receive the same color, the dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of $G$.

In this paper, we study the dominator chromatic (DC) number and domination number of classes of graphs obtained by performing the neighborhood corona [6, 8] on certain classes of graphs and also investigate bounds for these parameters. $K_n, P_n, C_n, S_{1,n}$ respectively denote the complete graph with $n$ vertices, the path on $n$ vertices, the cycle with $n$ vertices and the star graph with $n$ pendant vertices joined to a central vertex.

**Definition 1.1.** The neighborhood corona $G_1 \star G_2$ of two graphs $G_1$ and $G_2$ is the graph formed from one copy of $G_1$ and $|V(G_1)|$ copies of $G_2$ and joining the neighbors of the $i$th vertex of $G_1$ to every vertex in the $i$th copy of $G_2$.
In general $G_1 \star G_2$ is not commutative.
When $G_2 = K_1,\ G_1 \star G_2 = S(G_1)$ is the splitting graph defined in [16].
When $G_1 = K_1,\ G_1 \star G_2 = G_1 \cup G_2$.

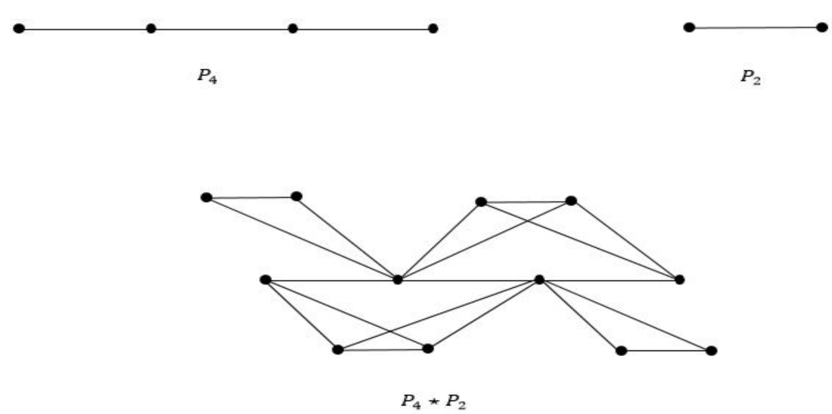


Figure 1. The neighborhood corona $P_4 \star P_2$

.

## 2. Domination number of neighborhood corona graphs

In this section we discuss the domination number of neighborhood corona of certain graph classes. We first obtain the domination number of the neighborhood corona of a path with any connected graph.

**Theorem 2.1.** *Let $P_n$ be a path with $n \geq 3$ vertices and $G$ be any connected graph with $m$ vertices. Then*

$$\gamma(P_n \star G) = \begin{cases} \frac{n+1}{2}, & \text{if } n \equiv 1 \ (mod\ 4) \\ \frac{n}{2}, & \text{if } n \equiv 0 (mod\ 4). \\ \lceil \frac{n-1}{2} \rceil + 1, & \text{otherwise} \end{cases}$$

*Proof.* Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of $P_n$ and $u_{i,j}$ be the vertices of G corresponding to $v_i$, where $i \in \{1, 2, \cdots, n\}$ and $j \in \{1, 2, \cdots, m\}$. The vertex set of the neighborhood corona graph $P_n \star G$ is $V(P_n \star G) = \{v_i : 1 \leq i \leq n\} \cup \{u_{i,j} : 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$. The minimum and maximum degree are $\delta = 2$ and $\Delta = 2m + 2$ respectively. The distance between $v_i$ and $u_{i,j}$ is 2. In order to construct a minimum dominating set of $P_n \star G$, we have to consider four cases:
Case(i): Let $n \equiv 1 \ (mod 4)$.
In this case, consider the subset $S = \{v_{k+2}, v_{k+3} \mid k = 0, 4, \cdots, n-5\} \cup \{v_{n-1}\}$ of $V(P_n \star G)$. By the definition of domination, each vertex in $P_n \star G$ is dominated by at least one vertex in the set S. Hence, the cardinality of $S$ is $\frac{n-1}{2} + 1 = \frac{n+1}{2}$.
Case (ii): Let $n \equiv 0 \ (mod 4)$.
In this case, consider the subset $S = \{v_{k+2}, v_{k+3} \mid k = 0, 4, \cdots, n-4\}$ of $V(P_n \star G)$. It is noted that, each vertex in $P_n \star G$ is dominated by at least one vertex in the set S. Thus, $\gamma(P_n \star G) = \frac{n}{2}$.
Case (iii): If $n \equiv 3 \ (mod 4)$, then consider the subset $S = \{v_{k+2}, v_{k+3} \mid k = 0, 4, \cdots, n-3\}$ of $V(P_n \star G)$. It is clear that each vertex of $P_n \star G$ is dominated by one of the elements in $S$. Thus, $\gamma(P_n \star G) = \lceil \frac{n-1}{2} \rceil + 1$.
Case (iv): If $n \equiv 2 \ (mod 4)$, then consider the subset $S = \{v_{k+2}, v_{k+3} \mid k = 0, 4, \cdots, n-4\} \cup \{v_{n-1}, v_n\}$ of $V(P_n \star G)$. Thus each vertex in $V(P_n \star G)$ is dominated by one of the vertices in $S$. Hence, we have $\gamma(P_n \star G) = \lceil \frac{n-1}{2} \rceil + 1$. Summing up Cases (i), (ii), (iii) and (iv), we get the theorem. □

We now obtain the domination number of the neighborhood corona of a cycle with any connected graph.

**Theorem 2.2.** *For $n \geq 3$,*

$$\gamma(C_n \star G) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \equiv 1 \ (mod\ 4) \\ \\ \dfrac{n}{2}, & \text{if } n \equiv 0 \ (mod\ 4) \\ \\ \left\lceil \dfrac{n-1}{2} \right\rceil + 1, & \text{otherwise} \end{cases}$$

*Proof.* The proof can be given as done in Theorem 2.1. □

The domination number of the neighborhood corona of a star graph with any connected graph is now obtained.

**Theorem 2.3.** *Let $S_{1,n}$ be a star graph with $n+1$ vertices and $G$ be any connected graph with $m$ vertices. Then $\gamma(S_{1,n} \star G) = 2$.*

*Proof.* Let $S \subseteq V(S_{1,n} \star G)$ be $\gamma-$set of $S_{1,n} \star G$. Since the center vertex $v$ of $S_{1,n}$ dominates many vertices in $S_{1,n} \star G$, $S$ must contain the vertex $v$. By the definition of domination, this vertex alone is not sufficient to dominate all the vertices of $S_{1,n} \star G$. So include in $S$, a vertex $c$ in $V(S_{1,n} \star G) - N[v]$. Thus each vertex in $V(S_{1,n} \star G)$ is dominated by one of the vertices in $S$. Hence, we have $\gamma(S_{1,n} \star G) = 2$. □

**Corollary 2.4.** *Let $W_{1,n}$ be the wheel graph with $n+1$ vertices, $K_m$ be the complete graph with $m$ vertices and $G$ be any connected graph with $t$ vertices. Then $\gamma(H \star G) = 2$, where $H$ is either $W_{1,n}$ or $K_n$*

*Proof.* The proof can be given as done in Theorem 2.3. □

## 3. DC-number of neighborhood corona of two graphs

In this section we first discuss bounds on the dominator chromatic (DC) number of neighborhood corona of any two graphs.

**Theorem 3.1.** *Let $G_1$ be a graph on $n$ vertices and $G_2$ be an arbitrary graph. Then*

$$\chi_d(G_1 \star G_2) \leq n + \chi(G_2)$$

*Proof.* First we assign distinct colors $a_1, a_2, \cdots, a_n$ to the vertices of $G_1$. Now for every copy of $G_2$, use the colors $b_1, b_2, \cdots, b_{\chi(G_2)}$ to its vertices. Each vertex of $G_1$ dominates one of its adjacent color classes or its own color class. Also each vertex of every copy of $G_2$ is adjacent to at least one vertex of $G_1$ and can use this color for dominator coloring. Note that this is a proper coloring and every vertex in the graph $G_1 \star G_2$ dominates at least one color class. So $\chi_d(G_1 \star G_2) \leq n + \chi(G_2)$. □

**Theorem 3.2.** *For any two graphs $G_1$ and $G_2$*

$$\chi_d(G_1 \star G_2) \geq \chi_d(G_1) + \chi(G_2)$$

*Proof.* We need at least $\chi_d(G_1)$ colors to color $G_1$ while $\chi(G_2)$ colors are enough to color $G_2$. So we need at least $\chi_d(G_1) + \chi(G_2)$ colors to color $G_1 \star G_2$. Thus we have $\chi_d(G_1 \star G_2) \geq \chi_d(G_1) + \chi(G_2)$.

For equality it suffices to consider the path $P_3$ as $G_1$ and $P_2$ as $G_2$. For the strict inequality it suffices to consider the path $P_5$ as $G_1$ and $P_2$ as $G_2$.
The following corollary is obtained from Theorem 3.1 and Theorem 3.2. □

**Corollary 3.3.** *Let $G_1$ be a graph on $n$ vertices and $G_2$ be an arbitrary graph. Then*

$$\chi_d(G_1) + \chi(G_2) \leq \chi_d(G_1 \star G_2) \leq n + \chi(G_2)$$

We now obtain the dominator chromatic number of the neighborhood corona of two paths.

**Theorem 3.4.** *Let $n, m \geq 2$. Then*

$$\chi_d(P_n \star P_m) = \lceil 2n/3 \rceil + 2.$$

*Proof.* Let $P_n$ be a path on $n$ vertices and $n \geq 2$. Let $P_m^i$, $1 \leq i \leq n$, be the $i$th copy of path $P_m$. Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each $i$, $1 \leq i \leq n$, let $V(P_m^i) = \{u_{ij} \mid 1 \leq j \leq m\}$ with $u_{i1}, u_{i2}, \cdots, u_{im}$ in order so that $V(P_n \star P_m) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ where each $i$, all the vertices in the $i$th copy of $P_m^i$, is adjacent with neighbors of $v_i, i = 1, 2, \cdots, n$. For a proper coloring to the vertices of $P_n \star P_m$ assign color 1 to $u_{ij}$, for odd $j$ and color 2 to $u_{ij}$, $1 \leq i \leq n$, for even $j$.

It can be seen that a minimum of $\lceil 2n/3 \rceil$ colors are required to color the remaining vertices of $v_i$, $1 \leq i \leq n$. It is clear that every vertex $u_{ij}$ of $P_m^i$, dominates the at least one color class of $v_i, 1 \leq i \leq n$. Thus $\chi_d(P_n \star P_m) = \lceil 2n/3 \rceil + 2$. □

We now obtain the dominator chromatic number of the neighborhood corona of a path and a cycle.

**Theorem 3.5.** *Let $P_n$ be a path on $n(\geq 2)$ vertices and $C_m$ be a cycle on $m(\geq 3)$ vertices. Then*

$$\chi_d(P_n \star C_m) = \begin{cases} \lceil 2n/3 \rceil + 2, & \textit{if } m \textit{ is even} \\ \lceil 2n/3 \rceil + 3, & \textit{if } m \textit{ is odd.} \end{cases}$$

*Proof.* Let $P_n$ be a path on $n$ vertices and $n \geq 2$. Let $C_m^i$, $1 \leq i \leq n$, be the $i$th copy of path $C_m$. Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each $i$, $1 \leq i \leq n$, let $V(C_m^i) = \{u_{ij} \mid 1 \leq j \leq m\}$ with $u_{i1}, u_{i2}, \cdots, u_{im}$ in order so that $V(P_n \star C_m) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ where for each $i$, all the vertices in the $i$th copy of $C_m^i$, are adjacent with neighbors of $v_i, i = 1, 2, \cdots, n$. Note that the minimum and maximum degree are $\delta(P_n \star C_m) = 3$ and $\Delta(P_n \star C_m) = 2m+2$. For assigning minimum number of colors to the vertices of $P_n \star C_m$ to obtain a dominator coloring, we consider the following cases:
Case (i): $m$ is even. Assign color 1 to the vertex $u_{ij}$ if $j$ is odd and color 2 if $j$ is even for all $i, 1 \leq i \leq n$. Now assign $\lceil 2n/3 \rceil$ new colors to the vertices of $P_i, 1 \leq i \leq n$.
Case (ii): $m$ is odd. For all $i, 1 \leq i \leq n$, assign to each vertex of $u_{ij}$ color 1 for odd $j, 1 \leq j \leq m-1$ and color 2 for even $j$. Assign color 3 to $u_{im}$ for all $i, 1 \leq i \leq n$.
Now assign $\lceil 2n/3 \rceil$ new colors to the vertices of $P_i, 1 \leq i \leq n$.
It can be seen that each vertex dominates at least one color class in each cases. Thus $\chi_d(P_n \star C_m) = 2 + \lceil 2n/3 \rceil$ in the case (i) while $\chi_d(P_n \star C_m) = 3 + \lceil 2n/3 \rceil$ in case(ii). □

We now obtain the dominator chromatic number of the neighborhood corona of a path and a complete graph.

**Theorem 3.6.** *Let $P_n$ be a path on $n(\geq 2)$ vertices and $K_m$ be a complete graph with $m$ vertices, where $n \geq m$. Then*

$$\chi_d(P_n \star K_m) = \lceil 2n/3 \rceil + m.$$

*Proof.* Let $K_m^i$, $1 \leq i \leq n$, be the $i$th copy of complete graph $K_m$ Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each $i$, $1 \leq i \leq n$, let $V(K_m^i) = \{u_{ij} \mid 1 \leq j \leq m\}$ so that $V(P_n \star K_m) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq m\}$ where, by definition, for each $i$, neighbors of $v_i, 1 \leq i \leq n$, are adjacent to every vertex $u_{ij}$ of $K_m^i$, the $i$th copy of $K_m$. For all $i, 1 \leq i \leq n$ assign color $c_j$ to $u_{ij}$ for $1 \leq j \leq m$ so that $m$ colors are used. Assign $\lceil 2n/3 \rceil$ new colors to the vertices of $P_i, 1 \leq i \leq n$. This yields a dominator coloring with a minimum number of colors. Thus $\chi_d(P_n \star K_m) = \lceil 2n/3 \rceil + m$ □

The following result can be proved as done in the proofs of the results dealt with earlier and so we omit the details of the proof.

**Theorem 3.7.** $(i)$ *Let $m, n \geq 2$ be positive integers. Then*

$$\chi_d((K_m \star P_n)) = m + 2.$$

$(ii)$ *Let $m, n \geq 3$ be positive integers. Then*

$$\chi_d(K_m \star C_n) = \begin{cases} m+2, & \text{if } m \text{ is even} \\ m+3, & \text{if } m \text{ is odd.} \end{cases}$$

## 4. Conclusion

In this paper, the dominator chromatic number $\chi_d(G_1 \star G_2)$ of neighborhood corona of some standard graphs $G_1$ and $G_2$ are computed. We have also obtained bounds on $\chi_d(G_1 \star G_2)$ in terms of chromatic number and dominator chromatic number of the individual graphs $G_1$ and $G_2$. It will be of interest to study dominator chromatic number of neighborhood corona of other classes of graphs not dealt with in this work.

**Acknowledgement.** The authors thank the reviewers and the editor for useful comments which helped to improve the presentation of the paper.

## References

1. A. Alqesmah, A. Alwardi and R. Rangarajan, *Edge injective domination of graphs*, Gulf Journal of Mathematics **5(2)** (2017), 46-55. https://doi.org/10.3390/math5030046
2. S. Arumugam, J. Bagga and K.R. Chandrasekar, *On Dominator Coloring in Graphs*, Proc. Indian Acad. Sci. **122** (2012), 561-571. https://doi.org/10.1007/s12044-012-0092-5
3. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier, North Holland, New York (1986).
4. B. Chaluvaraju and V. Chaitra, *Sign domination in Arithmetic graphs*, Gulf Journal of Mathematics **4(3)** (2016), 49-54. https://doi.org/10.56947/gjom.v4i3.73
5. M. Chellai and F. Maffray, *Dominator colorings in some classes of graphs*, Graphs combin. **28** (2012), 97-107.
6. F. Frucht, and F. Harary, *On the corona of two graphs*, Aequationes Math **4** (1970), 322-325. https://doi.org/10.1007/BF01844162
7. R. Gera, S. Horton, and C. Rasmussen, *Dominator Colorings and Safe Clique Partitions*, Congressus Numerantium **181** (2006), 19-32. https://doi.org/10.2306/scienceasia1513-1874.2006.32.181
8. I. Gopalapillai, *The spectrum of neighborhood corona of graphs*, Kragujevac Journal of Mathematics **35** (2011), 493–500. https://doi.org/10.1038/ijo.2011.4
9. T.W. Haynes, S.M. Hedetniemi, S.T. Hedetniemi, and M.A. Henning, *Domination is graphs applied to electic power networks*, SIAM J. Discrete Math. **15(4)** (2002), 519-529. https://doi.org/10.1137/S0895480100375831
10. S.M. Hedetniemi, S.T. Hedetniemi, A.A. Mcrae and J.R.S. Blair, *Dominator colorings of graphs*, Preprint, 2006.
11. T.W. Haynes, S.M. Hedetniemi, P.J. Slater, Fundamentals of Domination in Graphs, Marcel Dekkar, New York (1998).
12. K. Kavitha and N. G. David, *Dominator and Strong Dominator Coloring on Bistar Graph Families*, International Journal of Applied Mathematics **14(10)** (2012), 444-448.
13. K. Kavitha and N. G. David, *Dominator coloring of Central Graphs*, International Journal of Computer Applications **51(12)** (2012), 11-14. https://doi.org/10.5120/8093-1673

14. K. Kavitha and N. G. David, *Dominator Coloring of Some Classes of Graphs*, International Journal of Mathematical Archive **3(11)** (2012), 3954-3957.
15. S.R. Nayaka, A. Alwardi, Puttaswamy, *Transversal domination in graphs*, Gulf Journal of Mathematics **6(2)** (2018) 41-49. https://doi.org/10.1080/07468342.2017.1392101
16. E. Sampathkumar, H.B. Walikar, *On splitting graph of a graph*, J.Karnatak Univ. Sci. **25 and 26 (Combined)** (1980-81), 13-16. https://doi.org/10.3406/antaf.1980.1055

[1] Department of Mathematics, Madras Christian College, Chennai, 600 059, India.
*Email address*: sathishkumar@mcc.edu.in

[2] Honorary Visiting Professor, Department of Mathematics and Computer Science, Faculty of Science, Liverpool Hope University, Liverpool L16 9JD, UK
*Email address*: ngdmcc@gmail.com

[3] Honorary Visiting Professor, Department of Mathematics and Computer Science, Faculty of Science, Liverpool Hope University, Liverpool L16 9JD, UK
*Email address*: kgsmani1948@gmail.com