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SOME CURVATURE CHARACTERIZATIONS ON KENMOTSU METRIC SPACES

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ABSTRACT. The purpose of this study is to characterize Kenmotsu manifolds that satisfy specific curvature conditions. We give the Kenmotsu manifold curvature tensors satisfying the conditions $R \cdot W_5 = 0$, $R \cdot W_7 = 0$, $R \cdot W_9 = 0$ and $R \cdot W_0^{\star} = 0$. Also, we consider the Kenmotsu metric manifold a $W_0^{\star} - flat$ and a ϕ-$W_0^{\star} - flat$. As a result, M is an η-Einstein Kenmotsu metric manifold.

1. INTRODUCTION

In 1958, Bootby and Wong, from a topological standpoint, investigated odd dimensional manifolds with contact and almost contact structures [5]. Following then, in 1961, Sasaki and Hatakeyama used tensor calculus to re-examine [15]. After, Tanno characterized M as an almost contact metric manifold with the maximum dimension automorphism group [18]. In 1963, Kobayashi and Nomizu proved that any two simply linked complete Riemannian manifolds with constant curvature k are isometric. Following then, several authors have examined Kenmotsu manifolds in a variety of approaches, including [10].

In 1972, Kenmotsu manifold is a class of contact Riemannian manifolds studied by K. Kenmotsu [9]. If a Kenmotsu manifold satisfies the condition $R(X, Y) \cdot R = 0$, where R is the Riemannian curvature tensor and $R(X, Y)$ is the tensor algebra derivation at each point of the tangent space. S. K. Chaubey and R. H. Ojha, on the other hand, investigated the features of the Kenmotsu manifold with the $M-$projective curvature tensor and looked for basic definitions of the Kenmotsu and $\eta-$Einstein manifolds. They also looked at the $M-$projective curvature tensor in the Riemannian manifold and discovered a relationship between different curvature tensors [6].

Motivated from all the research papers mentioned above and specially from the work done by R.N. Singh et al [16]. The goal of this research is to investigate the properties of a certain curvature tensor in a Kenmotsu metric manifold. In this study, we look $R \cdot W_5 = 0$, $R \cdot W_7 = 0$, $R \cdot W_9 = 0$ and $R \cdot W_0^{\star} = 0$, where W_5, W_7, W_9, and $W_0^{\star}$ denote curvature tensors of manifold, respectively. We also

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examine Kenmotsu metric manifolds with $W_0^\star - flat$ and a ϕ-$W_0^\star - flat$. As a result, M is a $\eta-$Einstein Kenmotsu manifold.

2. Preliminaries

Let M be a $(2n+1)-$dimensional connected almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g), that is, ϕ is an $(1,1)$ tensor field, ξ is a vector field, η is a 1-form and the Riemanniann metric g satisfying

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\phi X) = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta\phi = 0 \tag{2.2}$$

for all $X, Y \in \chi(M)$[11]. Let g be Riemannian metric compatible with (ϕ, ξ, η), that is

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

or equivalently,

$$g(X, \phi Y) = -g(\phi X, Y) \quad \text{and} \quad g(X, \xi) = \eta(X) \tag{2.4}$$

for all $X, Y \in \chi(M)$[4]. If moreover,

$$(\nabla_X \phi)Y = -\eta(Y)\phi X - g(X, \phi Y)\xi, \tag{2.5}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.6}$$

where ∇ denotes the Riemannian connection of g hold, then (M, ϕ, ξ, η, g) is called an almost Kenmotsu manifold. An almost Kenmotsu manifold becomes a Kenmotsu manifold if

$$g(X, \phi Y) = d\eta(X, Y). \tag{2.7}$$

In a Kenmotsu manifold M, the following relation holds[9, 7]:

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{2.8}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.9}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.10}$$

$$S(X, \xi) = -2n\eta(X), \tag{2.11}$$

$$Q\xi = -2n\xi, \tag{2.12}$$

where R is the Riemannian curvature tensor and S is Ricci tensor defined by $S(X, Y) = g(QX, Y)$, where Q is Ricci operator. It yields to

$$S(\phi X, \phi Y) = S(X, Y) + 2n\eta(X)\eta(Y). \tag{2.13}$$

A Kenmotsu manifold M is said to be an $\eta-$Einstein manifold if its Ricci tensor S of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y) \tag{2.14}$$

for arbitrary vector fields X, Y; where a and b are functions on (M^{2n+1}, g). If $b = 0$, then $\eta-$ Einstein manifold becomes Einstein manifold[14, 9].

Let M be an $(2n+1)-$dimensional Kenmotsu manifold. The curvature tensor $\widetilde{R}$ of M with respect to the connection $\widetilde{\nabla}$ is defined by

$$\widetilde{R}(X, Y)Z = \widetilde{\nabla}_X \widetilde{\nabla}_Y Z - \widetilde{\nabla}_Y \widetilde{\nabla}_X Z - \widetilde{\nabla}_{[X,Y]} Z. \tag{2.15}$$

Then, in a Kenmotsu manifold, we have

$$\widetilde{R}(X,Y)Z = R(X,Y)Z + g(Y,Z)X - g(X,Z)Y, \tag{2.16}$$

where $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$, is the curvature tensor of M with respect to the connection ∇.

The concept of W_5-curvature tensor was defined by [13]. W_5-curvature tensor, W_7-curvature tensor, W_9-curvature tensor and $W_0^\star$-curvature tensor of a $(2n+1)$-dimensional Riemannian manifold are, respectively, defined as

$$W_5(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(X,Z)Y - g(X,Z)QY], \tag{2.17}$$

$$W_7(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - g(Y,Z)QX], \tag{2.18}$$

$$W_9(X,Y)Z = R(X,Y)Z + \frac{1}{2n}[S(X,Y)Z - g(Y,Z)QX], \tag{2.19}$$

$$W_0^\star(X,Y)Z = R(X,Y)Z + \frac{1}{2n}[S(Y,Z)X - g(X,Z)QY], \tag{2.20}$$

for all $X, Y, Z \in \chi(M)$[12, 13].

3. Some Curvature Characterizations On Kenmotsu Metric Spaces

In this section, we will give the main results for this paper.

Let M be $(2n+1)-$dimensional Kenmotsu metric manifold and we denote W_5-curvature tensor from (2.17), we have for later

$$W_5(X,Y)\xi = 2\eta(X)Y - \eta(Y)X + \frac{1}{2n}\eta(X)QY. \tag{3.1}$$

Putting $X = \xi$, in (3.1)

$$W_5(\xi,Y)\xi = 2Y - \eta(Y)\xi + \frac{1}{2n}QY. \tag{3.2}$$

From (2.18) and (2.9), we arrive

$$W_7(X,Y)\xi = \eta(X)Y + \frac{1}{2n}\eta(Y)QX. \tag{3.3}$$

and

$$W_7(\xi,Y)\xi = Y - \eta(Y)\xi. \tag{3.4}$$

Choosing $Z = \xi$, in (2.19), we obtain

$$W_9(X,Y)\xi = \eta(X)Y - \eta(Y)X + \frac{1}{2n}(S(X,Y)\xi - \eta(Y)QX). \tag{3.5}$$

In (3.5), it follows

$$W_9(\xi,Y)\xi = Y - \eta(Y)\xi. \tag{3.6}$$

In (2.20), choosing $Z = \xi$, we get

$$W_0^\star(X,Y)\xi = \eta(X)Y - 2\eta(Y)X - \frac{1}{2n}\eta(X)QY. \tag{3.7}$$

Setting $X = \xi$, in (3.7), we have

$$W_0^\star(\xi, Y)\xi = Y - 2\eta(Y)\xi - \frac{1}{2n}QY. \tag{3.8}$$

Theorem 3.1. *Let $M^{2n+1}(\phi, \xi, \eta, g)$ be a Kenmotsu manifold. Then M is a W_5 semi-symmetric if and only if M is an $\eta-$Einstein manifold.*

Proof. Suppose M is a W_5 semi-symmetric. This implies that

$$\begin{aligned}(R(X,Y)W_5)(U,W)Z &= R(X,Y)W_5(U,W)Z - W_5(R(X,Y)U,W)Z \\ &\quad -W_5(U,R(X,Y)W)Z \\ &\quad -W_5(U,W)R(X,Y)Z = 0,\end{aligned} \tag{3.9}$$

for any $X, Y, U, W, Z \in \chi(M)$. Taking $X = Z = \xi$ in (3.9), making use of (3.1), (2.9) and (2.10), for $p_1 = \frac{1}{2n}$, we have

$$\begin{aligned}(R(\xi,Y)W_5)(U,W)\xi &= R(\xi,Y)(2\eta(U)W - \eta(W)U + \eta(U)QW) \\ &\quad -W_5(\eta(U)Y - g(Y,U)\xi, W)\xi \\ &\quad -W_5(U, \eta(W)Y - g(Y,W)\xi)\xi \\ &\quad -W_5(U,W)(Y - \eta(Y)\xi) = 0.\end{aligned} \tag{3.10}$$

Taking into account (3.1), (3.2), (2.9) in (3.10), we obtain

$$\begin{aligned}&-W_5(U,W)Y - 2\eta(U)g(Y,W)\xi - 2n\eta(U)\eta(W)Y \\ &-\eta(U)S(Y,W)\xi + g(Y,U)W + p_1 g(Y,U)QW \\ &-\eta(U)\eta(W)QY - g(Y,W)U + \eta(U)g(Y,W)\xi \\ &-p_1 g(Y,W)QU = 0.\end{aligned} \tag{3.11}$$

Putting (2.17), (2.4) and choosing $U = \xi$, inner product both sides of (3.11) by $Z \in \chi(M)$, and finally using $W = \xi$, we arrive

$$S(Y,Z) = -2ng(Y,Z) + (2np_1 + 2n - 2)\eta(Y)\eta(Z) = 0, \tag{3.12}$$

and we conclude

$$S(Y,Z) = -2ng(Y,Z) + (2n-3)\eta(Y)\eta(Z).$$

So, M is an $\eta-$Einstein manifold. Conversely, let $M^{2n+1}(\phi, \xi, \eta, g)$ be an $\eta-$Einstein manifold i.e. $S(Y,Z) = -2ng(Y,Z) + (2n-3)\eta(Y)\eta(Z)$, then from (3.12), (3.11), (3.10) and (3.9), we have $R \cdot W_5 = 0$. □

Theorem 3.2. *Let $M^{2n+1}(\phi, \xi, \eta, g)$ be a Kenmotsu manifold. Then M is a W_7 semi-symmetric if and only if M is an Einstein manifold.*

Proof. Suppose that M is a W_7 semi-symmetric. This yields to

$$\begin{aligned}(R(X,Y)W_7)(U,W)Z &= R(X,Y)W_7(U,W)Z - W_7(R(X,Y)U,W)Z \\ &\quad -W_7(U,R(X,Y)W)Z \\ &\quad -W_7(U,W)R(X,Y)Z = 0,\end{aligned} \tag{3.13}$$

for any $X, Y, U, W, Z \in \chi(M)$. Taking $X = Z = \xi$ in (3.13) and using (3.3), (2.9), (2.10), for $p_1 = \frac{1}{2n}$, we obtain

$$\begin{aligned}(R(\xi,Y)W_7)(U,W)\xi &= R(\xi,Y)(\eta(U)W + p_1\eta(W)QU)\\ &\quad -W_7(\eta(U)Y) - g(Y,U)\xi, W)\xi\\ &\quad -W_7(U, \eta(W)Y - g(Y,W)\xi)\xi\\ &\quad -W_7(U,W)(Y - \eta(Y)\xi) = 0, \end{aligned} \tag{3.14}$$

and from (3.14), we arrive

$$\begin{aligned}&\eta(U)R(\xi,Y)W + p_1\eta(W)R(\xi,Y)QU - \eta(U)W_7(Y,W)\xi\\ &+g(Y,U)W_7(\xi,W)\xi - \eta(W)W_7(U,Y)\xi + g(Y,W)W_7(U,\xi)\xi\\ &-W_7(U,W)Y + \eta(Y)W_7(U,W)\xi = 0.\end{aligned} \tag{3.15}$$

Taking into account that (2.9), (2.10) and (3.3) in (3.15), we get

$$\begin{aligned}&-W_7(U,W)Y - 2np_1\eta(W)\eta(U)Y - p_1\eta(W)S(Y,U)\xi\\ &-p_1\eta(W)\eta(U)QY + g(Y,U)W - \eta(W)g(U,Y)\xi\\ &-p_1\eta(W)\eta(Y)QU - g(Y,W)U + p_1\eta(Y)\eta(U)QW = 0.\end{aligned} \tag{3.16}$$

Setting $U = \xi$, using (2.11), inner product both sides of (3.16) by $\xi \in \chi(M)$, we have

$$S(Y,W) = -2ng(Y,W).$$

Thus, M is an Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an Einstein manifold i.e. $S(Y,W) = -2ng(Y,W)$, then from (3.16), (3.15), (3.14) and (3.13), we have $R \cdot W_7 = 0$. □

Theorem 3.3. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a W_9 semi-symmetric if and only if M is an Einstein manifold.*

Proof. Suppose that M is a W_9 semi-symmetric. This means that

$$\begin{aligned}(R(X,Y)W_9)(U,W,Z) &= R(X,Y)W_9(U,W)Z - W_9(R(X,Y)U,W)Z\\ &\quad -W_9(U,R(X,Y)W)Z\\ &\quad -W_9(U,W)R(X,Y)Z = 0,\end{aligned} \tag{3.17}$$

for any $X, Y, U, W, Z \in \chi(M)$. Setting $X = Z = \xi$ in (3.17) and making use of (3.5), (2.9), for $p_1 = \frac{1}{2n}$, we obtain

$$\begin{aligned}(R(\xi,Y)W_9)(U,W)\xi &= R(\xi,Y)(\eta(U)W - \eta(W)U + p_1S(U,W)\xi\\ &\quad -p_1\eta(W)QU) - W_9(\eta(U)Y - g(Y,U)\xi, W)\xi\\ &\quad -W_9(U, \eta(W)Y - g(Y,W)\xi)\xi\\ &\quad -W_9(U,W)(Y - \eta(Y)\xi) = 0.\end{aligned} \tag{3.18}$$

Using (3.5), (3.6) and (2.9) in (3.18), we get

$$\begin{aligned}&W_9(U,W)Y - p_1S(U,W)Y - 2np_1\eta(W)\eta(U)Y\\ &+p_1\eta(U)S(Y,W)\xi - p_1\eta(W)\eta(U)QY - g(Y,U)W\\ &+g(Y,W)U = 0.\end{aligned} \tag{3.19}$$

Making use of (2.19) and choosing $U=\xi$, we have

$$p_1S(Y,W)\xi-\eta(W)Y-p_1\eta(W)QY+g(Y,W)\xi=0. \tag{3.20}$$

By using (2.11) and inner product both sides of (3.20) by $\xi\in\chi(M)$, we conclude

$$S(Y,W)=2ng(Y,W).$$

This tell us, M is an Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an Einstein manifold i.e. $S(Y,W)=2ng(Y,W)$, then from (3.20), (3.19), (3.18) and (3.17), we have $R\cdot W_9=0$ □

Theorem 3.4. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a $W_0^\star$ semi-symmetric if and only if M is an $\eta-$Einstein manifold.*

Proof. Suppose that M is a $W_0^\star$ semi-symmetric. This means that

$$\begin{aligned}(R(X,Y)W_0^\star)(U,W,Z) &= R(X,Y)W_0^\star(U,W)Z-W_0^\star(R(X,Y)U,W)Z\\ &\quad -W_0^\star(U,R(X,Y)W)Z\\ &\quad -W_0^\star(U,W)R(X,Y)Z=0,\end{aligned} \tag{3.21}$$

for any $X,Y,U,W,Z\in\chi(M)$. Setting $X=Z=\xi$ in (3.21) and making use of (3.7), (2.9), (2.10), for $p_1=\frac{1}{2n}$, we obtain

$$\begin{aligned}(R(\xi,Y)W_0^\star)(U,W)\xi &= R(\xi,Y)(\eta(U)W-2\eta(W)U-p_1\eta(U)QW)\\ &\quad -W_0^\star(\eta(U)Y-g(Y,U)\xi,W)\xi\\ &\quad -W_0^\star(U,\eta(W)Y-g(Y,W)\xi)\xi\\ &\quad -W_0^\star(U,W)(Y-\eta(Y)\xi)=0.\end{aligned} \tag{3.22}$$

Using (3.7) and (3.8) in (3.22), we get

$$\begin{aligned}&-W_0^\star(U,W)Y-\eta(U)g(Y,W)\xi+2np_1\eta(U)\eta(W)Y\\ &+p_1\eta(U)S(Y,W)\xi+g(Y,U)W-p_1g(Y,U)QW\\ &+p_1(U)\eta(W)QY-g(Y,W)U+2\eta(U)g(Y,W)\xi\\ &+p_1g(Y,W)QU=0.\end{aligned} \tag{3.23}$$

Using $U=\xi$, inner product both sides of (3.23) by $Z\in\chi(M)$, putting $W=\xi$ and making use of (2.20), we have

$$p_1S(Y,Z)+2np_1g(Y,Z)+(2-2np_1)\eta(Y)\eta(Z)=0. \tag{3.24}$$

and, we arrive

$$S(Y,Z)=-2n\left[g(Y,Z)+\eta(Y)\eta(Z)\right].$$

Thus, M is an $\eta-$Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an $\eta-$Einstein manifold i.e. $S(Y,Z)=-2n\left[g(Y,Z)+\eta(Y)\eta(Z)\right]$, then from (3.24), (3.23), (3.22) and (3.21), we have $R\cdot W_0^\star=0$. □

4. A $W_0^\star$-Flat Kenmotsu Manifold

In this section, we study a $W_0^\star-$flat Kenmotsu Manifold.

Theorem 4.1. *Let M be (2n+1)-dimensional a $W_0^\star$-Flat Kenmotsu Manifold. Then, M is an $\eta-$ Einstein manifold.*

Proof. A Kenmotsu manifold is said to be a $W_0^\star$-flat if

$$g(W_0^\star(X,Y)Z,W)=0. \tag{4.1}$$

where $X,Y,Z,W\in\chi(M)$.

Let a (2n+1)-dimensional Kenmotsu manifold M be $W_0^\star$-flat. Then using (2.20) in (4.1), we have

$$R(X,Y)Z=-\frac{1}{2n}[S(Y,Z)X-g(X,Z)QY. \tag{4.2}$$

Taking $Z=\xi$ and using (4.2) and (2.9) we get

$$2n\eta(X)Y-4n\eta(Y)X=\eta(X)QY. \tag{4.3}$$

Setting $X=\xi$ and inner product both sides of (4.3) by $V\in\chi(M)$, we arrive

$$S(Y,V)=2ng(Y,V)-4n\eta(Y)\eta(V).$$

Thus, M is an $\eta-$Einstein manifold. Conversely, let M be an $\eta-$Einstein manifold i.e. $S(Y,V)=2ng(Y,V)-4n\eta(Y)\eta(V)$, then from (4.3), (4.2) and (4.1), we have $g(W_0^\star(X,Y)Z,W)=0$. The proof is complete. □

5. A $\phi-W_0^\star$-Flat Kenmotsu Manifolds

Theorem 5.1. *Let M be (2n+1)-dimensional a $\phi-$ $W_0^\star-flat$ Kenmotsu manifold. Then, M is an $\eta-$ Einstein manifold.*

Proof. A Kenmotsu manifold is said to be a $\phi-$ $W_0^\star-flat$ if

$$g(W_0^\star(\phi X,\phi Y)\phi Z,\phi W)=0, \tag{5.1}$$

where $X,Y,Z,W\in\chi(M)$. So by the use of (2.20) $\phi-W_0^\star-flat$ means

$$\begin{aligned} R(\phi X,\phi Y,\phi Z,\phi W) &= -\frac{1}{2n}[S(\phi Y,\phi Z)g(\phi X,\phi W)\\ &\quad -g(\phi X,\phi Z)S(\phi Y,\phi W)]. \end{aligned} \tag{5.2}$$

Let $\{e_{i,}...,e_{2n},\xi\}$ be a local orthonormal basis of vector fields in M. Putting $X=W=e_i$ in (5.2) and summing up from 1 to $2n$, we have

$$\begin{aligned} \sum_{i=1}^{2n}R(\phi e_i,\phi Y,\phi Z,\phi e_i) &= -\frac{1}{2n}\sum_{i=1}^{2n}[S(\phi Y,\phi Z)g(\phi e_i,\phi e_i)\\ &\quad -g(\phi e_i,\phi Z)S(\phi Y,\phi e_i)]. \end{aligned} \tag{5.3}$$

It can be easily verify that

$$\sum_{i=1}^{2n}g(R(\phi e_i,\phi Y)\phi Z,\phi e_i)=S(\phi Y,\phi Z)+g(\phi Y,\phi Z), \tag{5.4}$$

$$\sum_{i=1}^{2n} S(\phi e_i, \phi e_i) = r + 2n, \tag{5.5}$$

$$\sum_{i=1}^{2n} S(\phi Y, \phi e_i) g(\phi e_i, \phi Z) = S(\phi Y, \phi Z), \tag{5.6}$$

$$\sum_{i=1}^{2n} g(\phi e_i, \phi e_i) = 2n \tag{5.7}$$

and

$$\sum_{i=1}^{2n} g(\phi Y, \phi e_i) g(\phi e_i, \phi Z) = g(\phi Y, \phi Z). \tag{5.8}$$

So by virtue of (5.4)-(5.8) the equation (5.3) can be written as

$$S(\phi Y, \phi Z) = \frac{2n}{4n-1} g(\phi Y, \phi Z). \tag{5.9}$$

Then by making use of (2.1) and (2.11), the equation (5.9) takes the form

$$S(Y, Z) = \frac{2n}{1-4n} g(Y, Z) - \frac{8n^2}{4n-1} \eta(Y)\eta(Z),$$

which implies from M is an $\eta-$Einstein manifold. Conversely, let M be an $\eta-$Einstein manifold i.e. $S(Y,Z) = \frac{2n}{1-4n} g(Y,Z) - \frac{8n^2}{4n-1} \eta(Y)\eta(Z)$, then from (5.9), (5.8),(5.7), (5.6), (5.5), (5.4), (5.3), (5.2) and (5.1), we have
$g(W_0^\star(\phi X, \phi Y)\phi Z, \phi W) = 0$. The proof is complete. □

Conclusion and Recommendations The tensors studied above can also be applied to other Manifolds.

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